Energetics of the Baroclinic Tide from the HRET Model

Edward D. Zaron

College of Earth, Oceanic, and Atmospheric Science
Oregon State University
edward.d.zaron@oregonstate.edu

OSTST 2020 Virtual Meeting
October 18–23, 2020

Zoom Drop-In Session

- I intend to be logged in: 8am to 10am PDT (UTC -07:00) on Wednesday, October 21.
- Equivalent times: 11am to 1pm EDT (Washington, DC), 5pm to 7pm CST (Paris)
- Please drop in if you want to say “hi” or to discuss the slides.
- https://oregonstate.zoom.us/j/93873108823?pwd=QWpSZFZzbXVLUEdwOTFobU9VNFpZUT09
- Password: 891682
- Meeting ID: 938 7310 8823

Introduction

Characteristics of the High-Resolution Empirical Tide (HRET) Model

- HRET is a “kinematic wave” model – it does not use wave dynamics per se.
- Mathematically, the baroclinic SSH field is represented as,

$$\eta(x,y) = \sum_{j=1}^{N}\sum_{k=0}^{2}\sum_{l=0}^{2-k}(a_{klj}x^k y^l \cos(\mathbf{k}_j \cdot (x,y)) + i b_{klj} x^k y^l \sin(\mathbf{k}_j \cdot (x,y))). \quad (1)$$

  The number of component waves, N, vector wavenumbers, $\mathbf{k}_j$, and the complex coefficients, $\{a_{klj}, b_{klj}\}$ are obtained by maximizing the goodness-of-fit to harmonic constants determined from along-track altimetry within a prescribed data window, centered on $(x,y) = (0,0)$ in the local coordinate system.
- The above representation corresponds to a sum of waves with phase propagation along directions, $\mathbf{k}_j$, with quadratic modulation of the wave amplitudes.

Characteristics of the High-Resolution Empirical Tide (HRET) Model

- Unlike most other altimeter-derived models of the baroclinic tides, the wavelength of each component wave, $L_j = 2\pi/|\mathbf{k}_j|$, is inferred from altimetry.

- The quantities $\{a_{klj}, b_{klj}, \mathbf{k}_j, N\}$ are computed on a coarse $1/4^o$ grid from data within overlapping 250 km analysis windows (except for S_2, which uses a larger window).

- The η fields comprising the HRET model are represented on a fine $1/20^o$ grid by smoothly patching the local solutions together.

Relevant Questions

1. Does it make sense to compute velocity, $\mathbf{u}$, from η using wave dynamics? (YES, see Zaron 2019, Baroclinic tidal sea level from exact-repeat mission altimetry. *J. Phys. Oceanogr.*, 49(1):193–210.)
2. Can the HRET solution be represented in terms of baroclinic modes? (YES, partly answered below.)
3. Do the wavelengths estimated from altimetry correspond to the baroclinic modes inferred from theory, based on climatological stratification and depth? (Yes, within a few percent.)
4. Two expressions for the wave energy flux may be used, $c_g\mathbf{E}$ (group speed times wave energy) and $\mathbf{u}p$ (velocity times baroclinic pressure anomaly), do they agree? (Depends on how uniform the wave field is.)
5. What does the wave energy flux look like? Is it consistent with independent information? (See below.)

HRET vs. Theoretical Phase Speed

Comparing observed vs. theoretical wave properties

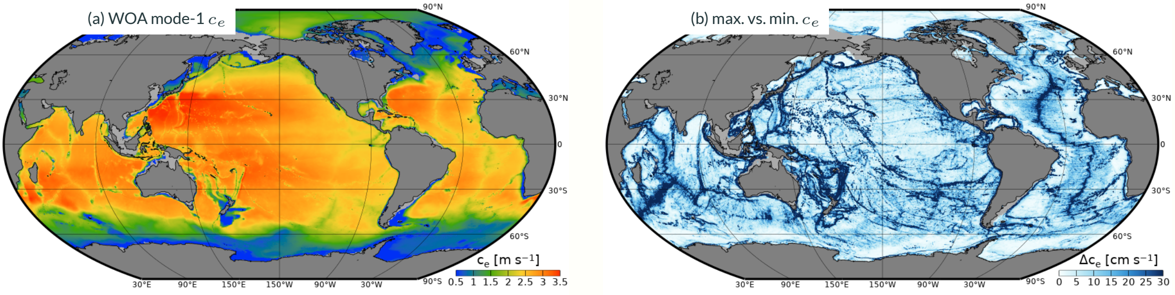
- Theory of small-amplitude inertia gravity waves over a *flat bottom* can be used to predict the wave phase speed $c_p^{(n)} = \omega/|k_n|$.
- The phase speed is related to the eigenspeed, $c_e^{(n)}$,

$$c_p^{(n)} = \frac{\omega}{(\omega^2 - f^2)^{1/2}} c_e^{(n)}. \tag{2}$$

- The eigenspeed is computed from the eigenvalue of a Sturm-Liouville equation involving the stratification, $N^2(z)$, and the water depth, H.

HRET provides estimates of the wavenumbers which are converted to equivalent eigenspeed for comparison with the theory. N^2 is derived from the WOA climatology, and H is derived from GEBCO2020.

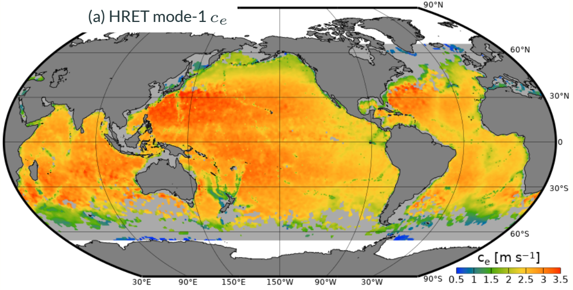
Theoretical eigenspeed from WOA and GEBCO

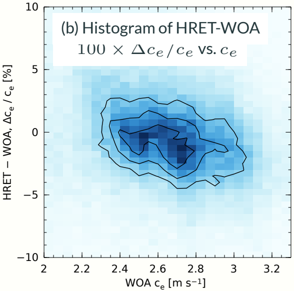


(a) Mode-1 eigenspeed computed using the median depth within $1/4^o$ grid cells. Grey cells indicate water depth less than 500 m. (b) Difference in mode-1 eigenspeed computed using the maximum depth versus the minimum depth within grid cells.

Uncertainty about averaging scale of H leads to non-trivial uncertainty in the theoretical mode speed.

Observed eigenspeed from HRET





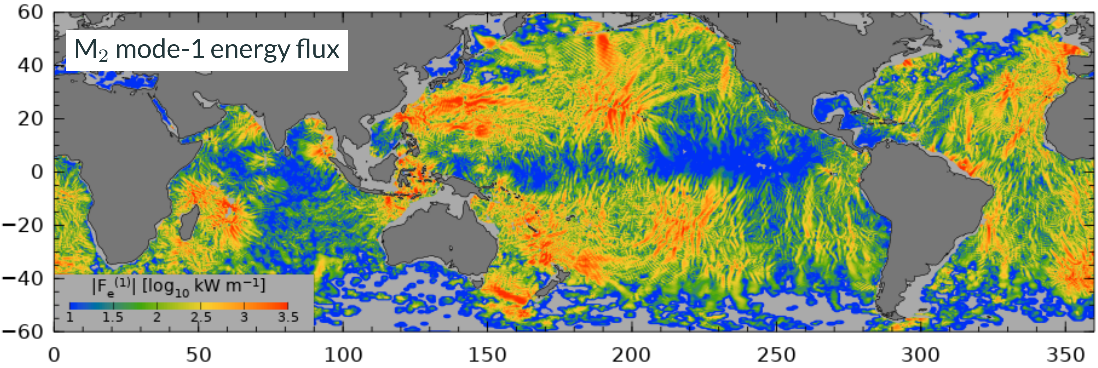
(a) Mode-1 eigenspeed estimated from altimetry, HRET M_2 solution. Grey cells indicate water depth less than 500m or cells where the mode-1 eigenspeed could not be identified from altimetry. (b) Two-dimensional histogram of the fractional difference between HRET and WOA eigenspeeds (y-axis) versus WOA eigenspeed (x-axis). Contours indicate 40%, 60%, and 80% of maximum counts per cell.

Comments on observed vs. theoretical eigenspeed

- Small scale noise in the HRET eigenspeed suggests it might be wise to smooth it before the wave fitting. I'll try this in HRETv9.
- What explains the slight difference between the observed and theoretical eigenspeeds?
    1. Biased estimator in HRET? (Maybe.)
    2. Biased estimate of N^2 due to vertical resolution of WOA? (No. Difference is of opposite sense.)
    3. Climatological N^2 represents a different time period than altimetry? (Maybe. WOA waves are faster; WOA has stronger stratification than inferred from HRET.)
    4. Bottom boundary condition assumed in theory is wrong? (An interesting idea.)
    5. The observed waves "feel" the minimum depth rather than the mean or median depth. (Hmmm.)

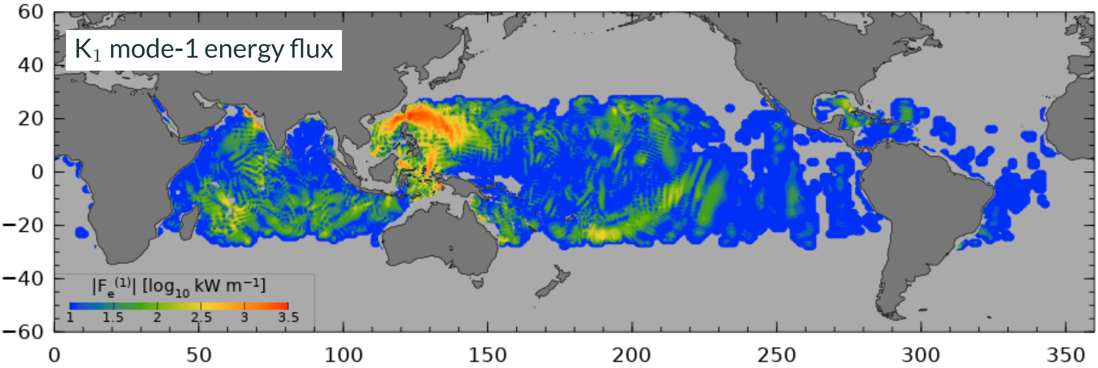
Energy Diagnostics of HRET

Energy Flux: $\mathbf{F}_e = \mathbf{u}p$



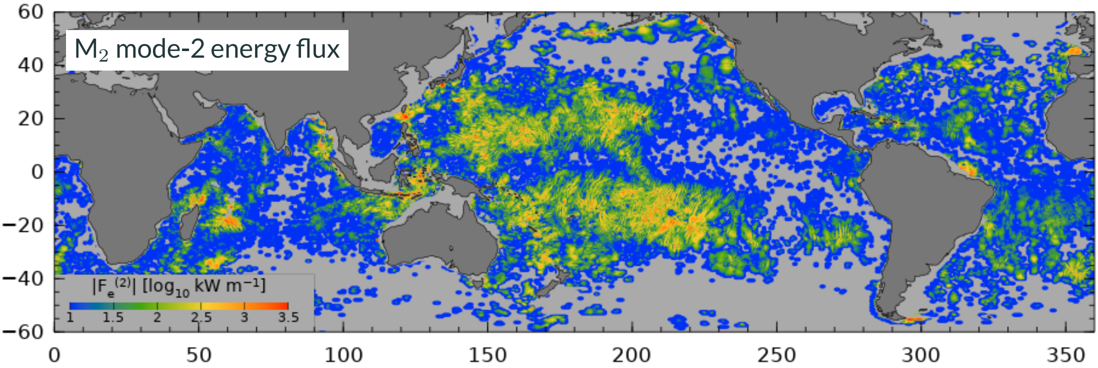
Oceanic areas colored dark gray represent regions where depth is less than 500m or where no mode-1 waves were identified from altimetry.

Energy Flux: $\mathbf{F}_e = \mathbf{u}p$

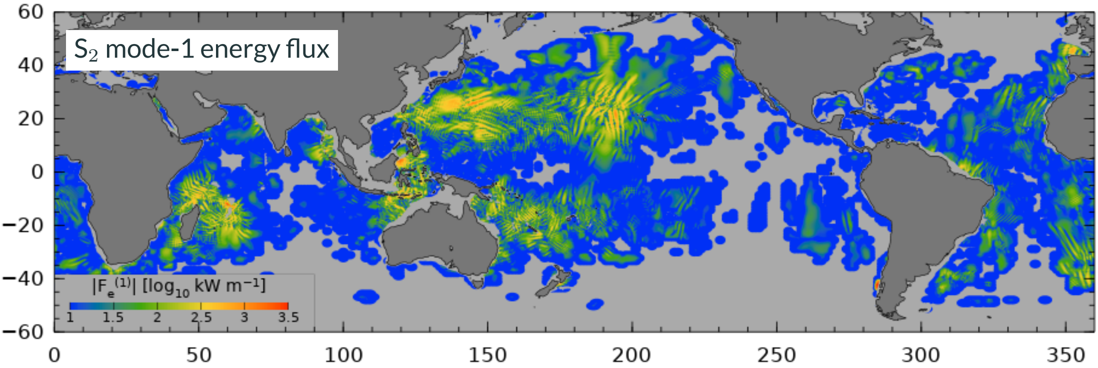


Oceanic areas colored dark gray represent regions where depth is less than 500m or where no mode-1 waves were identified from altimetry.

Energy Flux: $\mathbf{F}_e = \mathbf{u}p$



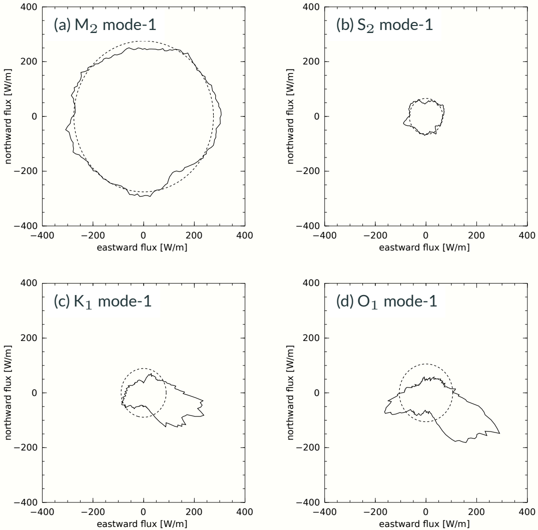
Energy Flux: $\mathbf{F}_e = \mathbf{u}p$



Directional Distribution of Energy Flux

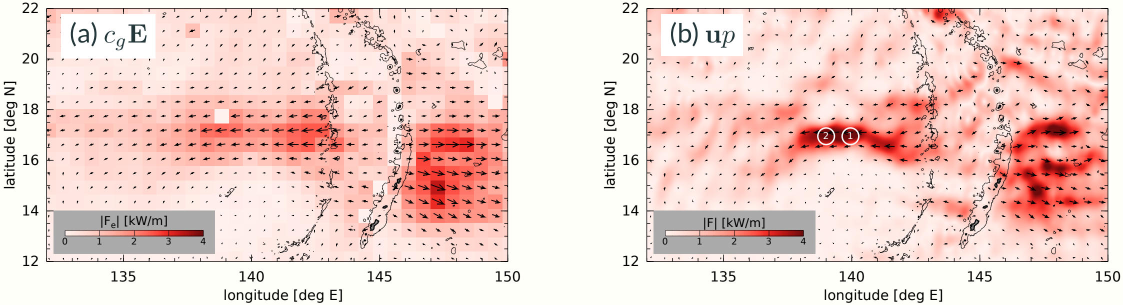
M_2 and S_2 energy flux is nearly isotropic. The mean flux (and energy) in S_2 is smaller than would be expected from the ratio of the M_2 and S_2 forcing.

Energy fluxes of the K_1 and O_1 tides are dominated by the source in Luzon Strait.



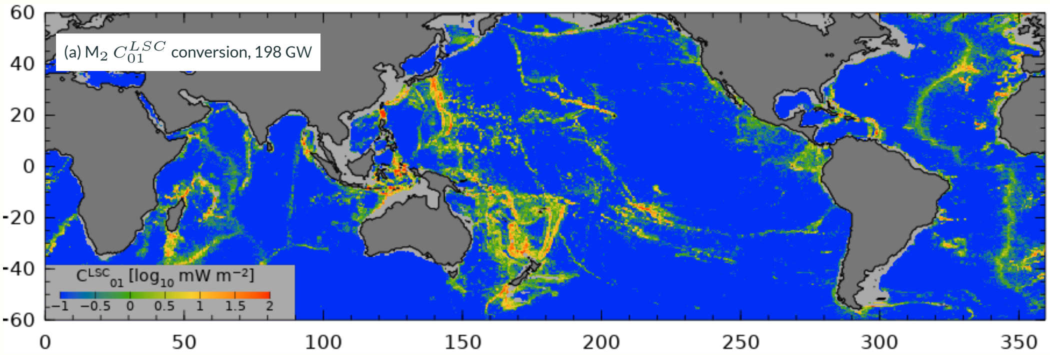
Comparison of $c_g\mathrm{E}$ and $\mathbf{u}p$ Flux Estimates

Wave focus in the Western Pacific: Zhao and D'Asaro (2011) A perfect focus of the internal tide from the Mariana Arc. *Geophys. Res. Lett.*, 38, L14 609.



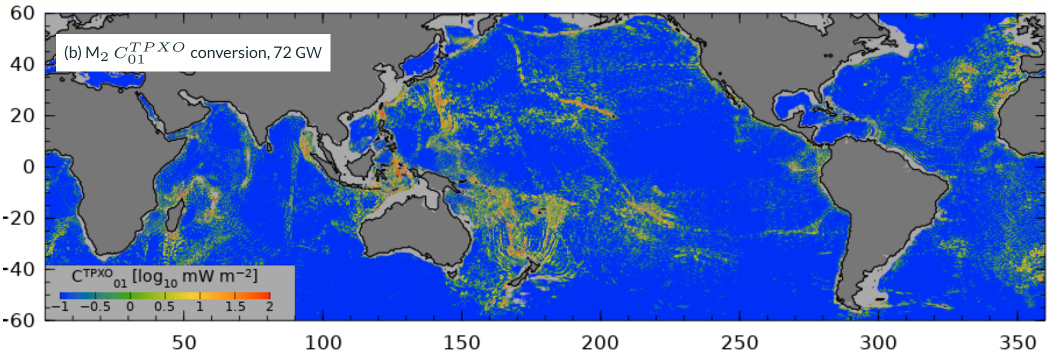
Mode-1 M_2 energy flux near the Mariana Arc. The location of maximum SSH (labelled point 1) occurs upstream of the location of the maximum energy flux (labelled point 2).

Barotropic-to-Baroclinic Conversion



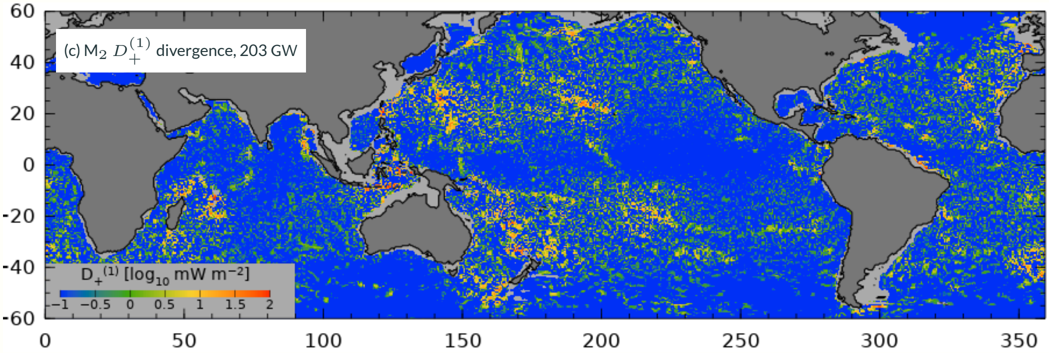
M_2 barotropic-to-baroclinic-mode-1 conversion from theory valid for linear waves at sub-critical topography, based on TPXO7, stratification, and topography (de Lavergne, C., et al, 2019: Toward global maps of internal tide energy sinks. *Ocean Mod.*, 137, 52–75.).

Barotropic-to-Baroclinic Conversion



M_2 barotropic-to-baroclinic-mode-1 conversion evaluated from TPXO9-Atlas and the HRET solution as $\overline{wp}$, the vertical velocity caused by the cross-isobath barotropic flow times the baroclinic pressure anomaly at the bottom.

Barotropic-to-Baroclinic Conversion



M$_2$ mode-1 energy flux divergence, $\nabla \cdot (\mathbf{u}p)$, positive values only. In spite of noise, the area integral is dominated by "hotspots" and agrees well with deLavergne et al.

Tabular Summary

Tide	KE [PJ]	PE [PJ]	E [PJ]	C_{0n}^{LSC} [GW]	$D_{+}^{(n)}$ [GW]	τ [days]
mode $n = 1$:						
M_2	27.3	15.7	43.0	198	203	2.5
S_2	2.1	1.5	3.5	50	16	2.5
K_1	2.7	1.1	3.8	14	14	3.1
O_1	1.9	0.6	2.6	-	13	2.3
mode $n = 2$:						
M_2	8.4	6.8	15.1	152	61	2.9

(keep paging for commentary)

Tabular Summary

Tide	KE [PJ]	PE [PJ]	E [PJ]	C_{0n}^{LSC} [GW]	$D_+^{(n)}$ [GW]	τ [days]
mode $n = 1$:						
M_2	27.3	15.7	43.0	198	203	2.5
S_2	2.1	1.5	3.5	50	16	2.5
K_1	2.7	1.1	3.8	14	14	3.1
O_1	1.9	0.6	2.6	-	13	2.3
mode $n = 2$:						
M_2	8.4	6.8	15.1	152	61	2.9

The energy estimates highlight the fact that the M_2 waves dominate the energy in the baroclinic tidal fields.

We would expect S_2 to contain about 10 PJ of energy. HRET is coming up short since fewer altimeters can measure S_2, compared to M_2.

Tabular Summary

Tide	KE [PJ]	PE [PJ]	E [PJ]	C_{0n}^{LSC} [GW]	$D_+^{(n)}$ [GW]	τ [days]
mode $n = 1$:						
M_2	27.3	15.7	43.0	198	203	2.5
S_2	2.1	1.5	3.5	50	16	2.5
K_1	2.7	1.1	3.8	14	14	3.1
O_1	1.9	0.6	2.6	-	13	2.3
mode $n = 2$:						
M_2	8.4	6.8	15.1	152	61	2.9

C_{0n}^{LSC} is the integrated conversion from deLavergne et al, and $D_+^{(n)}$ is the integral of positive values of $\nabla \cdot (\mathbf{u}p)$ from HRET.

The agreement of these values for M_2 and K_1 mode-1 is encouraging, but it may be a coincidence since C_{0n}^{LSC} is very sensitive to the minimum depth criterion used in its definition.

Note the values disagree for M_2 mode-2. HRET is likely to be missing considerable signal.

Tabular Summary

Tide	KE [PJ]	PE [PJ]	E [PJ]	C_{0n}^{LSC} [GW]	$D_+^{(n)}$ [GW]	τ [days]
mode $n=1$:						
M_2	27.3	15.7	43.0	198	203	2.5
S_2	2.1	1.5	3.5	50	16	2.5
K_1	2.7	1.1	3.8	14	14	3.1
O_1	1.9	0.6	2.6	-	13	2.3
mode $n=2$:						
M_2	8.4	6.8	15.1	152	61	2.9

$\tau = E/D_+^{(n)}$ is an energy residence time. For M_2 mode-1, τ agrees with a completely independent estimate in Zaron (2019). The fact that S_2 agrees with M_2 is probably a coincidence, since the S_2 energy is too low.

It is hard to assess the significance of τ for the other waves without additional independent data.

Conclusions

Conclusions

1. Energetics of the mode-1 M_2 solution in HRETv8.1 appear to be consistent with independent estimates.
2. Maps of mode-1 K_1 and O_1 energy fluxes appear reasonable, but integrated energy depends a lot on details of η at the edges of the waveguide.
3. The poorer quality of the S_2 solution is exhibited in energy diagnostics. This is due to lack of data from sun-synchronous missions.
4. Higher modes and smaller-scales of the mode-1 solutions at topographic features are probably not well-constrained by altimetry.
5. Next version of HRET should be coming in 2021.

THANK YOU – THE END