Non-stationary Internal Tides Observed Using Dual-Satellite Altimetry

Edward D. Zaron (Portland State University)

Summary

- Dual-satellite crossover data is used to quantify stationary and non-stationary tides from time-lagged mean square sea-surface height (SSH) differences.
- It is necessary to remove independent estimates of the stationary tide and mesoscale SSH variance in order to observe the tiny residual variance attributed to the internal tide.
- For the semidiurnal tidal band, the stationary tidal variance (which is comprised of both internal tide and barotropic tide model error) is approximately 0.73cm^2, and the non-stationary variance is about 0.33cm^2, about 1/3 of the total.
- The temporal correlation of the non-stationary tide is examined by complex demodulation and found to be oscillatory with first zero-crossing at 400 hours (17 days).
- Results from 4 years of Jason-2 (J2) and CryoSat-2 (C2) are summarized below for lags from 1 hour to 1440 hours (60 days).

$\widehat{\Gamma}(\tau)$, the mean square time-lagged crossover SSH difference

A quantity analogous to the variogram is computed using time-lagged measurements of SSH at orbit crossover points,

$$\widehat{\Gamma}(\tau_k) = N_k^{-1} \sum_{ij} (h^{(J2)}(t_i) - h^{(C2)}(t_j))^2 I_{ijk} \tag{1}$$

- $h^{(J2)}$ and $h^{(C2)}$ are sea-surface heights for the Jason-2 and CryoSat-2 missions, respectively;
- Orbit, MSS, and path-delay corrections are appied according to the GDR-D standards;
- The indicator function, I_{ijk}, is non-zero only at orbit crossover points and when $\tau_k - \Delta t < |t_i - t_j| < \tau_k + \Delta t$, where $2\Delta t = 1$h bins are used;
- The normalization $N_k = \sum I_{ijk}$ is the count of data within each lag bin, it is approximately 5000;
- Data are used from the latitude range $\pm 50°$, water depth greater than 3000m, distance from the coast greater than 60km, and dates within 2010-01-01 to 2014-05-30;
- The quantity $\widehat{\Gamma}$ may be related to the single-point two-time covariance function of SSH, here it is referred to as the SSH *structure function*.

Reducing sample variance with a mesoscale correction

The sample variance of $\widehat{\Gamma}(\tau_k)$ is related to $\mu(\tau_k)$, the 4th moment of the SSH increments,

$$\mu(\tau_k) = N_k^{-1} \sum_{ij} (h^{(J2)}(t_i) - h^{(C2)}(t_j))^4 I_{ijk}. \tag{2}$$

In terms of μ and $\widehat{\Gamma}$, the sample variance is

$$\sigma^2_{\widehat{\Gamma}}(\tau_k) = N_k^{-1} \left(\mu(\tau_k) - \frac{N_k - 3}{N_k - 1} \widehat{\Gamma}^2(\tau_k) \right), \tag{3}$$

- To observe the influence of internal tides on the structure function it is necessary to reduce the sample variance of $\widehat{\Gamma}$ by removing as many sources of SSH variance as possible prior to computing the structure function.
- Stationary tides (using GOT4.10) and an estimate of the mesoscale are subtracted from the SSH prior to computation of $\widehat{\Gamma}$.
- The mesoscale correction is obtained from the delayed time multi-satellite gridded sea level anomaly maps provided by Ssalto/Duacs and distributed by AVISO.
- The mesoscale-corrected structure function is denoted $\Gamma(\tau)$, below.

A model for $\Gamma(\tau)$

The SSH structure function for a signal of the form,

$$h(t) = A(1 + \alpha(t))\cos(\omega t + \phi(t)), \tag{4}$$

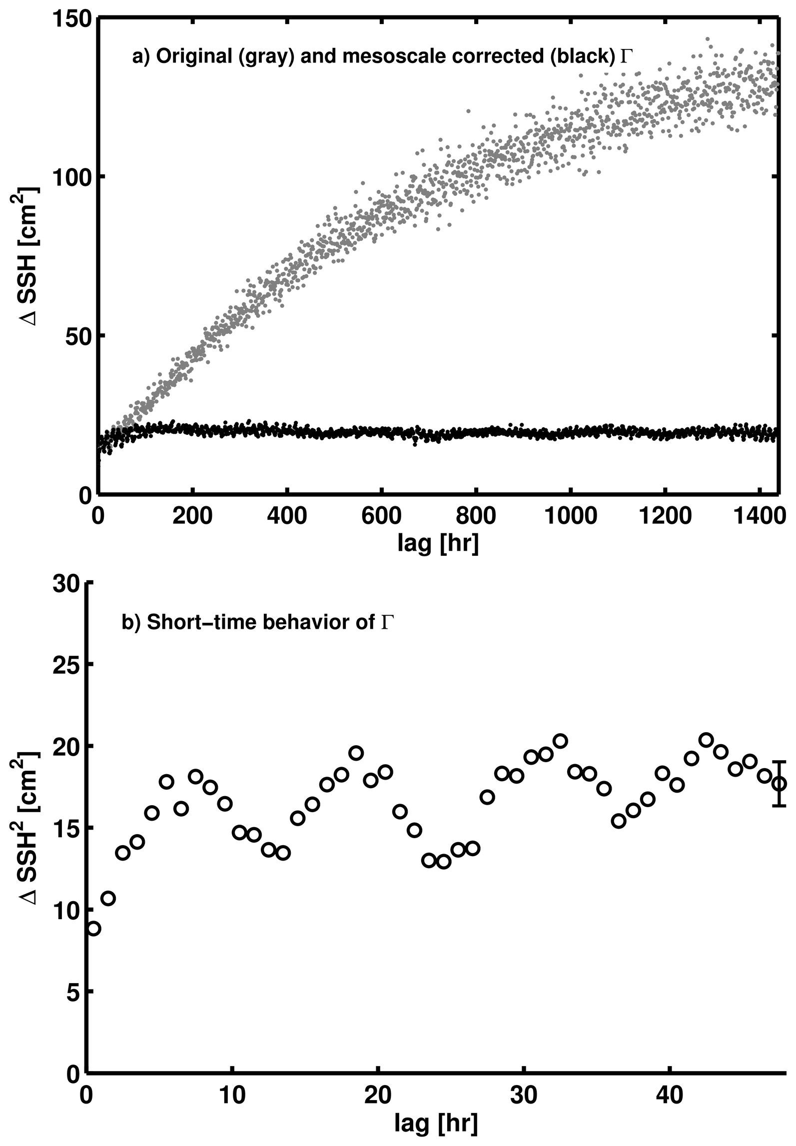
is given by $\gamma(\tau) = \langle (h(t+\tau) - h(t))^2 \rangle$, where

$$\begin{aligned} \gamma(\tau)/A^2 = &\ (1 - \cos(\omega\tau))(1 + \frac{3\sigma_\phi^4}{4}) \\ &+ (\sigma_\phi^2 - C_{\phi\phi}(\tau))\cos(\omega\tau) \\ &+ (\sigma_\alpha^2 - C_{\alpha\alpha}(\tau)\cos(\omega\tau)) \\ &- \frac{1}{2}(\sigma_\alpha^2 + \frac{\sigma_\phi^2}{2} + \frac{3\sigma_\phi^4}{4})\sin^2(\omega\tau), \end{aligned} \tag{5}$$

- $\alpha(t)$ and $\phi(t)$ are (non-dimensional) amplitude and phase modulations, respectively, which are assumed small;
- $C_{\alpha\alpha}(\tau)$ and $C_{\phi\phi}(\tau)$ are the time-lagged auto-covariance functions of $\alpha(t)$ and $\phi(t)$;
- The variances of the modulations are given by $\sigma_\alpha^2 = C_{\alpha\alpha}(0)$ and $\sigma_\phi^2 = C_{\phi\phi}(0)$.
- $G(\tau)$ is modeled as a sum of $\gamma_i(\tau)$ sub-models for 7 major tides, plus terms for the measurement error, a trend, and rapid and slow AR(1) processes.

Result 1: The mesoscale correction is highly effective a explaining variance for lags greater than 150hr.

$\Gamma(\tau)$ is the mesoscale-corrected structure function computed by subtracting the Ssalto/Duacs delayed time multi-satellite gridded SSH maps from $h^{(J2)}$ and $h^{(C2)}$.



$\Gamma_m(\tau)$ regression coefficients.

Coefficient	Process	Value [cm^2]
σ_0^2	inst. noise	5.2
α_0^2	trend (week^{-1})	-0.1
σ_1^2	$\lambda_1 = 3$h AR(1)	2.4
σ_2^2	$\lambda_2 = 40$h AR(1)	4.4
A_1^2	M_2 tide (12.42h)	0.26
A_2^2	S_2 tide (12.00h)	0.10
A_3^2	N_2 tide (12.66h)	0.07
A_4^2	μ_2 tide (12.87h)	0.17
A_5^2	L_2 tide (12.19h)	0.07
A_6^2	K_1 tide (23.93h)	0.14
A_7^2	O_1 tide (25.82h)	0.03
A_8^2	J2 repeat (9.9 days)	0.37

Figure 1: Behavior of $\widehat{\Gamma}(\tau)$ and $\Gamma(\tau)$. a) Removal of the mesoscale signal from the SSH structure function, $\widehat{\Gamma}(\tau)$ (gray), leaves much smaller and less noisy values for subsequent analysis of $\Gamma(\tau)$ (black). b) The behavior of $\Gamma(\tau)$ for $0 \le \tau \le 48$h shows a rapid rise over about 6 hours, followed by oscillations of approximately 12h period. Standard errors are similar at all lags and are shown for $\tau = 48$h.

Result 2: The influence of tides is evident in the mesoscale-corrected structure function, $\Gamma(\tau)$.

- Least-squares regression has been used to estimate the model parameters describing the stationary tides and other processes listed in the table above.
- Tide signals in $\Gamma(\tau)$ consist of internal tides as well as barotropic tide model error.

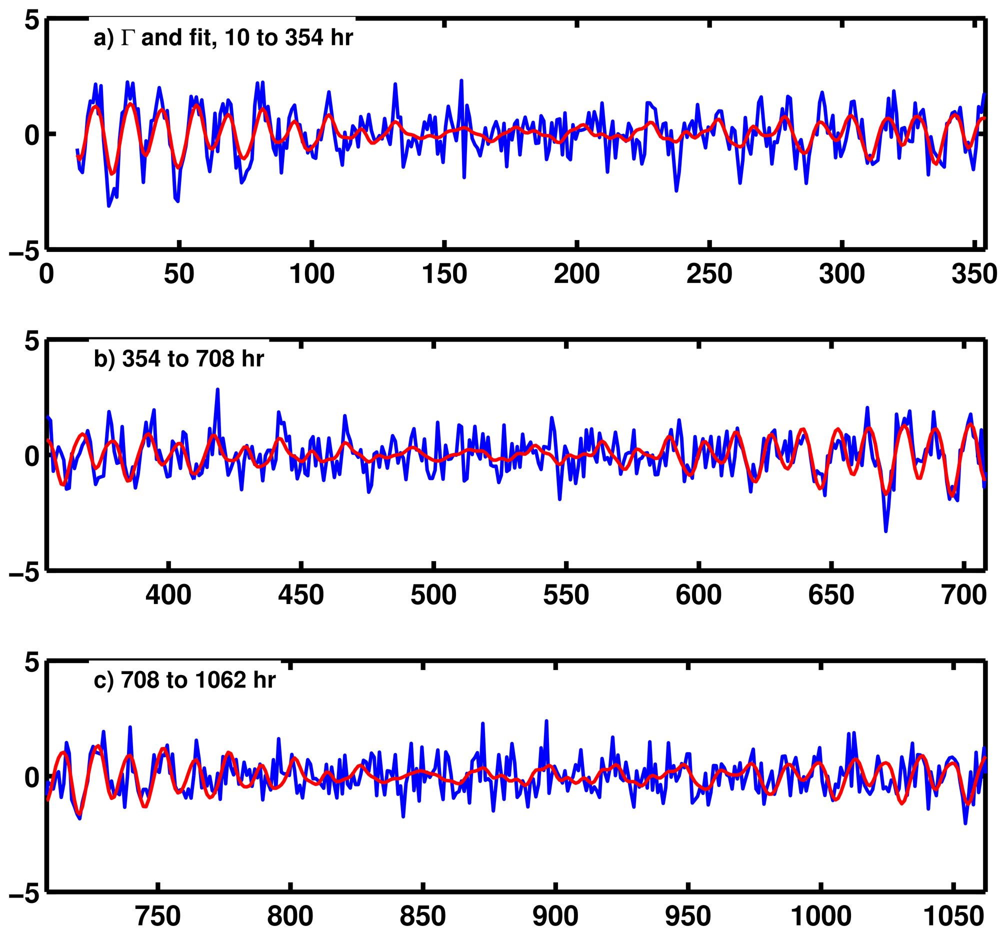


Figure 2: High-passed $\Gamma(\tau)$ (blue) and $\Gamma_0(\tau)$ (red) a model for $\Gamma(\tau)$ which includes only stationary tides. Panels show three different ranges of lag, τ, each a complete spring-neap cycle (354.4h).

Result 3: Stationary tides explain about 70% of the tide-band variance.

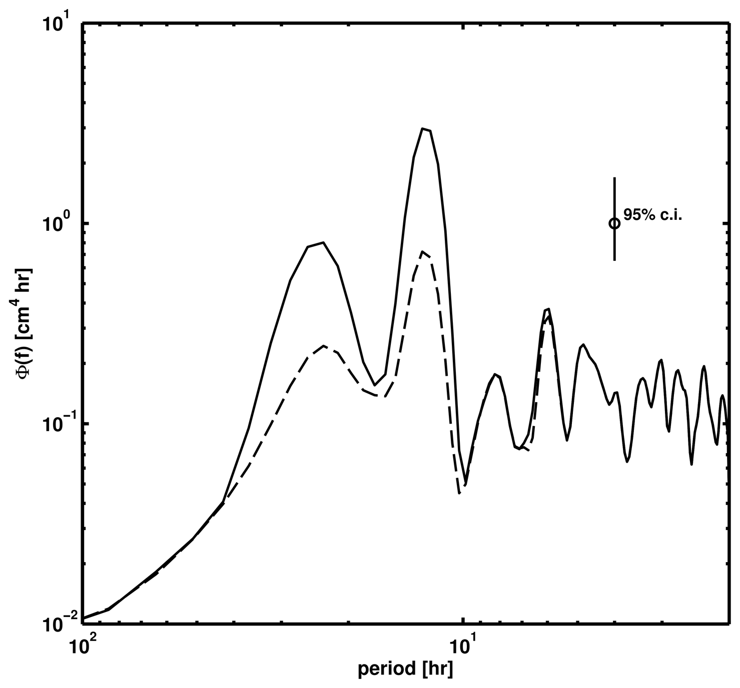


Figure 3: Power spectral density of high-passed $\Gamma(\tau)$ (solid) and nonstationary residual, $\Gamma(\tau) - \Gamma_0(\tau)$ (dashed).

Result 4: The decorrelation time of the non-stationary tide is about 17 days.

Complex demodulation of the residual, $\Gamma(\tau) - \Gamma_0(\tau)$, is used to identify the time-lagged covariance of semidiurnal band SSH from the coefficient of $\cos(\omega\tau)$, $C_{\alpha\alpha}(\tau) + C_{\phi\phi}(\tau)$ (eqn. 5). Note that it is not possible to distinguish phase and amplitude modulations at this level of analysis.

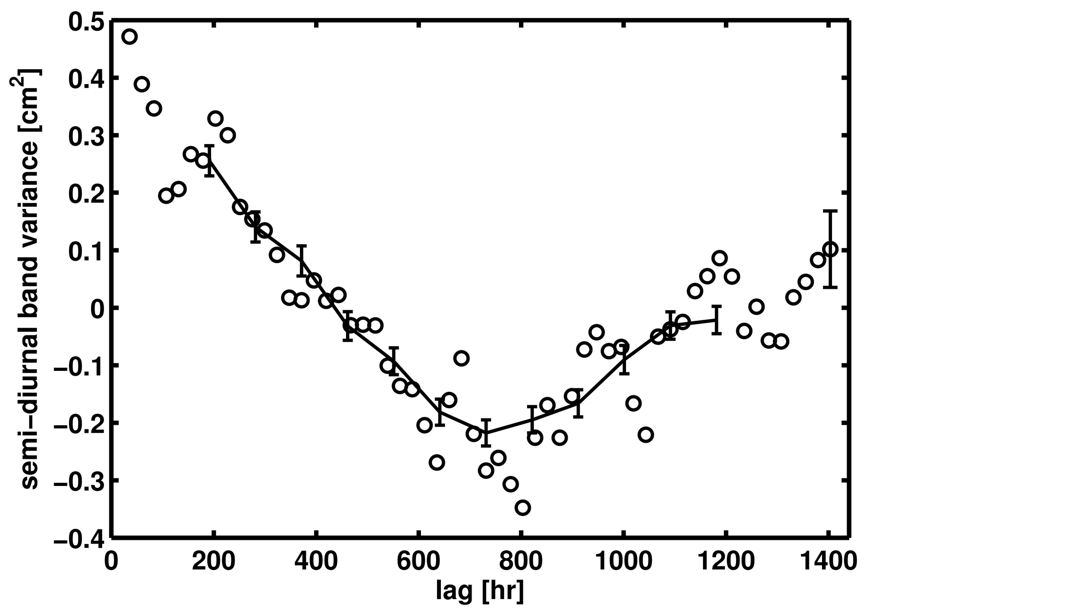


Figure 4: Complex demodulation of semidiurnal band variance. Complex demodulation of the high-passed semidiurnal band residual, $\Gamma(\tau) - \Gamma_0(\tau)$, at the M_2 and K_1 frequencies is performed within 48-hour windows (circles) and within 360-hour windows (solid line). Due to overlap of data windows, every 5th demodulate is statistically independent.

Further work in progress

- Analysis of almost 7 years of Jason-1 and Envisat, May 2002 to Jan 2009, for lags from 1 hour to 1 year is being conducted.
- Although there are almost 25000 cross-overs per 1 hr bin in this larger data set, combined spatio-temporal aliasing is problematic.
- The goal is to develop a spatially resolved map of non-stationary internal tides.

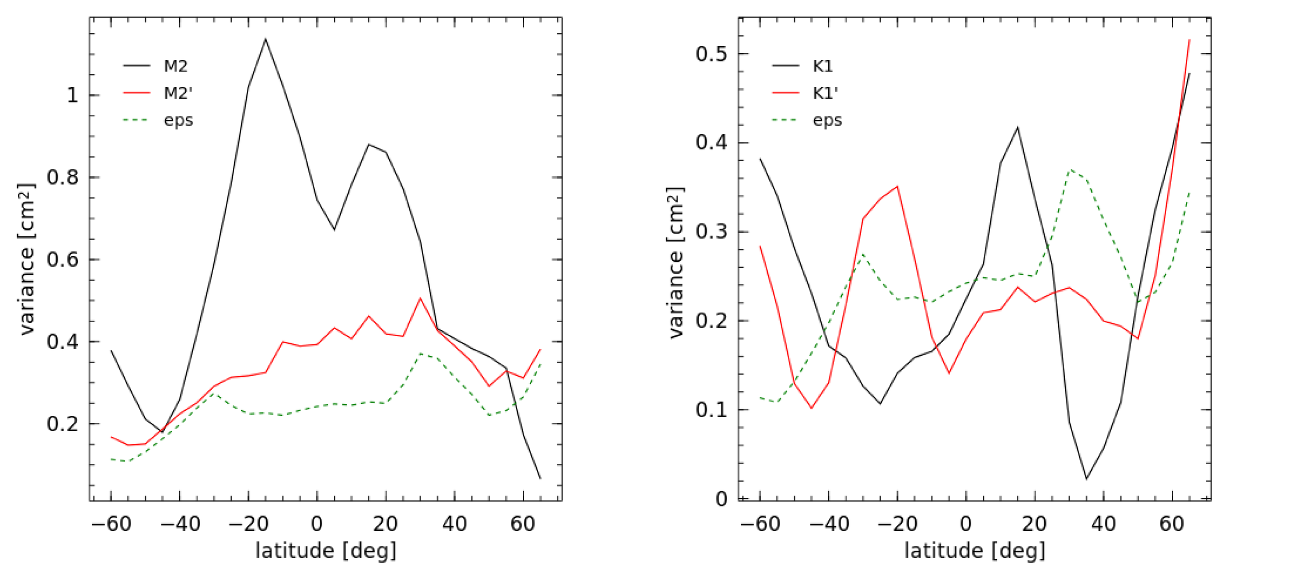


Figure 5: Analysis of $\Gamma(\tau)$ for semi-diurnal band (left) and diurnal band (right) SSH variance from Jason-1 and Envisat data within latitude bands, in water depth greater than 2000m and more than 60km from the coast. Solid black line is stationary tidal variance obtained by model fit to $\Gamma(\tau)$, red line is non-stationary tidal variance obtained from tide-band estimate of $\Gamma(\tau) - \Gamma_0(\tau)$ variance, and the dashed green line is an error estimate for the non-stationary variance.

Conclusions

- Dual satellite crossover data have been used to compute the second-order temporal structure function of SSH, $\Gamma(\tau)$, in the deep ocean for lags from 1 to 60 days.
- The Ssalto/Duacs multi-satellite mesoscale estimate resolves mesoscale signals with time scales longer than about 5 days, and provides a very effective "mesoscale correction" for studying tides.
- About 70% of the tidal variance in the $\Gamma(\tau)$ function is stationary (phase-locked with the astronomical tidal forcing).
- The decorrelation time of the non-stationary semidiurnal tides is about 17 days.
- Because the decorrelation time of the tide (17 days) is longer than the shortest temporal scale of signals in the Ssalto/Duacs mesoscale estimate (5 days), the latter may provide the basis for a non-stationary tidal correction for the SWOT mission.
- Ongoing analysis of a larger Jason-1/Envisat data set is aimed at identifying geographic variability of non-stationary internal tides.

Reference

E. D. Zaron, 2015: Non-stationary Internal Tides Observed Using Dual-Satellite Altimetry. *J. Phys. Oceanogr.*, 45, 2239–2246. `http://dx.doi.org/10.1175/JPO-D-15-0020.1`

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Edward D. Zaron
Department of Civil and Environmental Engineering
Portland State University
ezaron@pdx.edu