Handling Minor Tidal Constituents:
To Infer or Not

Richard Ray
NASA Goddard Space Flight Center

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Tidal constituents used in our GDRs

GOT4, TPXO8, etc. are distributed with these tides

Q_1 O_1 P_1 K_1

N_2 M_2 S_2 K_2

(+ a few others)

GOT4 height predictions also account for these 16 tides

$2Q_1$ σ_1 ρ_1 M_1 χ_1

π_1 φ_1 θ_1 J_1 OO_1

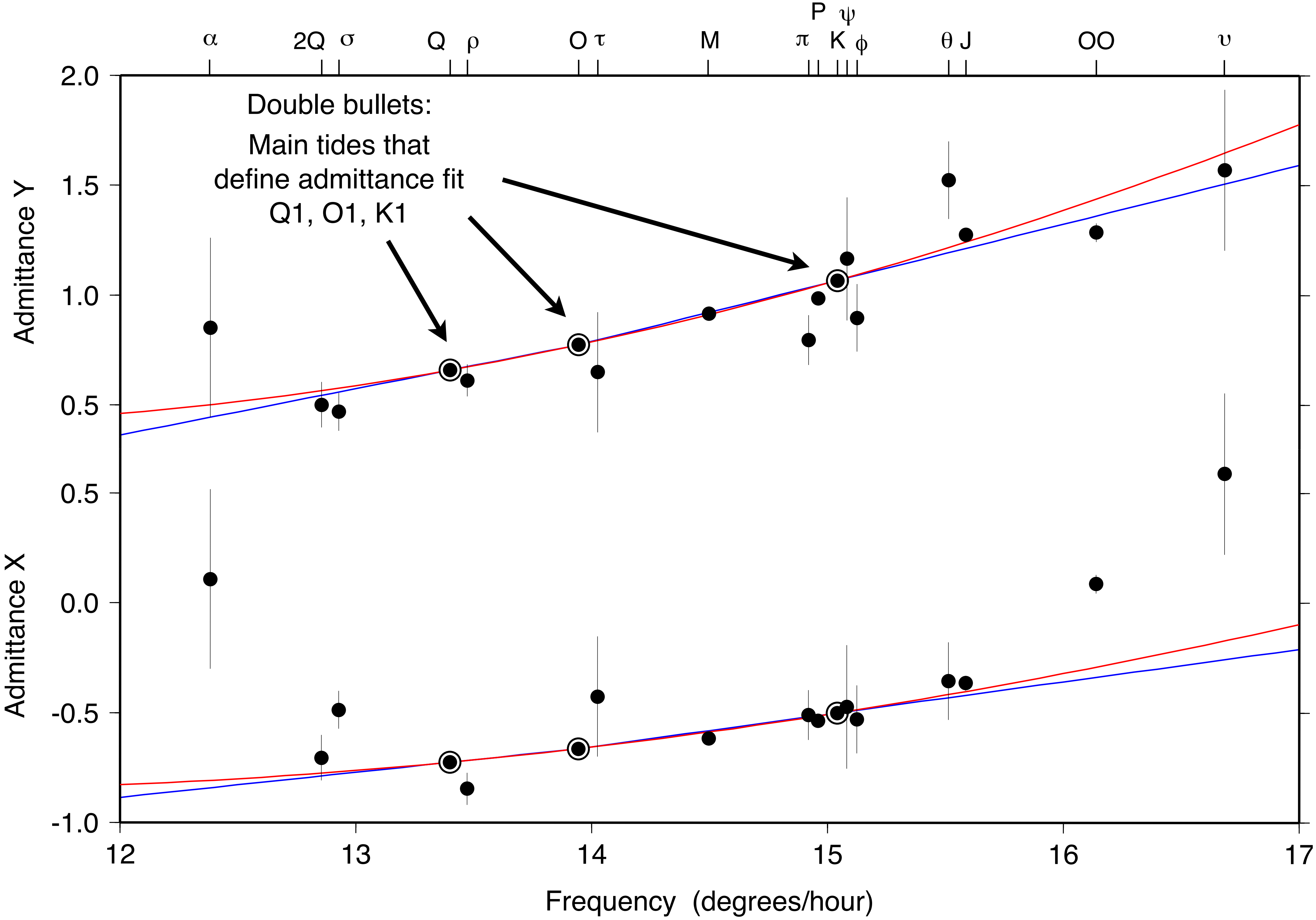
$2N_2$ μ_2 ν_2 λ_2 L_2 T_2

The tide prediction software accounts for minor tides "on the fly" by inferring their constant from major tides.

This exploits the (usual) smoothness of admittances in deep water.

$$\text{Admittance} = \frac{\text{Observed tide}}{\text{Amplitude in tidal potential}}$$

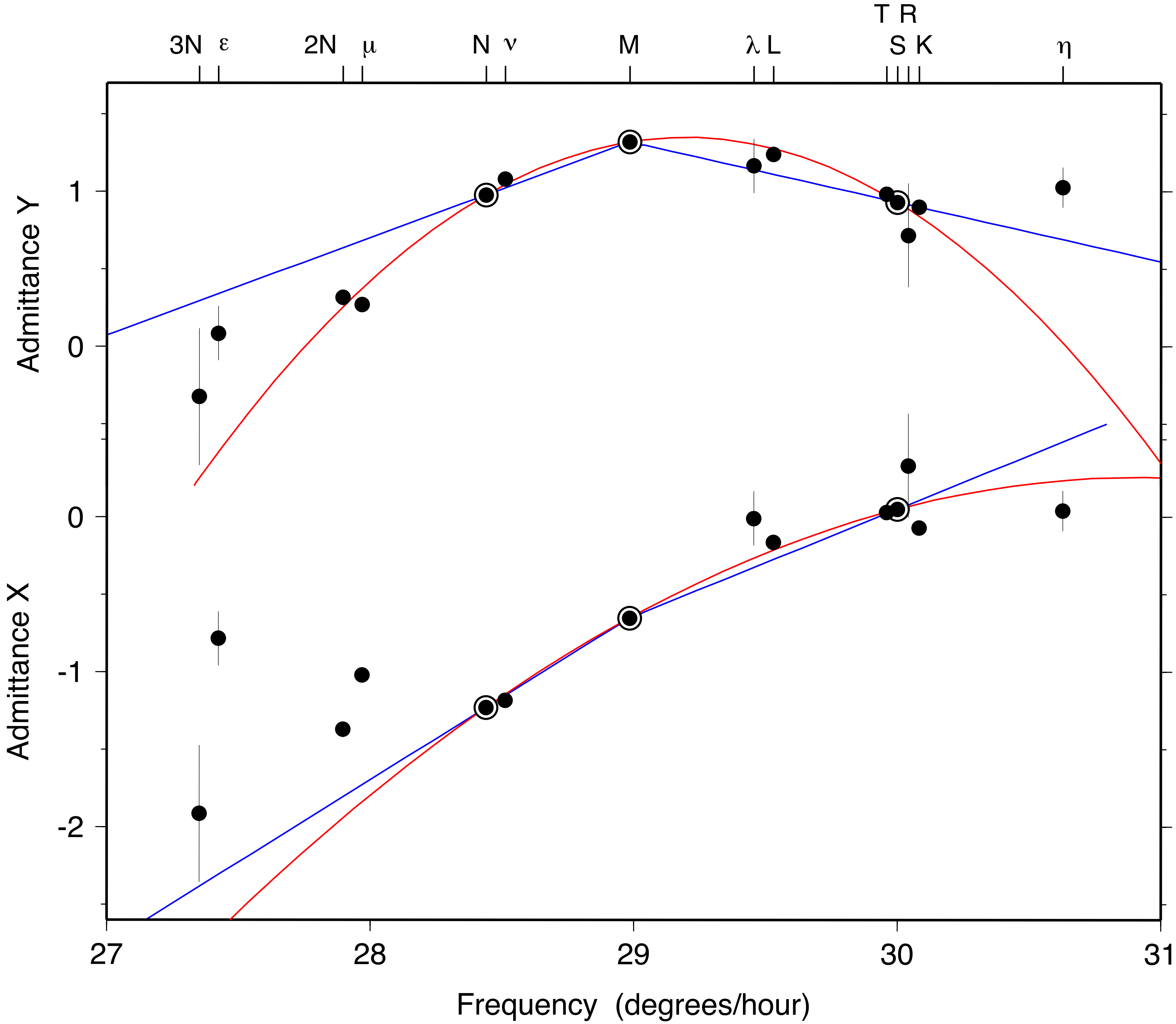
Diurnal Admittances for DART Station 46419



Blue line: linear fit to admittances (used by GOT, TPXO)

Red line: Fourier series fit to admittances (used by Munk-Cartwright, orthotides)

Semi-diurnal Admittances for DART Station 46419



Double bullets:

Main tides that define admittance fit

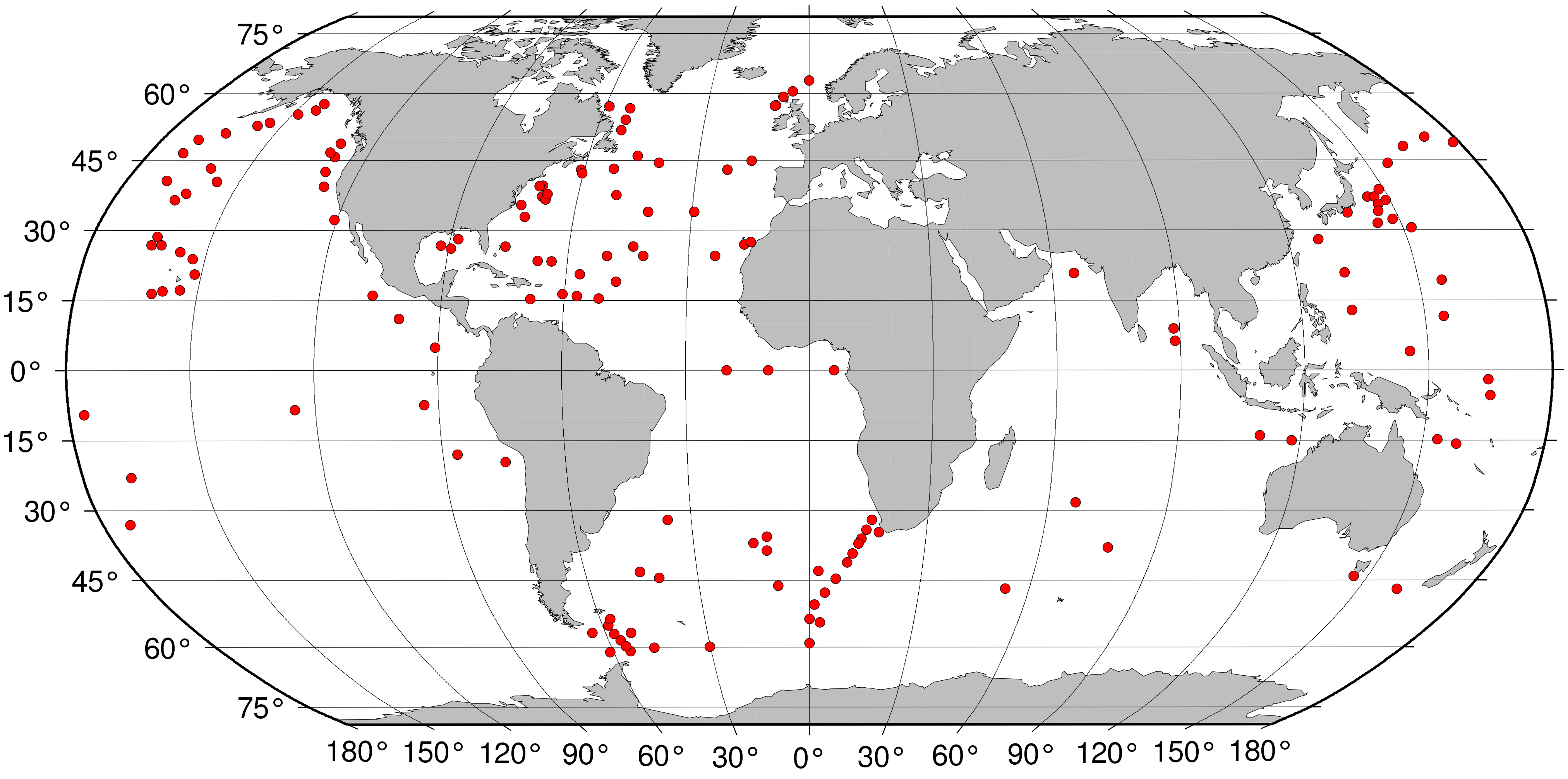
N2 M2 S2

Blue line: linear fit to admittances

Red line: Fourier series fit to admittances

Approach:

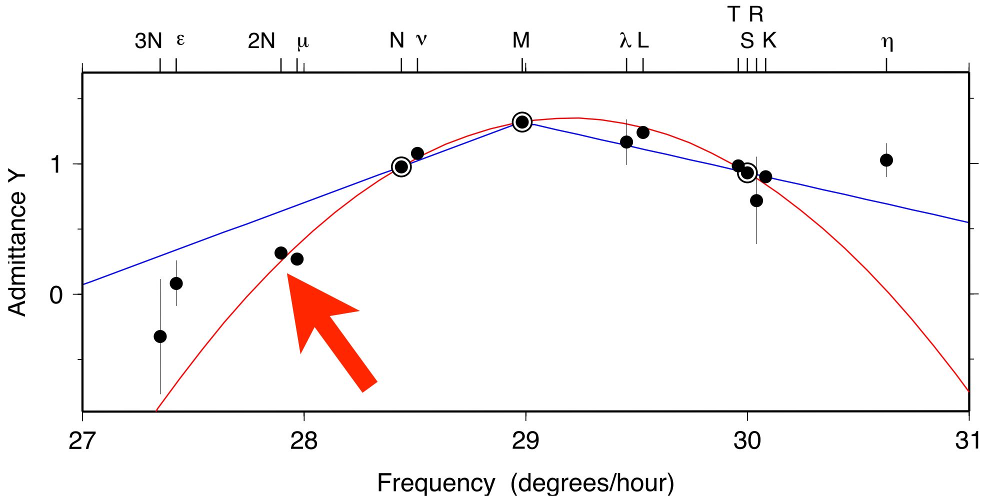
Use 151 bottom-pressure “ground truth” stations to assess how well admittance interpolation works.



R. Ray, “Precise comparisons of bottom-pressure and altimetric ocean tides,”
JGR: Oceans, **118**, 4570–4584.

Warning: Take care because FES2014 assimilated some of these data!

Tests of $2N_2$



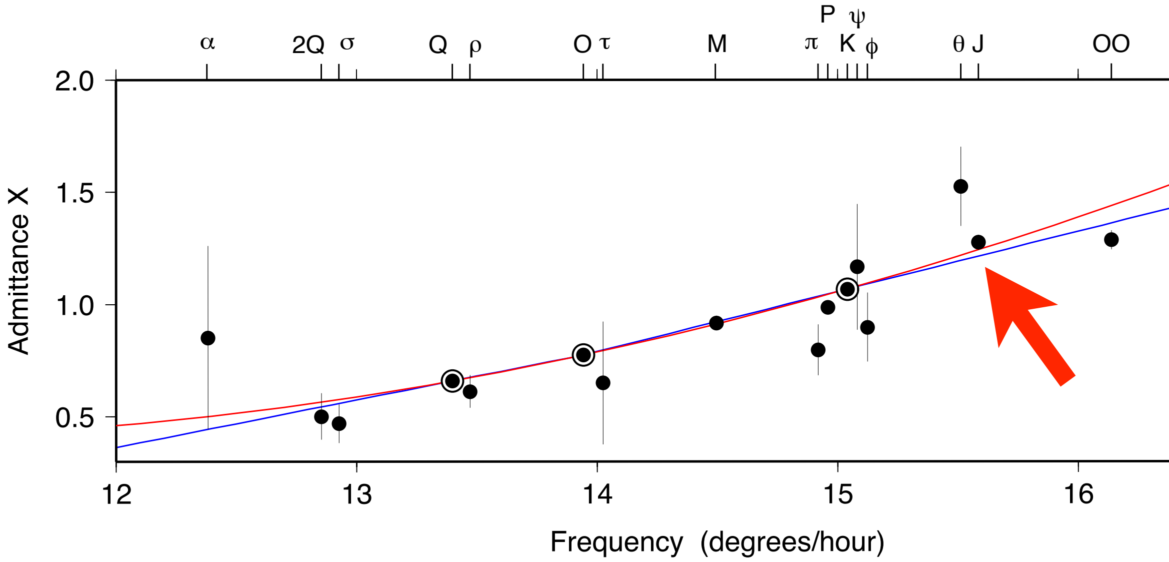
RMS Difference (mm) with BPR Tides

	FES04	FES14	EOT11a
Direct	1.22	0.83	1.15
Inferred (linear)	2.54	2.70	2.71
Inferred (fourier)	2.81	3.08	3.10

Bootstrap standard error on RMS is ~0.15 cm; RMS signal is 8.0 mm.

GOT, TPXO, & others should attempt direct solutions for 2N2.

Tests of J_1



RMS Difference (mm) with BPR Tides

	FES12	FES14
Direct	1.31	4.50
Inferred (linear)	1.77	1.71
Inferred (fourier)	1.86	1.75

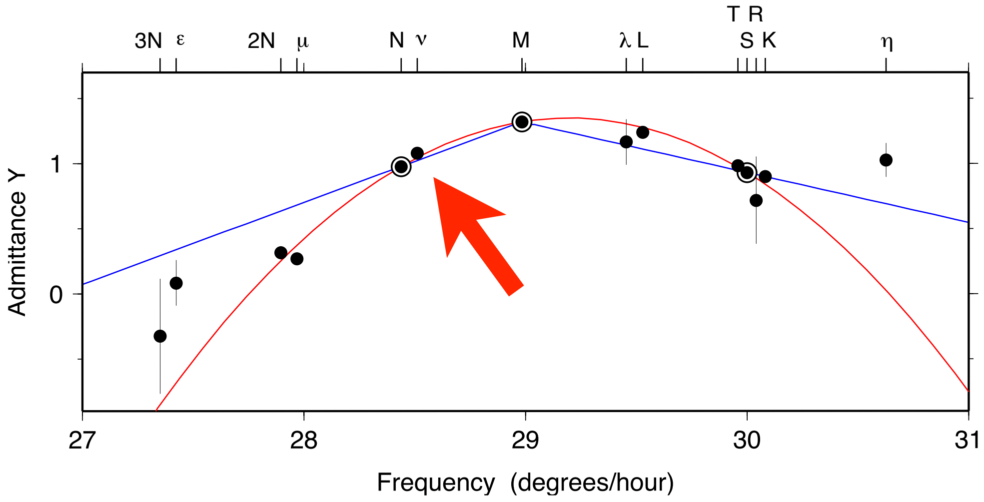
FES14 is pure hydrodynamic

Bootstrap standard error on RMS is ~0.10 cm; RMS signal is 7.2 mm.

GOT, TPXO, & others should attempt direct solutions for J1.

FES14 should use data assimilation for J1.

Tests of ν_2



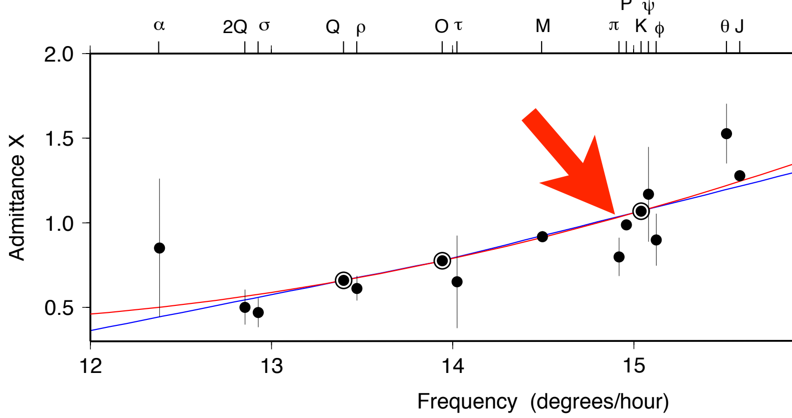
RMS Difference (mm) with BPR Tides

	FES12	FES14
Direct	1.29	0.73
Inferred (linear)	0.78	0.70

Bootstrap standard error on RMS is ~0.08 cm; RMS signal is 11.3 mm.

Inferred nu2 is more accurate than directly estimated nu2 (because inference is based on nearby, accurate N2.

Tests of P_1



RMS Difference (mm) with BPR Tides

	GOT99.2	GOT4.10	FES04	FES12	HAM12	TPXO6.2	TPXO8
Direct	2.54	–	3.25	3.55	1.99	1.80	1.80
Inferred, w/o FCN	2.31	2.45	2.50	2.49	2.23	2.15	2.19
Inferred, w/ FCN	1.99	1.91	2.24	2.10	1.91	1.76	2.06

Bootstrap standard error on RMS is ~0.10 mm; RMS signal is 39.9 mm.

All models except TPXO8 are better when P1 is inferred.

(although HAM12, TPXO6 not significant).

FES14 not shown because it assimilated test data (rms = 1.39 cm).

CONCLUSIONS

- For small tides near larger, well-determined tides (e.g., nu2), inference may be more accurate than direct estimation.
- For “large” tides on the edges of tidal bands (2N2, J1), direct solutions can be more accurate than inferred (extrapolated) solutions.
- For the large P1 tide, inference is more accurate than direct estimation for all models except TPXO8, if FCN accounted for.
- What about “large” tides in middle of band (L2, M1) ?

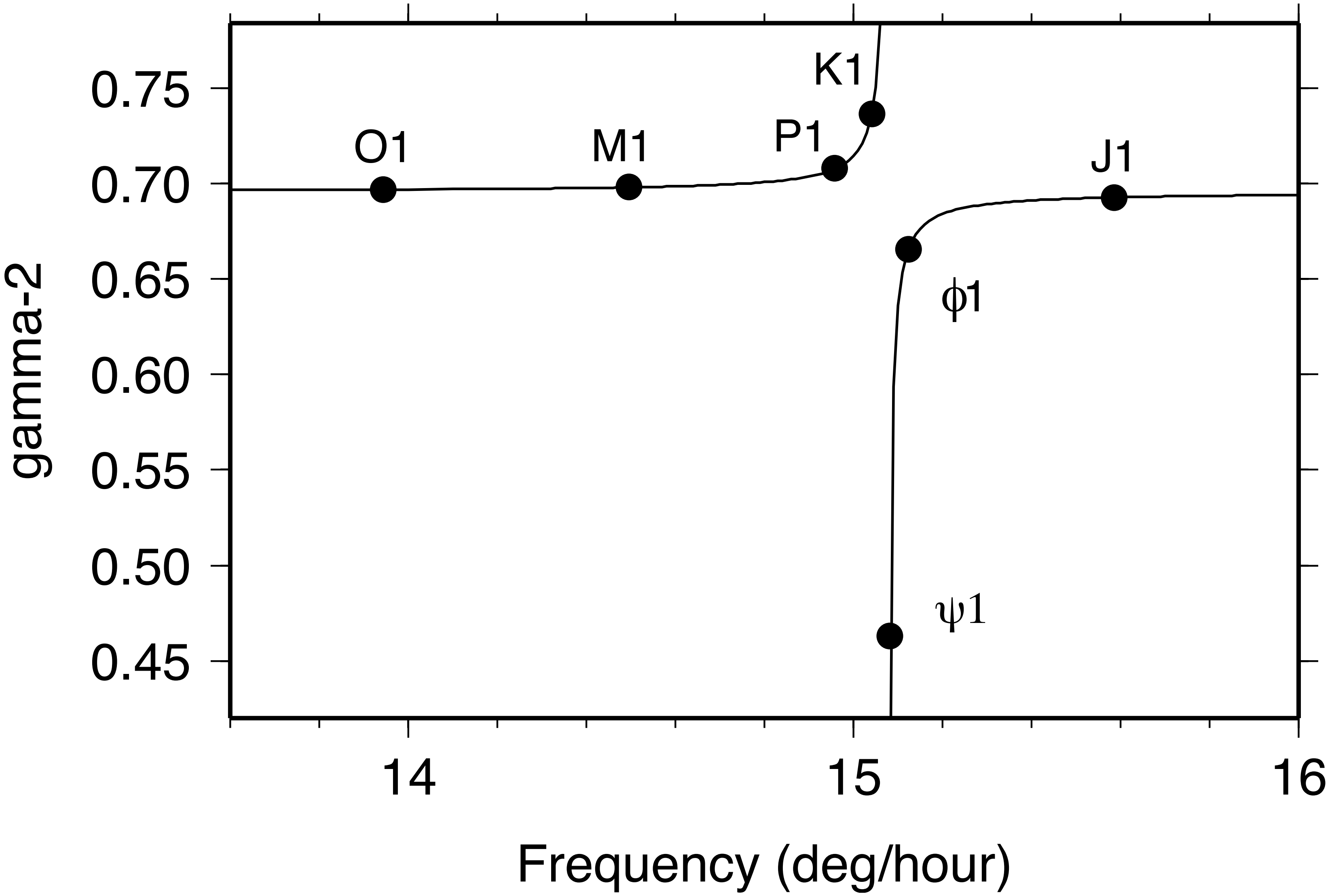
Next GOT, TPXO models need to estimate 2N2, J1. Maybe OO1?

Next FES model needs data constraints on J1.

Next FES model might rely on inference for nu2. Maybe mu2? L2?

For Free Core Nutation business, see:
Ray, “On tidal inference in the diurnal band,” *JTech*, under review.

Free Core Nutation Resonance in Love Numbers



In the dynamical equations of motion, the tidal potential is scaled by the Love number γ_2, which has a resonance between K1 and psi1.

This perturbs oceanic tides, as first predicted by Wahr & Sasao (1981).

How to Infer P1 from K1?

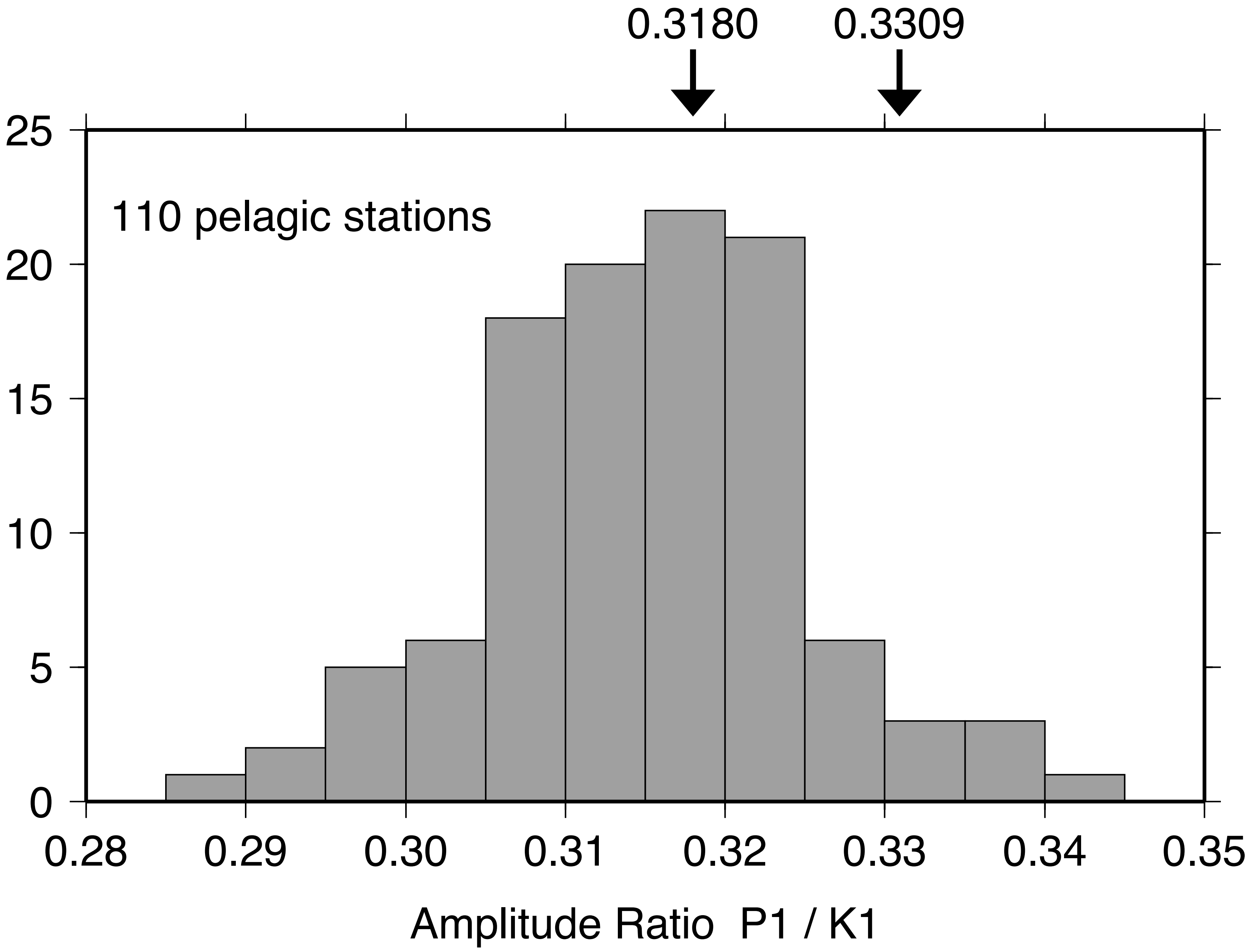
$$\text{P1 / K1 amplitude ratio} = \frac{\text{P1 tidal potential}}{\text{K1 tidal potential}} = 0.3309$$

All tide textbooks say to use this to infer P1 from K1

$$\text{P1 / K1 amplitude ratio} = \frac{\text{P1 tidal potential}}{\text{K1 tidal potential}} \times \frac{\gamma(\text{P1})}{\gamma(\text{K1})} = 0.3180$$

Accounting for FCN resonance suggests this.

“An Improbable Observation of the Diurnal Core Resonance”*



For details: Ray, “On tidal inference in the diurnal band,” *JTech*, under review.

*** in the words of Duncan Agnew, who just reported the same effect in the old IHO tide gauges.**