

I. OVERVIEW

It is widely acknowledged that the Black Sea level budget is dominated by freshwater fluxes. However, satellite altimetry observations have shown that the non-seasonal sea level in the Black Sea is coherent with that in the Aegean and Marmara Seas (Fig. 1 and 2a), but lags behind them by 10-40 days at sub-annual periods (Fig. 2c). This leads to a hypothesis that the Black Sea elevation responds to synoptic sea level changes in the Mediterranean (Aegean Sea), which experiences almost basin-uniform fluctuations. The hypothesis is tested with a linear analytical model. We find that the Mediterranean sea level fluctuations are the dominant forcing mechanism on synoptic time scales, while sea level pressure and freshwater fluxes also make a significant contribution.

II. OBSERVATIONS

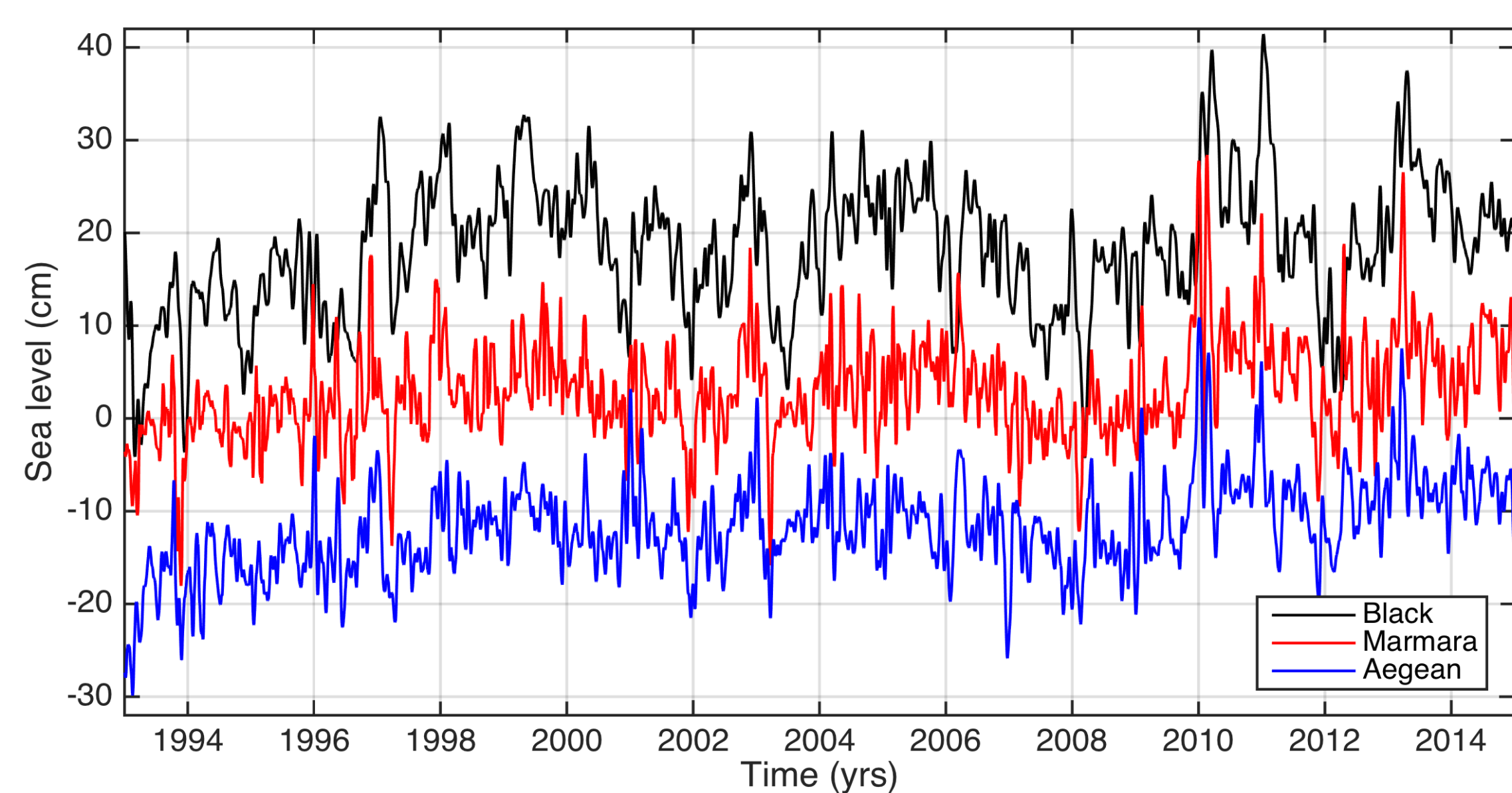
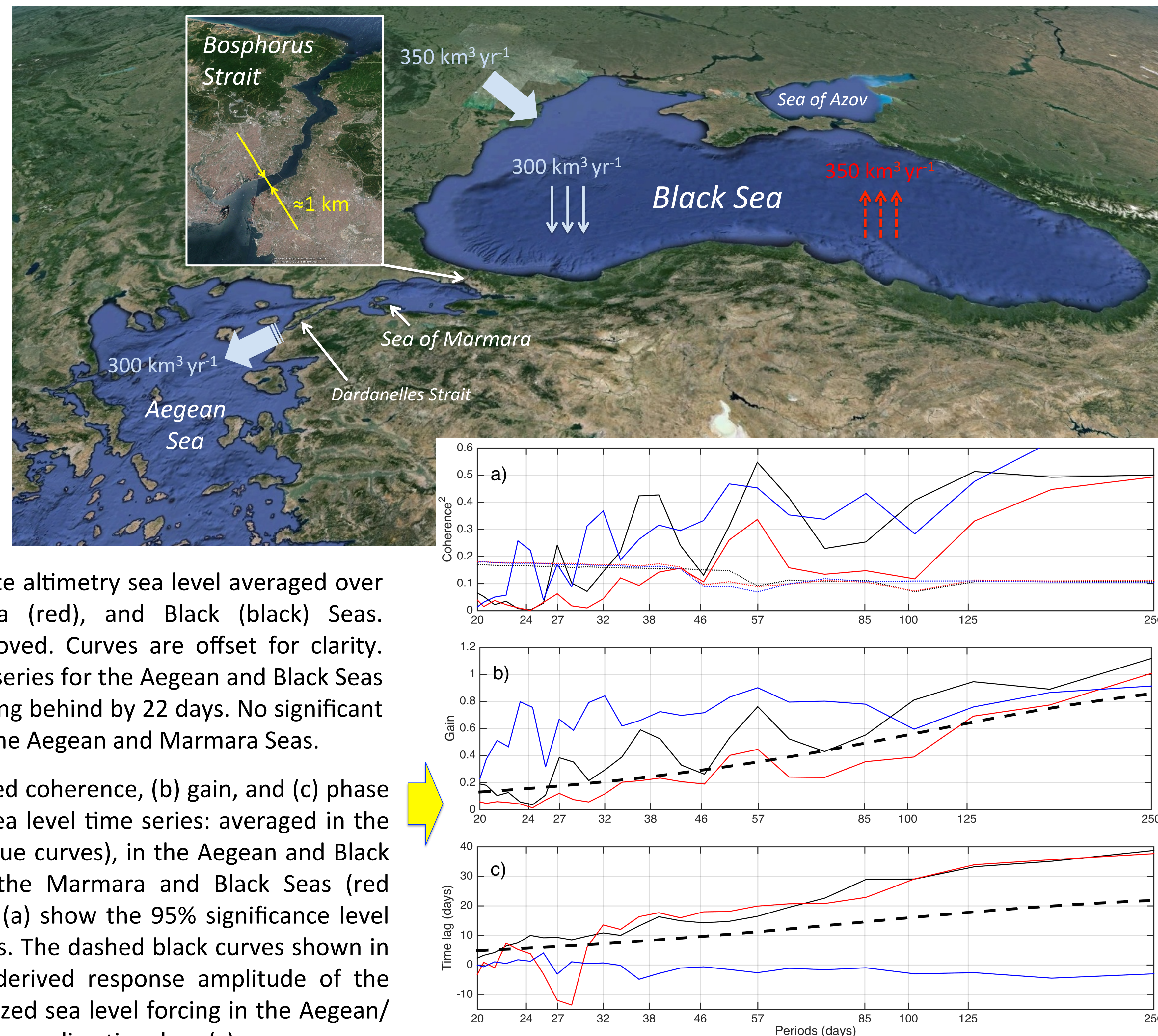


Figure 1. Time series of satellite altimetry sea level averaged over the Aegean (blue), Marmara (red), and Black (black) Seas. Seasonal cycle has been removed. Curves are offset for clarity. Correlation between the time series for the Aegean and Black Seas is 0.67 with the Black Sea lagging behind by 22 days. No significant time lag is detected between the Aegean and Marmara Seas.

Figure 2. (a) Magnitude squared coherence, (b) gain, and (c) phase (days) between the pairs of sea level time series: averaged in the Aegean and Marmara Seas (blue curves), in the Aegean and Black Seas (black curves), and in the Marmara and Black Seas (red curves). The dotted curves in (a) show the 95% significance level for each pair of the time series. The dashed black curves shown in (b) and (c): the analytically derived response amplitude of the Black Sea elevation to normalized sea level forcing in the Aegean/Marmara Sea (b) and the corresponding time lags (c).



III. ANALYTICAL MODEL

Consider the response of a single basin (the Black Sea) of area S_1 , separated by a strait (Bosphorus) of effective depth H_s from a water body (Marmara and Aegean Seas) which experiences oscillations of sea level near the strait given by $\text{Re}[\eta_0 e^{i\omega t}]$ (Fig. 3):

$$\frac{\partial u_s}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{\tau_s}{H_s \rho} - F_s$$

Along-strait barotropic motion ignoring advective and Coriolis terms

$$P_0 = P_a + \rho g \eta_0$$

Assumptions: P_a varies uniformly over the Black Sea, the along-strait SLP gradient is negligible, η_0 responds to P_a isostatically due to the free connection to the wider Mediterranean.

$$P_1 = P_a + \rho g \eta_1$$

$$\eta_0 = \eta_0' - P_a / \rho g$$

$$S_1 \frac{\partial \eta_1}{\partial t} = A_s u_s + Q_{fw}$$

The Black Sea level budget ignoring the steric effects

VARIABLES

u_s - barotropic flow along the strait
 P - subsurface pressure
 P_a - atmospheric pressure
 τ_s - along-strait wind stress
 ρ - density
 $F_s = \lambda_s u_s$, where λ_s is a friction coefficient
 S_1 - surface area of the Black Sea
 A_s - cross-sectional area of Bosphorus Str.
 Q_{fw} - net freshwater flux into the Black Sea
 L_s - length of the strait
 ω - frequency of fluctuations

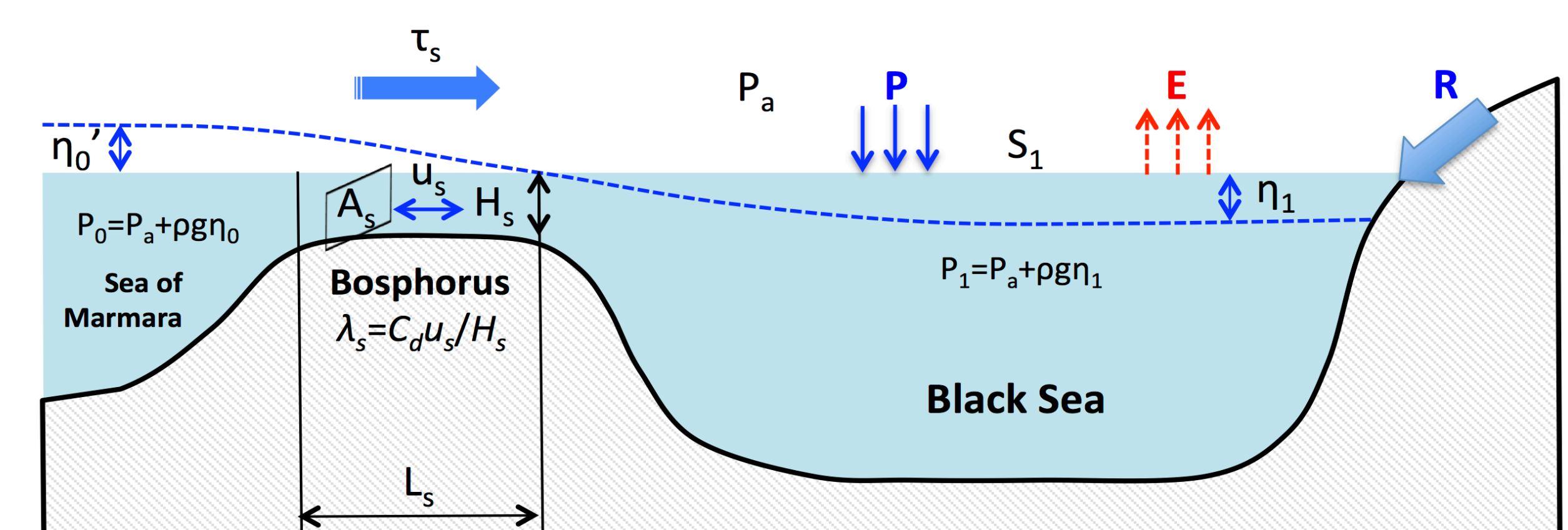


Figure 3. Schematic of the Black Sea with area S_1 connected to the Sea of Marmara by the Bosphorus Strait. Abbreviations: P – precipitation, E – evaporation, R – river runoff.

Letting u_s, η, P_a, τ_s behave as $e^{i\omega t}$, and approximating $d/dx \approx 1/L_s$

$$\eta_1 = \left[\eta_0' - \frac{P_a}{\rho g} + \frac{\tau_s L_s}{\rho g H_s} + \frac{Q_{fw} L_s}{g A_s} (-i\omega + \lambda_s) \right] / \left[1 - \omega^2 \frac{S_1 L_s}{A_s g} - i\omega \lambda_s \frac{S_1 L_s}{A_s g} \right]$$

If $P_a = \tau_s = Q_{fw} = 0$ \Rightarrow $\eta_1 = \eta_0' / (1 - \omega^2 / \omega_r^2 - i\omega \lambda_s / \omega_r^2)$, where $\omega_r = \sqrt{A_s g / S_1 L_s}$ is Helmholtz frequency.

For the geometrical characteristics of the Black Sea (Sea of Marmara) and the Bosphorus Strait (Dardanelles Strait) the corresponding Helmholtz period is about 17 (3) days. Using the equation in blue we estimate the dardic amplitudes and phases of the Black Sea response to an increase of sea level in the Sea of Marmara by 10 cm ($\eta_0' = 10$) (Fig. 4). The analytic amplitude (normalized) and phase are compared to the observed gain and phase (Fig. 2, black dashed curves). Reasonable values for standard deviations of the other forcing terms are $\sigma(P_a) = 240$ Pa (2.4 mbar), $\sigma(\tau_s) = 0.1$ N m⁻², and $\sigma(Q_{fw}) = 7 \times 10^3$ m³ s⁻¹. We use these values in the equation in red and consider each forcing term individually by setting the other terms to zero, with $\lambda_s = 3.8 \times 10^{-5}$ s⁻¹ (Fig. 4a, dashed curves).

IV. CONCLUSIONS

- The analytical model is able to partially explain the amplitude and phase of the observed Black Sea elevation with respect to sea level changes in the Aegean and Marmara Seas (Fig. 2b and 2c);
- While the Aegean/Marmara sea level is the dominant forcing mechanism on synoptic time scales, the individual response to P_a over the Black Sea and the net freshwater flux into the Black Sea is considerable (up to 2-3 cm) and the response to the along-strait wind stress is small (<1 cm) (Fig. 4a);
- The response phases to individual forcing terms at $\lambda_s = 3.8 \times 10^{-5}$ s⁻¹ are the same (Fig. 4b, solid black curve);
- Useful predictions of the Black Sea elevation in response to the sea level fluctuations in the Mediterranean, Aegean, and Marmara Seas can be made from a few weeks to a month in advance, which may be of societal and economic value for the region.

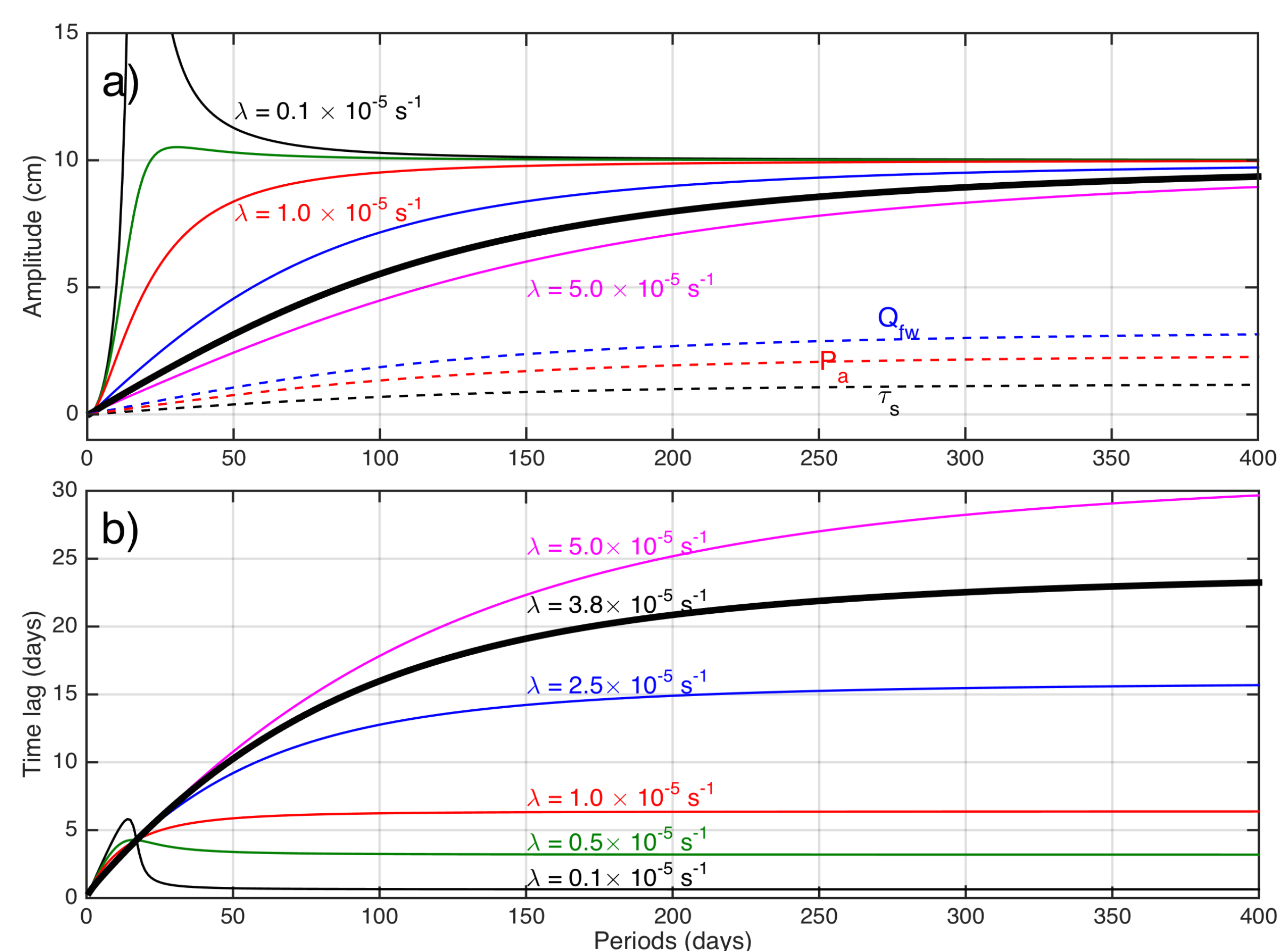


Figure 4. (a) Amplitude (cm) and (b) time lag (days) of the response of the Black Sea elevation to an increase of sea level in the Aegean/Marmara Sea by 10 cm, computed using the equation in blue for a number of friction coefficients. The dashed curves in (a) show the response amplitudes to individual forcing terms in the equation in red at $\lambda_s = 3.8 \times 10^{-5}$ s⁻¹: the net freshwater flux (blue), P_a over the Black Sea (red), and the along-strait wind stress (black).