

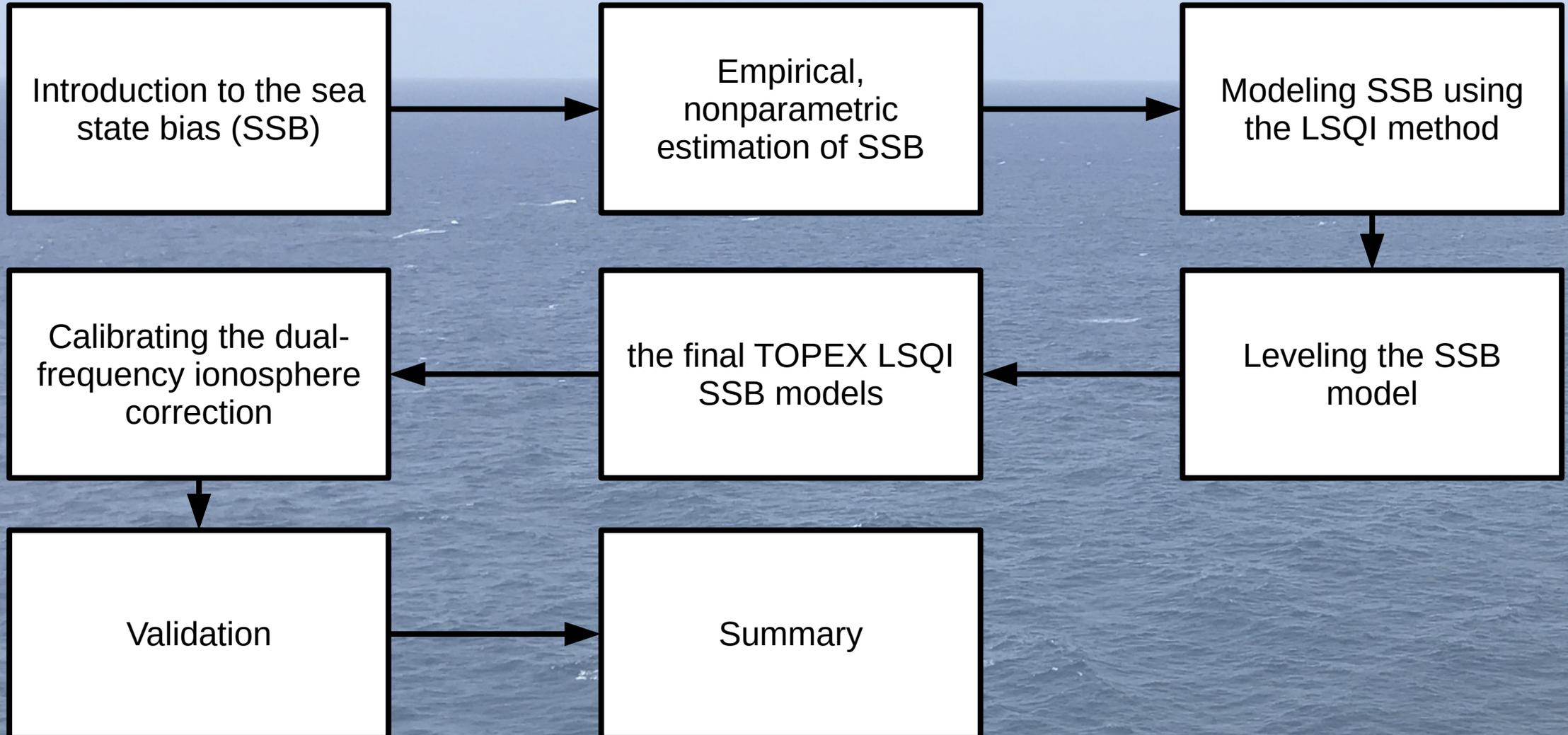
Estimating the sea state bias for TOPEX

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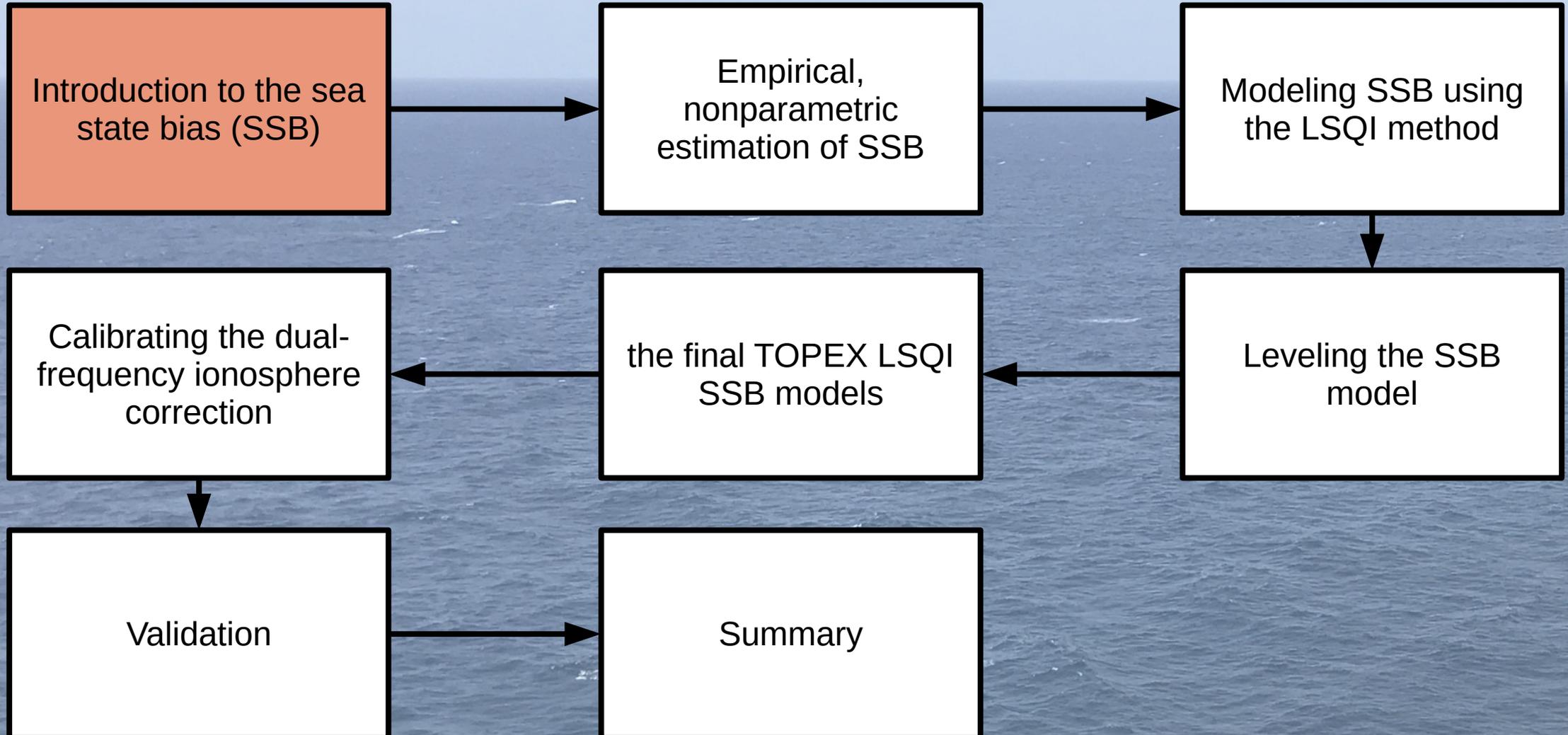
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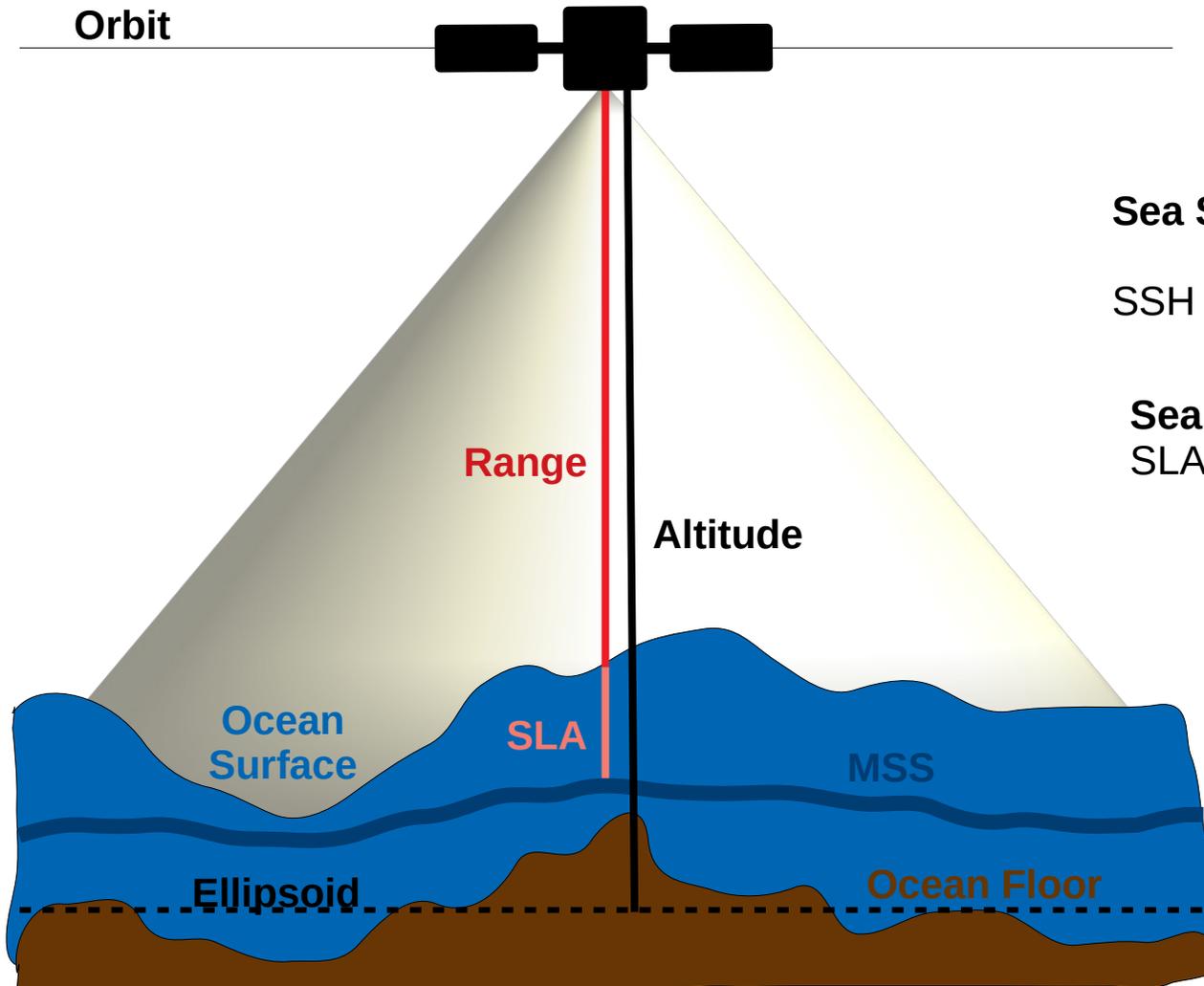
Presentation Outline



Presentation Outline



Sea surface height and sea level anomaly



Corrected Range:

$$R_{\text{corr}} = \text{Range} + \text{Wet Troposphere Correction} \\ + \text{Dry Troposphere Correction} \\ + \text{Ionosphere Correction} \\ + \text{Sea State Bias}$$

Sea Surface Height: geocentric height of the sea surface above the reference ellipsoid

$$\text{SSH} = \text{Altitude} - R_{\text{corr}}$$

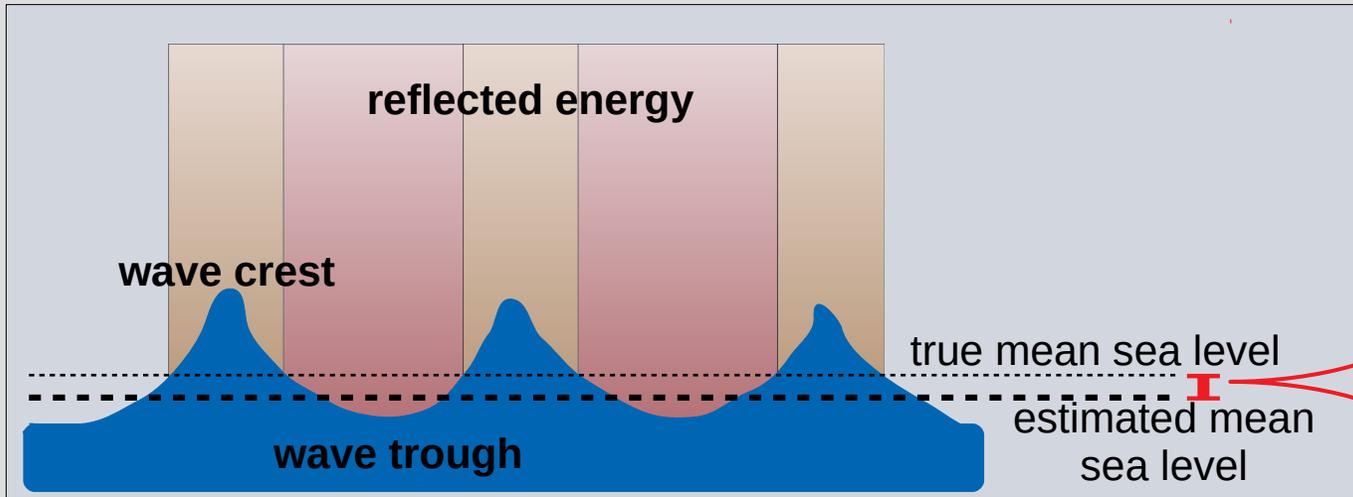
Sea Level Anomaly: deviations from the mean sea surface

$$\text{SLA} = \text{SSH} - \text{MSS}$$

- Solid Earth Tide Height
- Pole Tide Height
- Geocentric Ocean Tide Height
- Inverted Barometer Height Correction
- HF Fluctuations of the Sea Surface Topography

Sea State Bias (SSB)

- an altimeter range delay that places the estimated mean sea level below the true mean sea level
- remains the largest error in the SLA error budget



sea state bias:

electromagnetic bias (EMB):

- wave troughs provide stronger reflections than wave crests

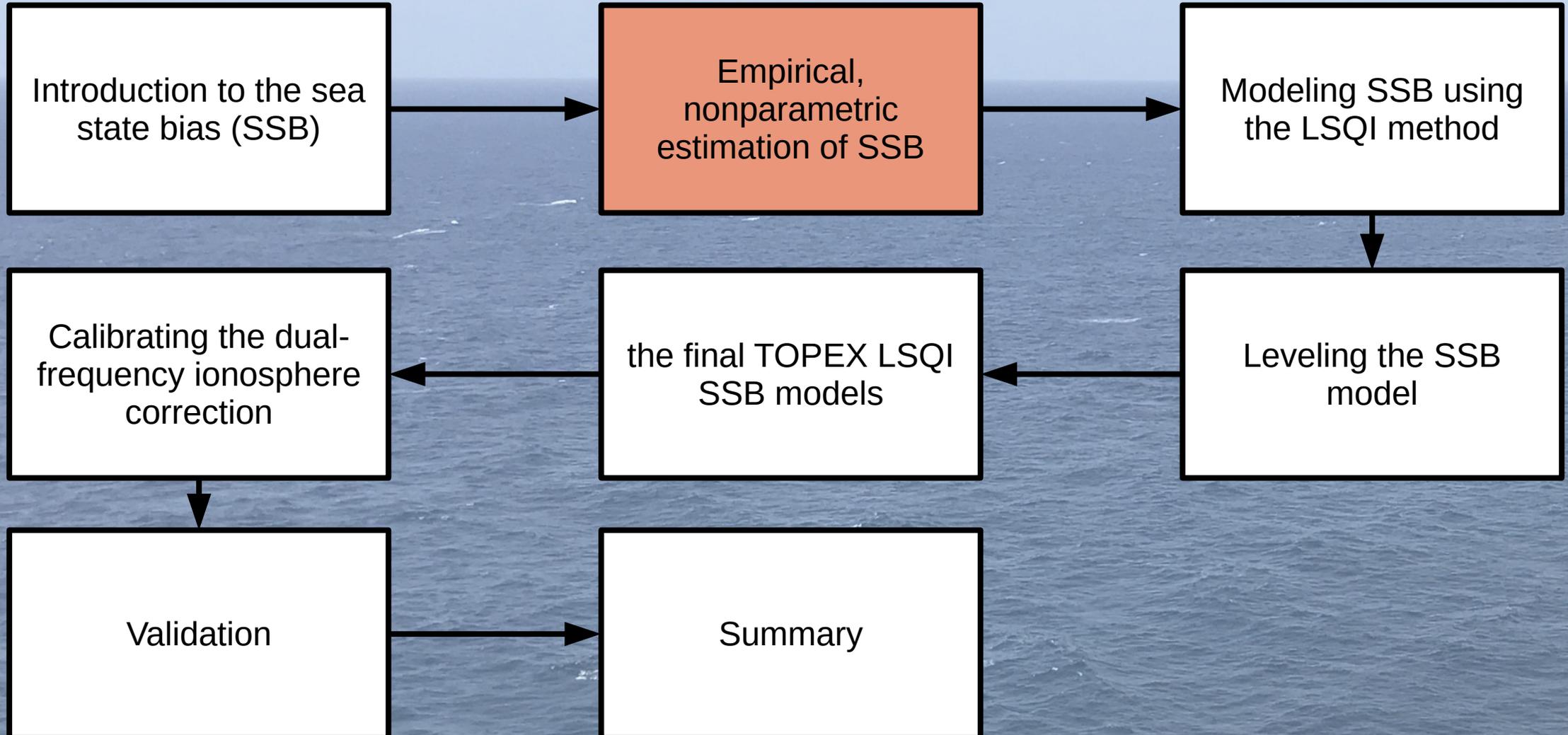
skewness bias:

- more surface area below the true mean sea level than above

tracker bias:

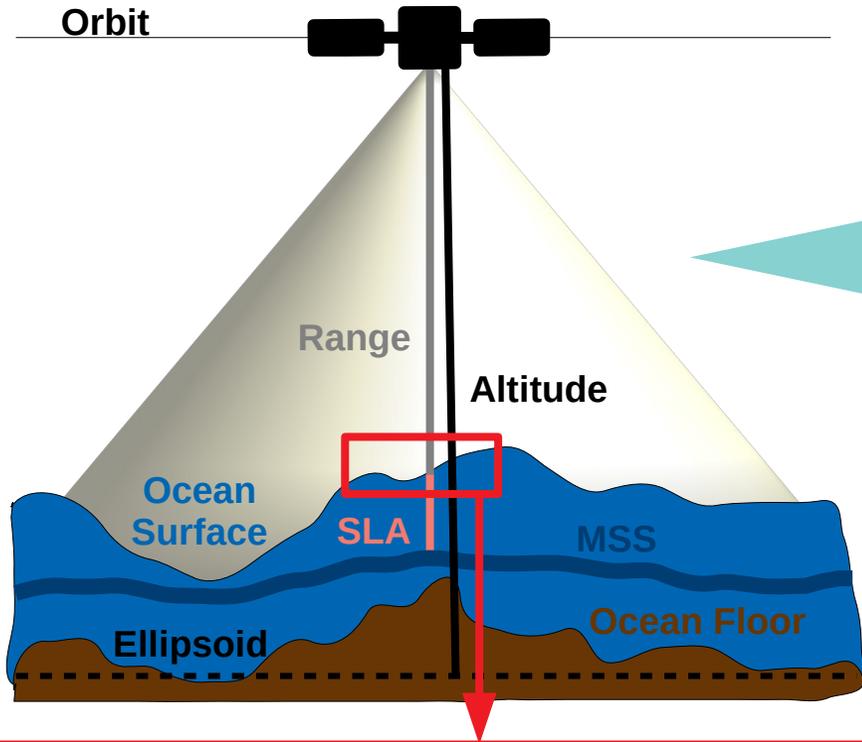
- instrumental effects and assumptions made by the retracking algorithm used to fit the return signal.

Presentation Outline



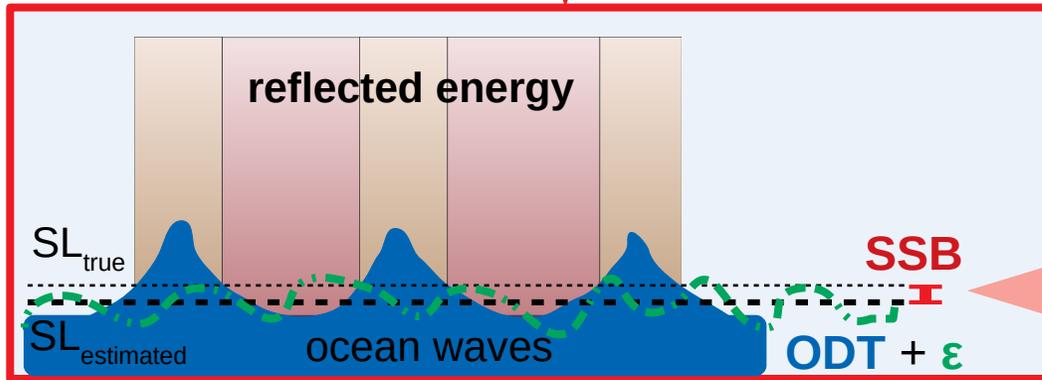
Empirical measurements

used to overcome the challenges of physical SSB models



Sea Level Anomaly:

- SLA = Altitude
- Range
- Wet Troposphere Correction
- Dry Troposphere Correction
- Ionosphere Correction (use GIM in uSLA [see below])
- Sea State Bias
- MSS
- Solid Earth Tide Height
- Pole Tide Height
- Geocentric Ocean Tide Height
- Inverted Barometer Height Correction
- HF Fluctuations of the Sea Surface Topography



uncorrected Sea Level Anomaly: SLA uncorrected for SSB

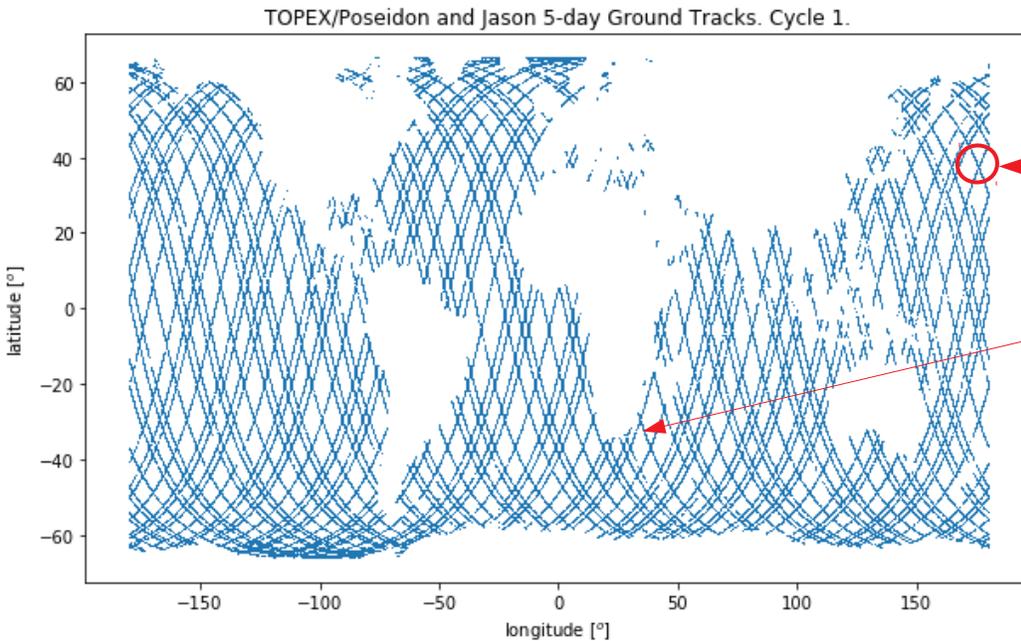
$$uSLA = SSB + ODT + \epsilon$$

contains signal due to the ocean dynamic topography (ODT), as well as range correction errors, orbit computation error and noise (ϵ)

Empirical measurements

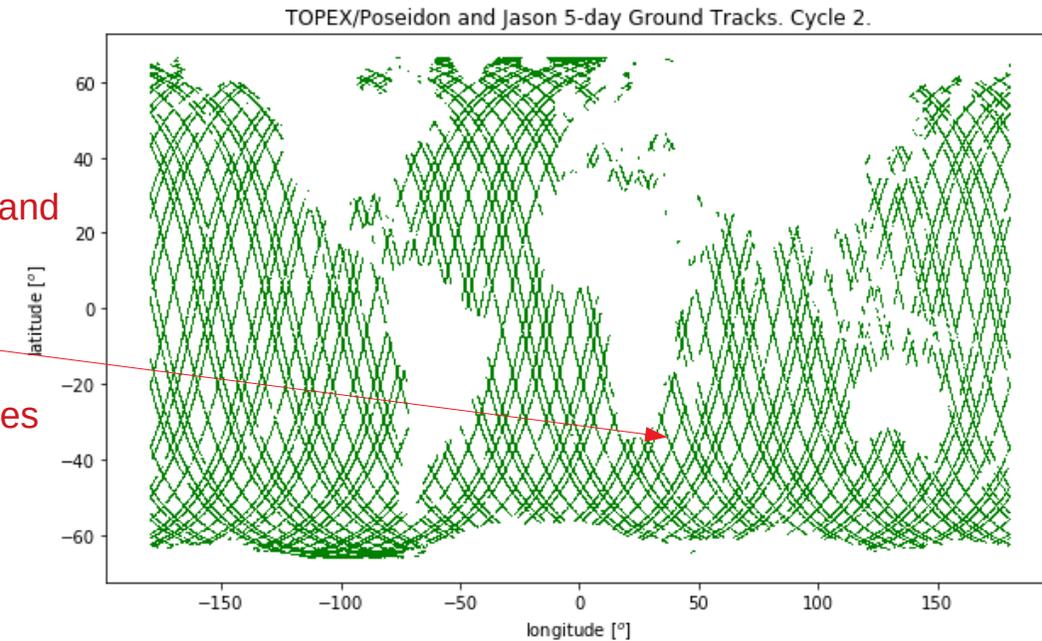
Measurement approach:	Absolute	Difference
Observation Equation:	$\beta = uSLA = \text{SSB} + \text{ODT} + \varepsilon$	$\beta = uSLA_{t_2} - uSLA_{t_1} = (\text{SSB}_{t_2} - \text{SSB}_{t_1}) + \Delta\text{ODT} + \Delta\varepsilon$
Assumptions:	$E[\text{ODT}] = 0, E[\varepsilon] = 0, E[\beta] = E[\text{SSB}]$	$E[\Delta\text{ODT}] = 0, E[\Delta\varepsilon] = 0, E[\beta] = E[\text{SSB}_{t_2} - \text{SSB}_{t_1}]$

Difference Approach: measurement taken at different times (t1 and t2) but at the same location.



crossover:
intersecting ascending and descending passes

collinear:
overlapping passes of subsequent repeat cycles



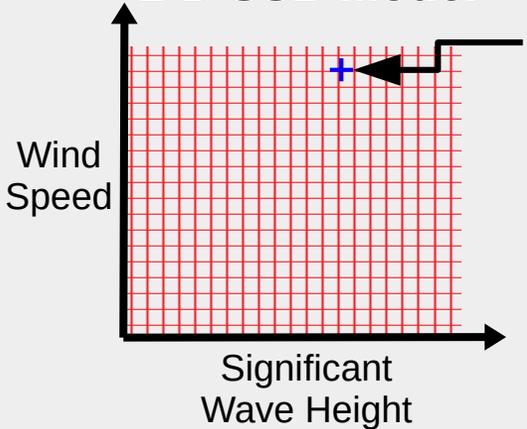
Nonparametric estimation

fits a function to the data without relying on a finite set of parameters to capture all there is to know about the data

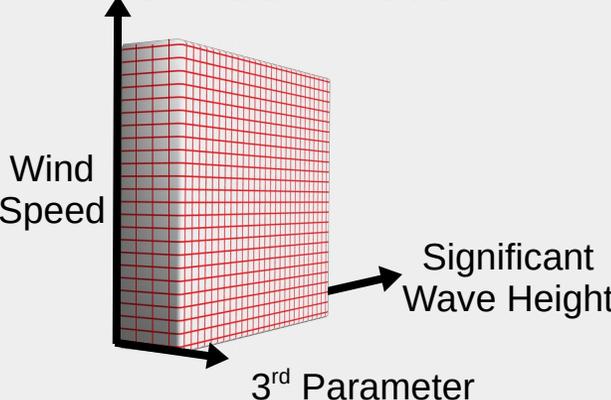
SSB nonparametric model, $\varphi(\mathbf{x})$
SSB is a nonspecified function (φ) of a vector (\mathbf{x}) containing M independent variables that can be written as;
$$\text{SSB} = \varphi(\mathbf{x})$$

Independent variables, \mathbf{x}
Sea state and sea state variability characteristics:
wave height, wave period, wave power spectrum, wind speed, swell

2-D SSB Model



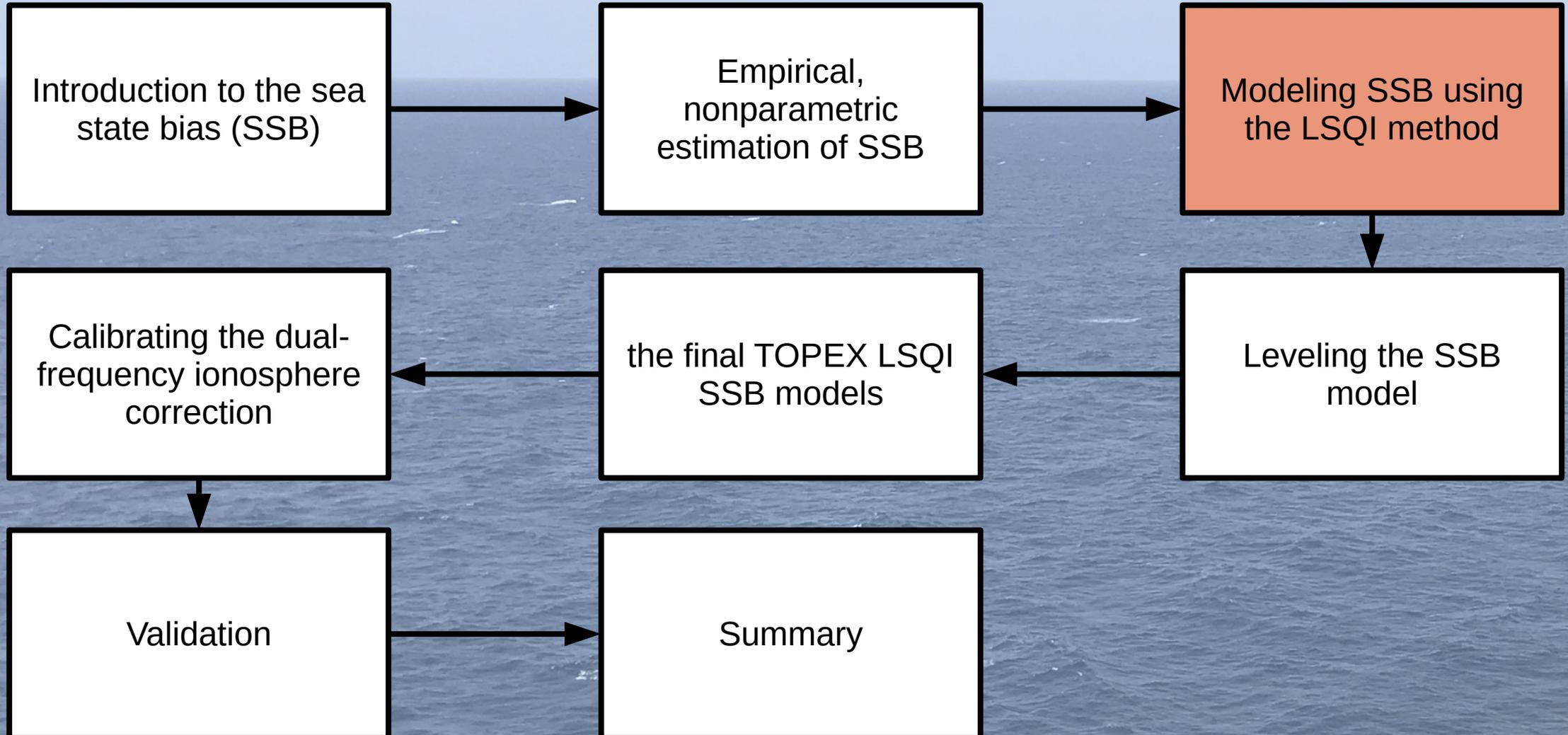
3-D SSB Model



$\varphi(\mathbf{x})$ SSB models are typically provided in the form of either a 2-D or 3-D look-up table.

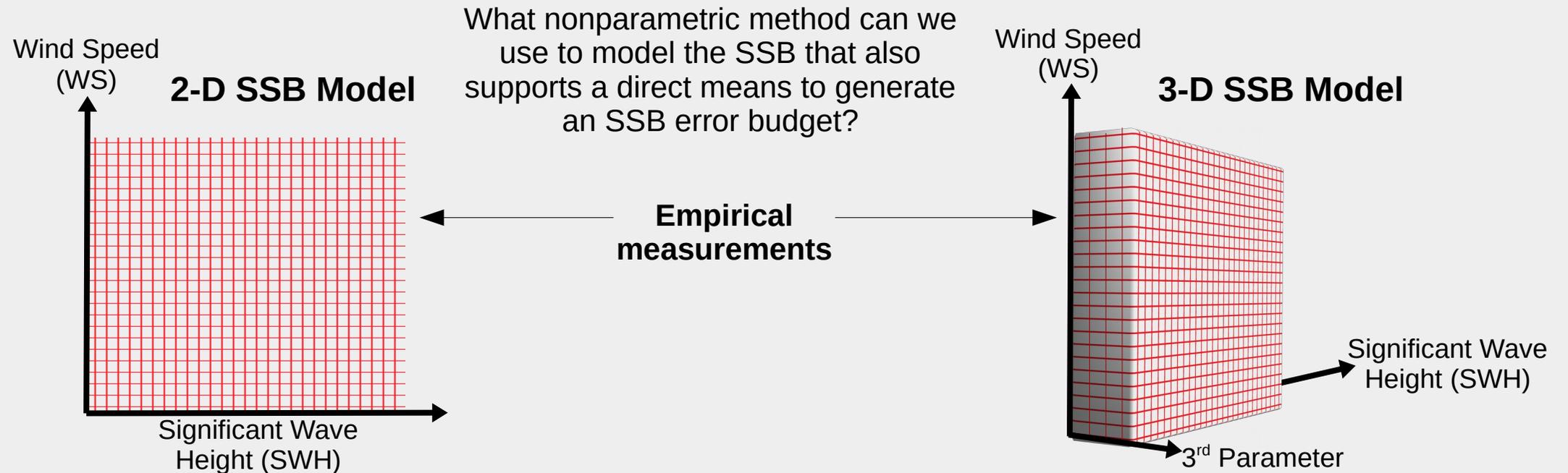
Sea state bias estimates are found by interpolating the model at point $(\text{SWH}_i, \text{WS}_i)$.

Presentation Outline



SSB model development

Even though nonparametric SSB modeling methods are already available, the goal here is to develop a simple and efficient method that provides a means to further investigate the largest error in the SLA error budget.



Bilinear interpolation of a point $[swh_i, ws_i]$ requires the application of bilinear weights to each of the four surrounding parameters.

Why not use the weights and inversely solve for the parameters?

Least squares interpolation (LSQI) method

By pre-constructing the SWH and WS bins, we can determine the weights assigned to each observation, i.

$$\text{e.g., } \delta_{i,00} = (\text{swh}_i - \text{swh}_0) * (\text{ws}_i - \text{ws}_0) / (\Delta \text{swh} * \Delta \text{ws})$$

So long as there are more observations than parameters (i.e. nodes in the model), we can solve for the SSB model using least squares.

SSB modeling using LSQI:

K known observations (~ 500,000 obs/cycle)

P=MxN unknown parameters (~ 4000 nodes)

$$\beta_{x0,y0} = \delta_{0,00} \Phi_{00} + \delta_{0,10} \Phi_{10} + \delta_{0,01} \Phi_{01} + \delta_{0,11} \Phi_{11}$$

$$\beta_{x1,y1} = \delta_{1,00} \Phi_{00} + \delta_{1,10} \Phi_{10} + \delta_{1,01} \Phi_{01} + \delta_{1,11} \Phi_{11}$$

$$\beta_{x2,y2} = \delta_{2,00} \Phi_{00} + \delta_{2,10} \Phi_{10} + \delta_{2,01} \Phi_{01} + \delta_{2,11} \Phi_{11}$$

$$\beta_{x3,y3} = \delta_{3,00} \Phi_{00} + \delta_{3,10} \Phi_{10} + \delta_{3,01} \Phi_{01} + \delta_{3,11} \Phi_{11}$$

⋮

$$\beta_{xk,yk} = \delta_{k,N-1,M-1} \Phi_{N-1,M-1} + \delta_{k,N,M-1} \Phi_{N,M-1} + \delta_{k,N-1,M} \Phi_{N-1,M} + \delta_{k,NM} \Phi_{NM}$$

↓

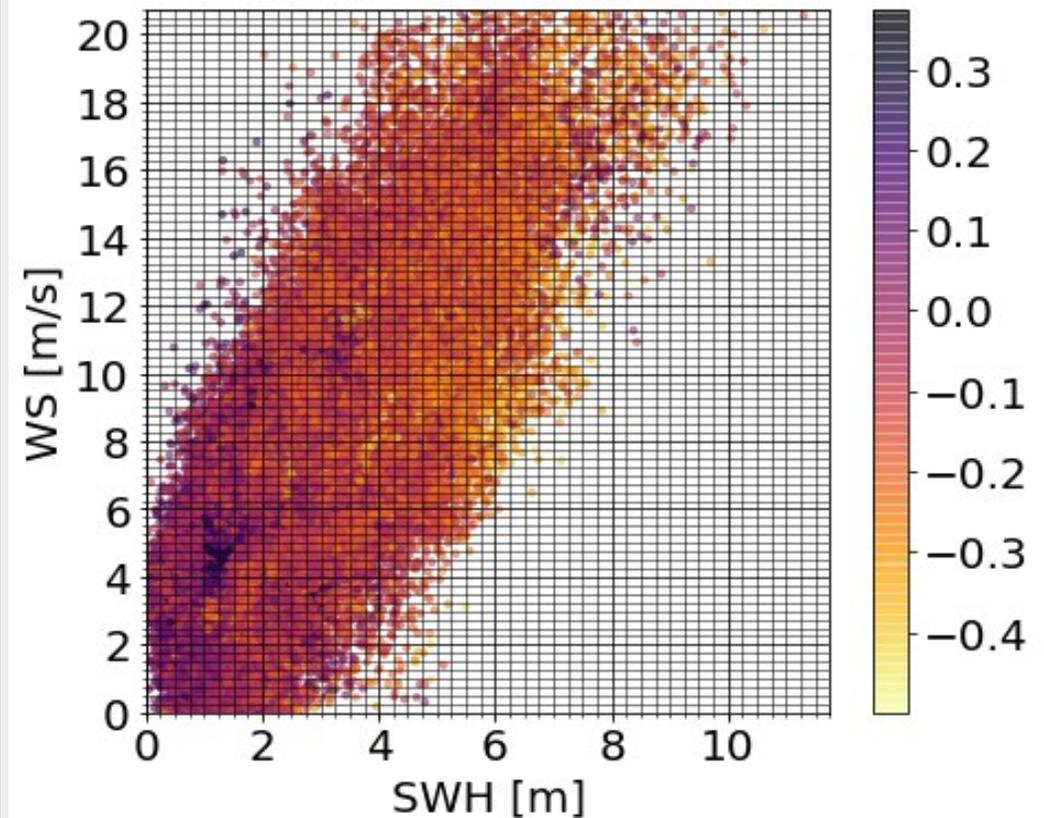
$$\Phi = (\delta^T \delta)^{-1} \delta^T \beta$$

which can be accumulated cycle-by-cycle

↓

$$\Phi = (\sum(\delta^T \delta))^{-1} \sum(\delta^T \beta)$$

SLA Uncorrected for SSB



Model observables

SSB modeling using LSQI:

K known observations (~ 500,000 obs/cycle)
P=MxN unknown parameters (~ 4000 nodes)

$$\begin{aligned}\beta_{x_0,y_0} &= \delta_{0,00} \Phi_{00} + \delta_{0,1} \\ \beta_{x_1,y_1} &= \delta_{1,00} \Phi_{00} + \delta_{1,1} \\ \beta_{x_2,y_2} &= \delta_{2,00} \Phi_{00} + \delta_{2,1} \\ \beta_{x_3,y_3} &= \delta_{3,00} \Phi_{00} + \delta_{3,1}\end{aligned}$$

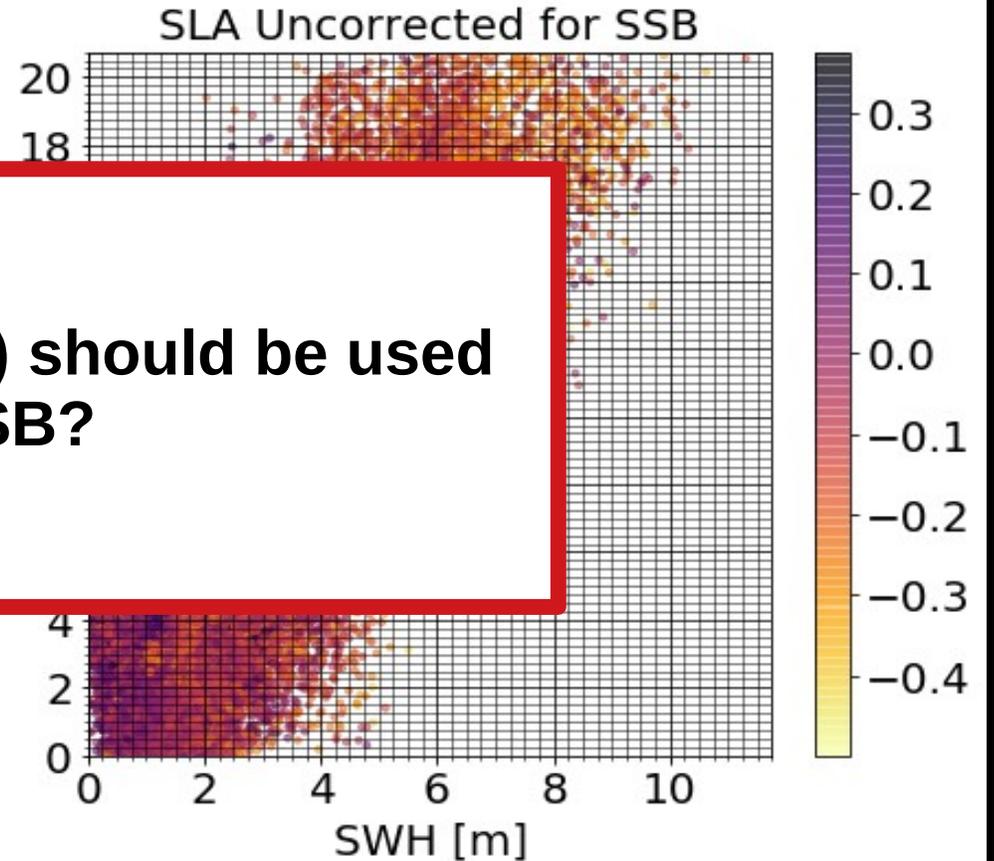
$$\beta_{x_k,y_k} = \delta_{k,N-1,M-1} \Phi_{N-1,M-1} + \delta_{k,N}$$

which measurements (β) should be used to model SSB?

$$\Phi = (\delta^T \delta)^{-1} \delta^T \beta$$

which can be accumulated cycle-by-cycle

$$\Phi = (\sum (\delta^T \delta))^{-1} \sum (\delta^T \beta)$$



Absolute Measurements

Direct

$$\beta = u\text{SLA} = \text{SSB} + \text{ODT} + \varepsilon$$

Advantages:

- high number of observations per cycle
- contains full SSB signal

Disadvantages:

- contains ODT signal and SL correction error

Difference Measurements

Crossover

$$\beta = \Delta u\text{SLA}_{\text{des-asc}} = \Delta \text{SSB}_{\text{des-asc}} + \Delta \text{ODT}_{\text{des-asc}} + \Delta \varepsilon_{\text{des-asc}}$$

Advantages:

- reduces ODT signal and SL correction error
- time gap, Δt , less than 10 days

Disadvantages:

- low number of observations per cycle
- contains SSB differences

Collinear

$$\beta = \Delta u\text{SLA}_{\text{p2-p1}} = \Delta \text{SSB}_{\text{p2-p1}} + \Delta \text{ODT}_{\text{p2-p1}} + \Delta \varepsilon_{\text{p2-p1}}$$

Advantages:

- reduces ODT signal and SL correction error
- high number of observations per cycle

Disadvantages:

- time gap, Δt , only equal to 10 days
- contains SSB differences

Joint

$$\beta = [w(\Delta t_{\text{des-asc}}) * (\Delta \text{SSB}_{\text{des-asc}} + \Delta \text{ODT}_{\text{des-asc}} + \Delta \varepsilon_{\text{des-asc}}), w(\Delta t_{\text{p2-p1}}) * (\Delta \text{SSB}_{\text{p2-p1}} + \Delta \text{ODT}_{\text{p2-p1}} + \Delta \varepsilon_{\text{p2-p1}})]$$

where $w(\Delta t)$ is a time-dependent weight in favor of lower Δt that also considers the significant difference in the number of observations between crossover and collinear measurements.

$$w(\Delta t) = 230 * e^{-|\Delta t| / 3} + 120$$

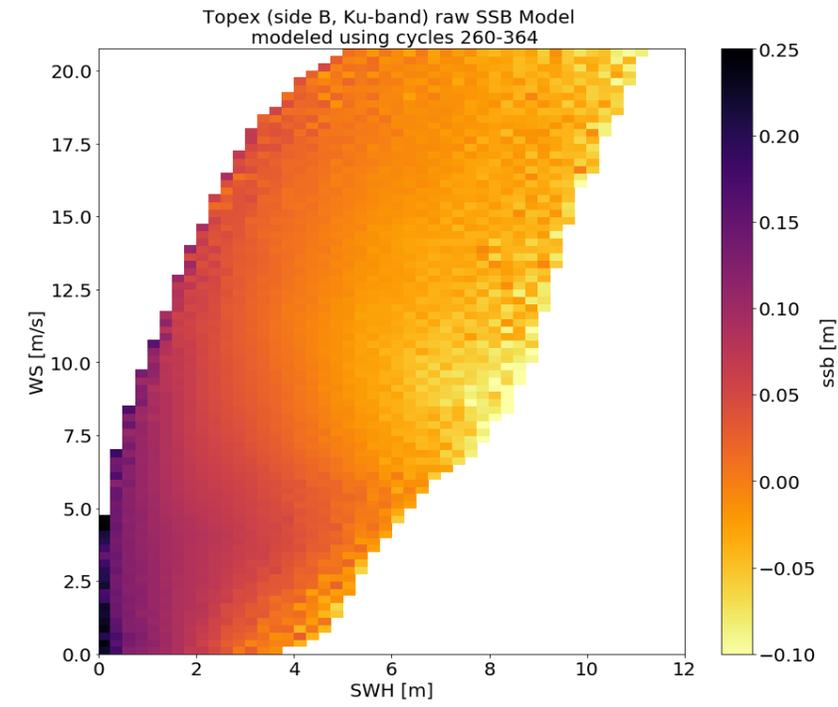
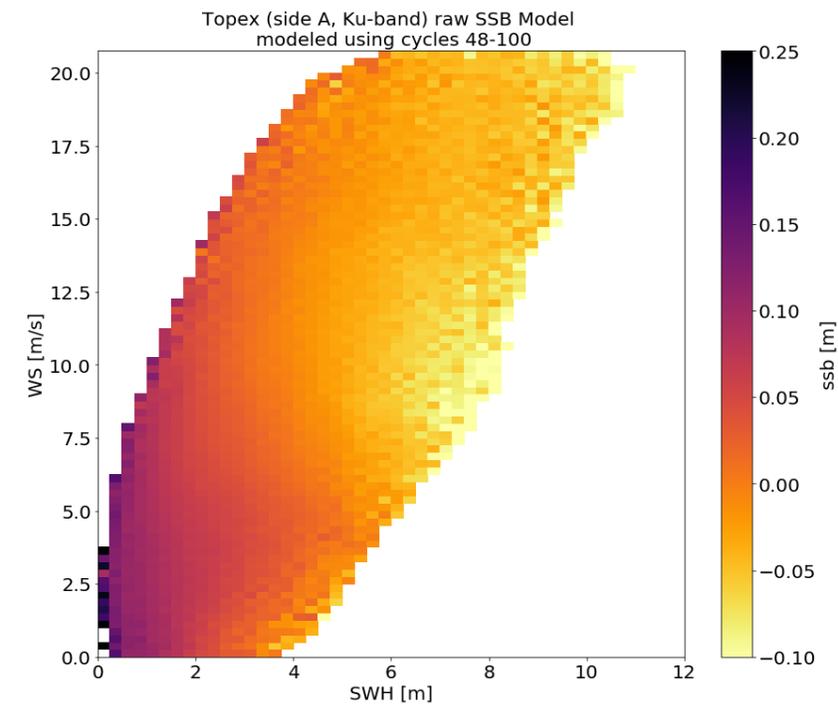
This combination utilizes the temporal resolution of crossover differences, as well as the spatial resolution of collinear differences.

Raw SSB model

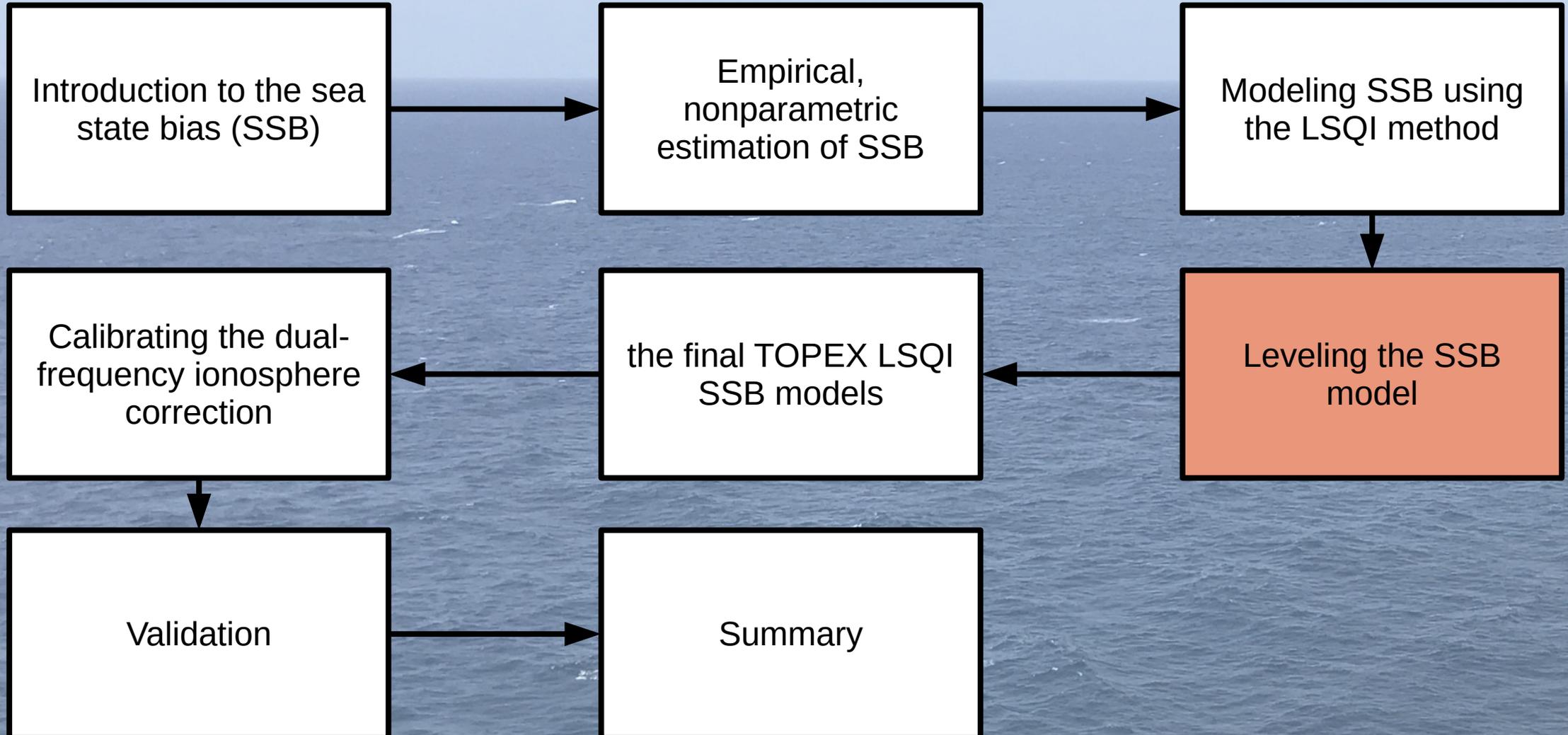
The LSQI modeling approach first provides a raw SSB model output using *joint measurements* that have been filtered based upon surface and echo classification, rain and ice flags, variable limits along with their respective flags, as well as a 4-sigma iterative outlier detection of uSLA.

The raw model is the direct result of the empirical measurements **without leveling nor smoothing/extrapolation**, and is important in that it supports a direct means to investigate SSB estimation error.

Note: Both of the TOPEX models below were developed using retracked altimeter data (Desjonqueres, 2019).



Presentation Outline

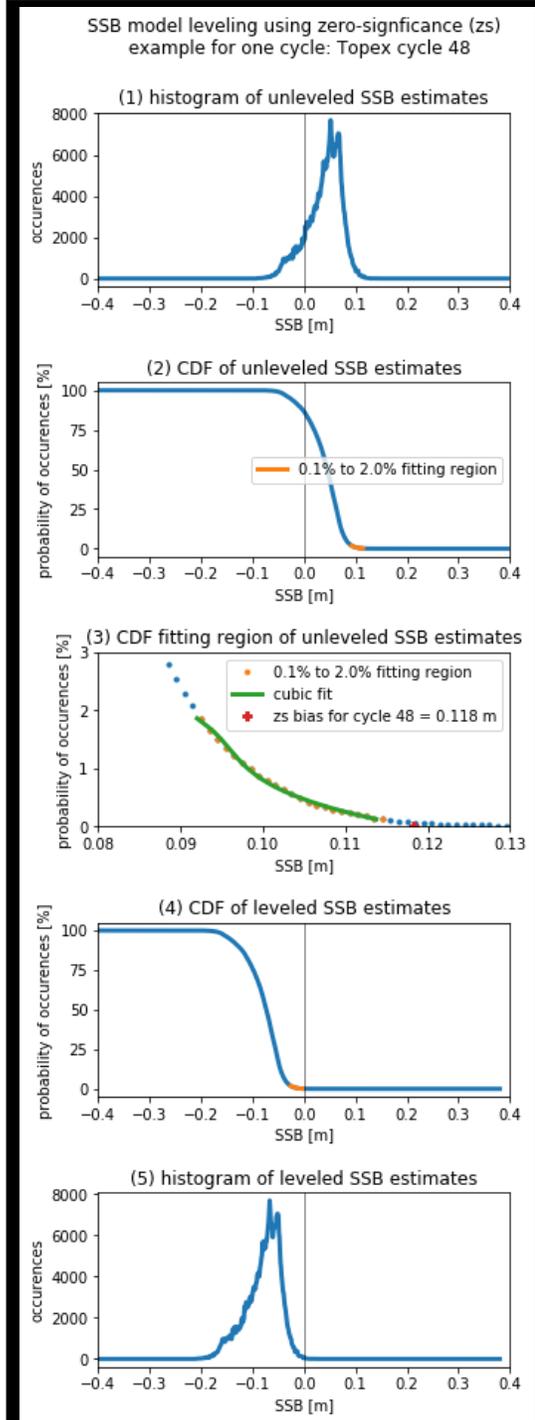
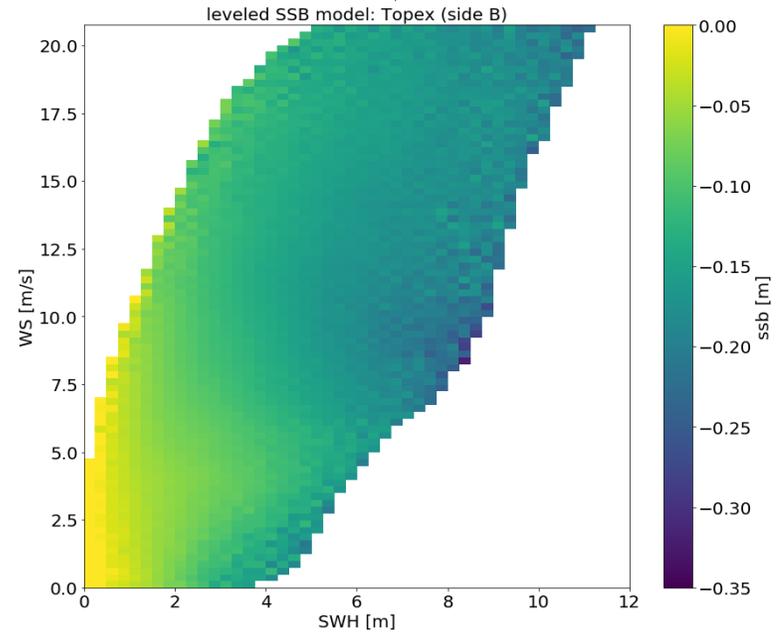
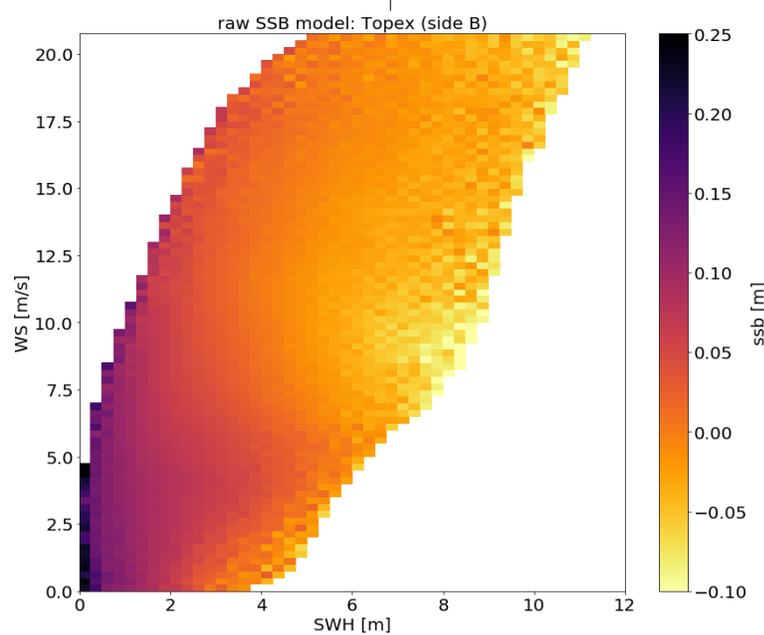


Leveling the SSB model

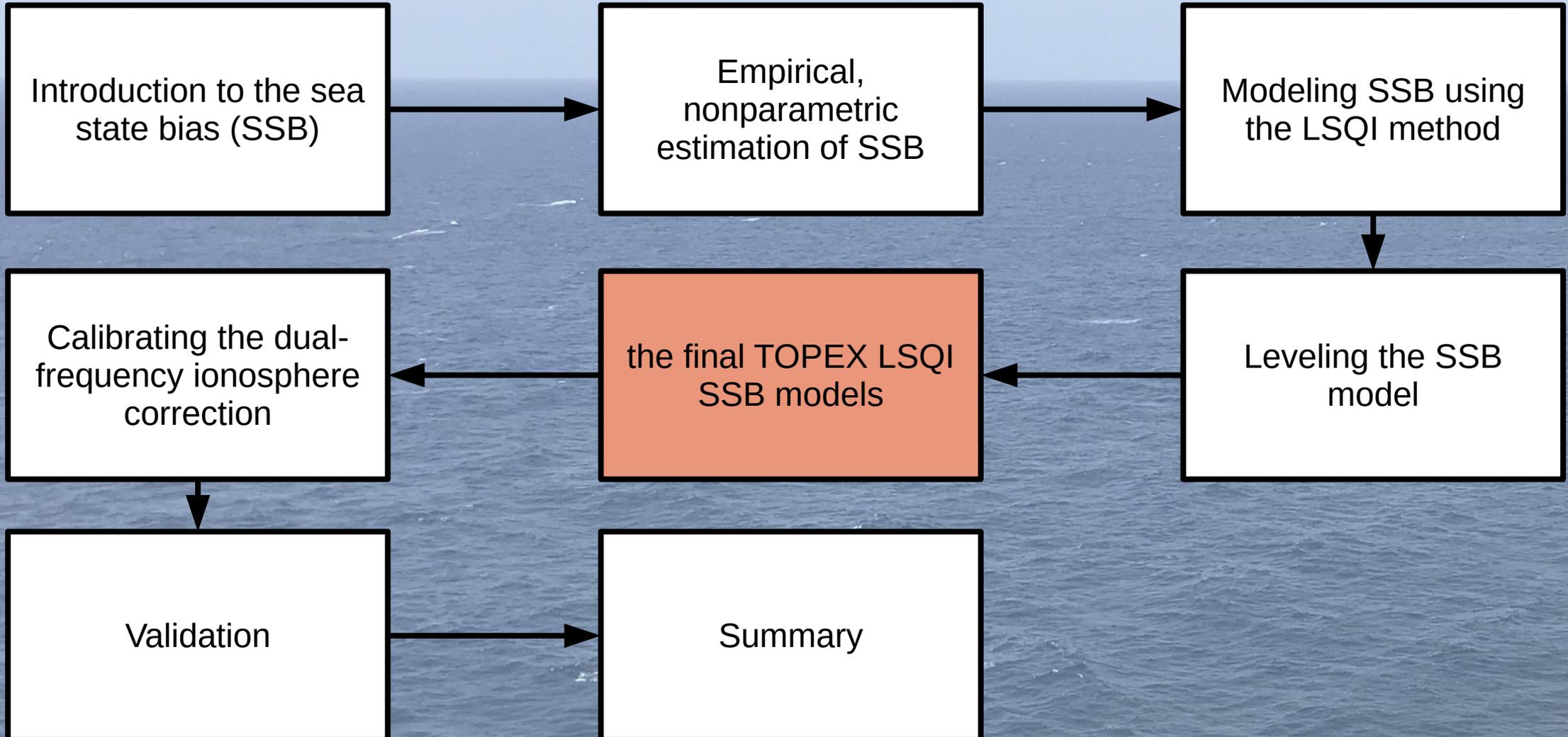
Using difference measurements determines the model to within a constant, and therefore a global shift must be applied.

Zero-significance (zs) approach: under the assumption that a true SSB estimate at 0 m SWH and 0m/s WS would equal zero, fit a polynomial to the region of the SSB distribution that would provide the most consistent bias estimate and evaluate the polynomial at 0-percentile (analogous to Ruf [2000] application to radiometer calibration).

the global bias applied to the model is the mean of all zero-significance estimates of each cycle used to develop the model.
 TOPEX (side A), Ku: $\Phi_{zs} = \sum \Phi_{zs,i}$ where $i = [48:100] = 12.3$ cm
 TOPEX (side B), Ku: $\Phi_{zs} = \sum \Phi_{zs,i}$ where $i = [280:364] = 14.2$ cm



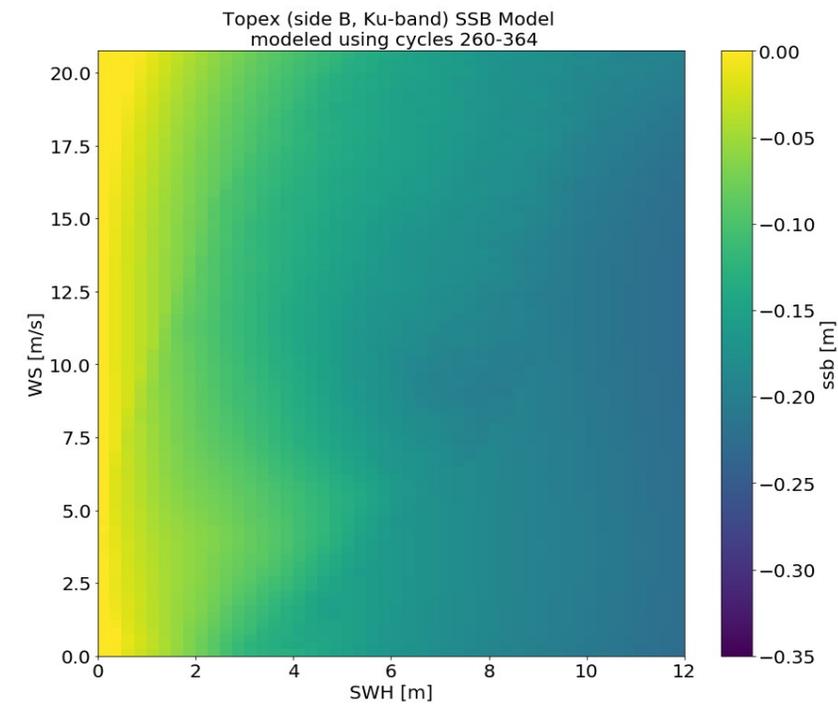
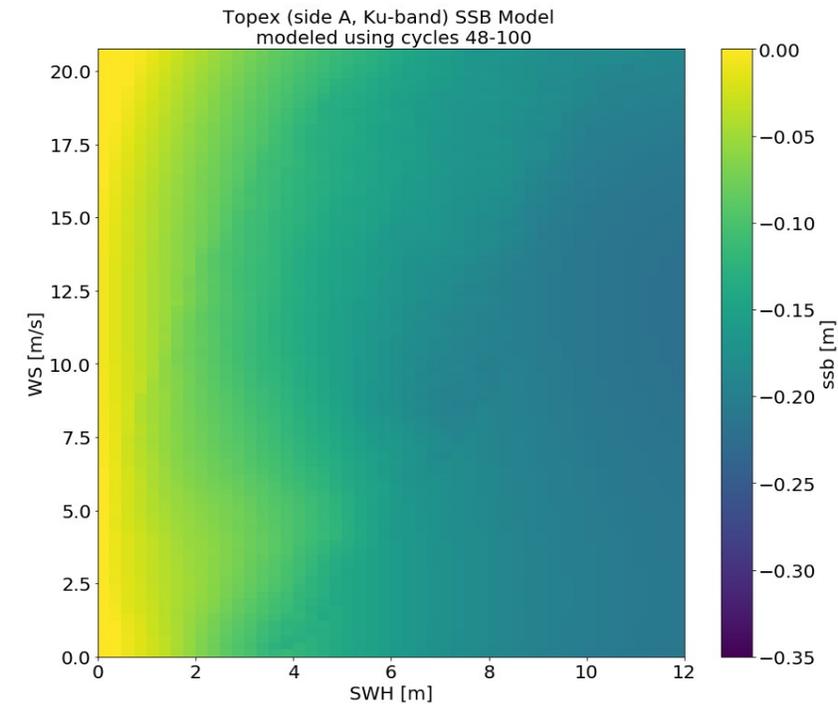
Presentation Outline



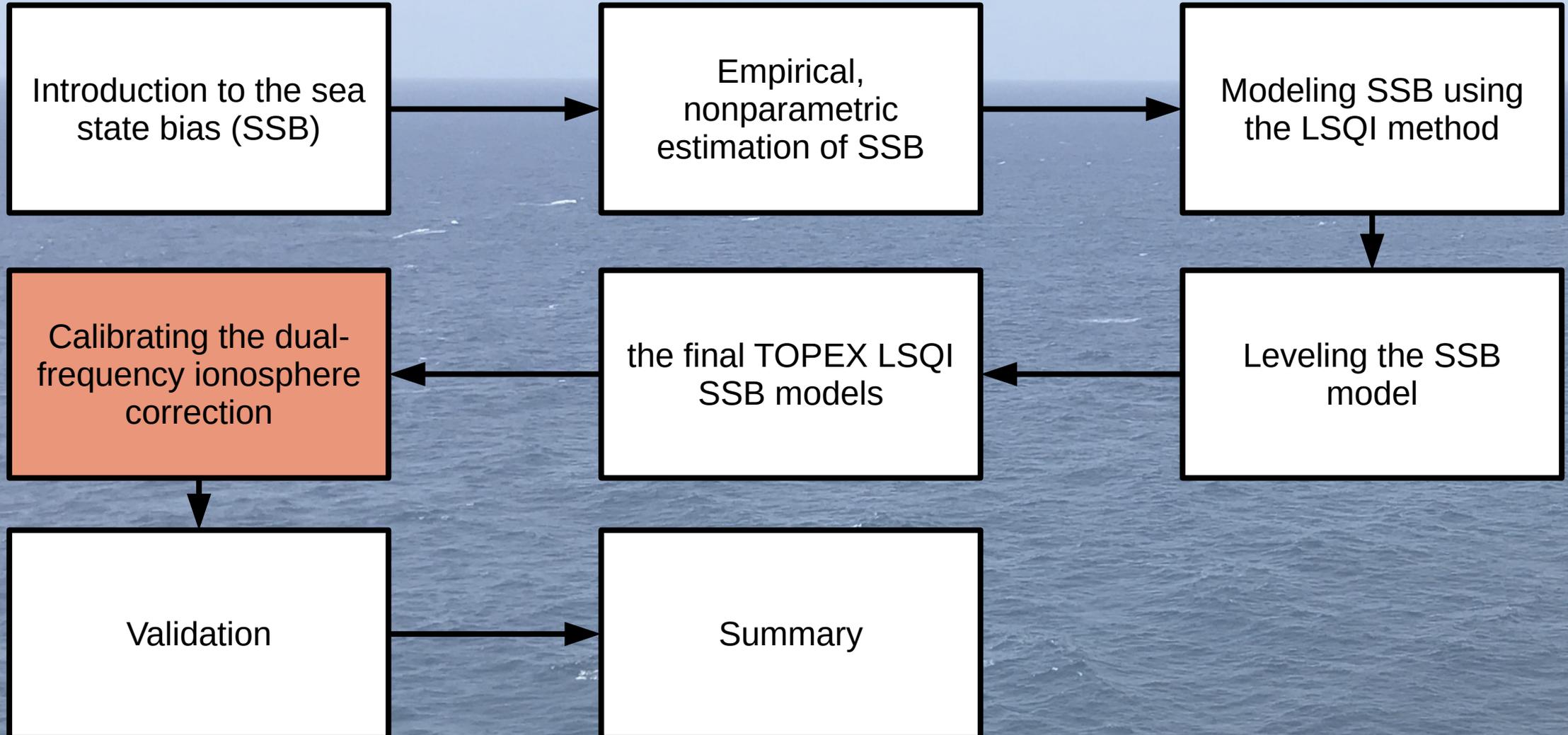
LSQI models

The final TOPEX SSB models are obtained in three steps:

- 1) derive the raw SSB model using the LSQI approach with the joint measurements
 - cycles 48 – 100 are used to create the Topex (side A) model
 - cycles 280 – 364 are used to create the Topex (side B) model
- 2) level the the raw SSB model with the zero-significance method
- 3) smooth and extrapolate the raw, leveled SSB model using a distance-error-weighted average and a parametric fit.



Presentation Outline



Summary of the ionosphere correction

Ionosphere correction as a function of total electron content (TEC):

$$I_f = -40.3 \cdot \text{TEC} \cdot f_f^2$$

$$I_{ku} = -0.2187 \cdot \text{TEC in cm (TEC in TECU = } 1e16 \text{ el/m}^2)$$

$$I_c = -1.4347 \cdot \text{TEC in cm (TEC in TECU = } 1e16 \text{ el/m}^2)$$

Global Ionospheric Maps (GIM): snapshots of global TEC using data from the GPS network

Dual-frequency (DF) ionosphere correction: altimeter-derived range correction due to ionosphere

$$I_{rel} = I_c - I_{ku} = (R_{ku} + \text{SSB}_{ku}) - (R_c + \text{SSB}_c) = -1.2160 \cdot \text{TEC in cm}$$

$$I_{ku} = 0.1798 \cdot [(R_{ku} + \text{SSB}_{ku}) - (R_c + \text{SSB}_c)] \text{ in m}$$

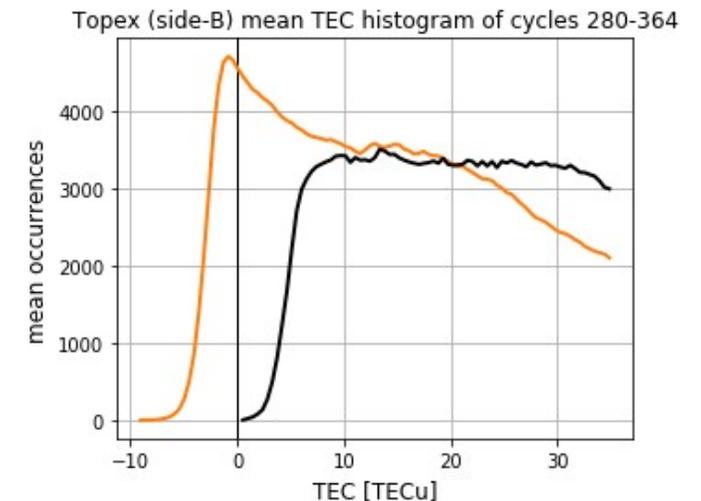
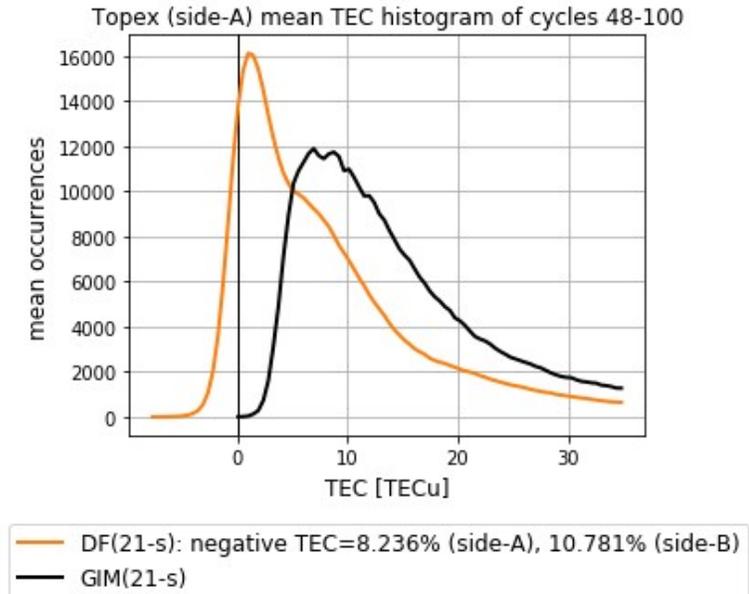
$$I_c = 1.1798 \cdot [(R_{ku} + \text{SSB}_{ku}) - (R_c + \text{SSB}_c)] \text{ in m}$$

Averaged ionosphere correction:

Imel, 1994: 21-second running window to reduce the effects of altimeter noise within the ionosphere correction.

GIM vs. DF:

GIM is not as accurate as DF, however it is not susceptible to range noise.



Uncovering the ionosphere correction bias

Roughly 10% of dual-frequency (DF) TEC values are negative.



Since negative TEC values are not physically possible, this suggests that there exists an **unexplained relative bias** ($\Delta\text{TEC}_{\text{DF}}$) between Ku and C band range + SSB.

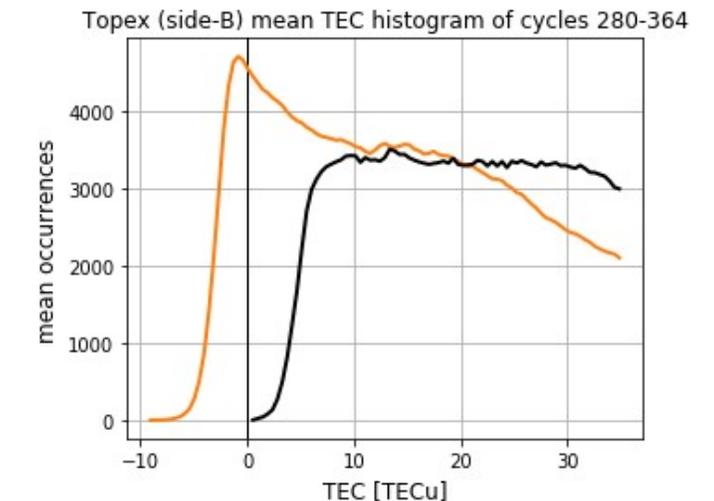
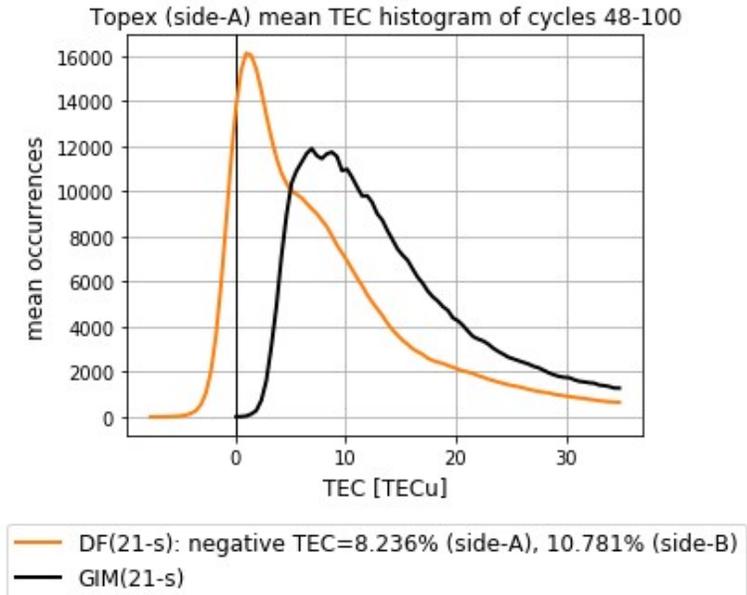


From the I_{rel} equation on the previous slide, we can write-out the relative range+SSB bias in terms of dual-frequency-derived TEC ($\Delta\text{TEC}_{\text{DF}}$) as:

$$I_{\text{rel}} = [(R_{\text{ku}} + \text{SSB}_{\text{ku}}) - (R_{\text{c}} + \text{SSB}_{\text{c}})] + [(\Delta R_{\text{ku}} + \Delta \text{SSB}_{\text{ku}}) - (\Delta R_{\text{c}} + \Delta \text{SSB}_{\text{c}})] \\ = -1.2160 * (\text{TEC}_{\text{DF}} + \Delta \text{TEC}_{\text{DF}})$$

in cm

with TEC in TECu = $1\text{e}16$ el/m²



Calibrating the ionosphere correction

for each cycle in N total cycles:

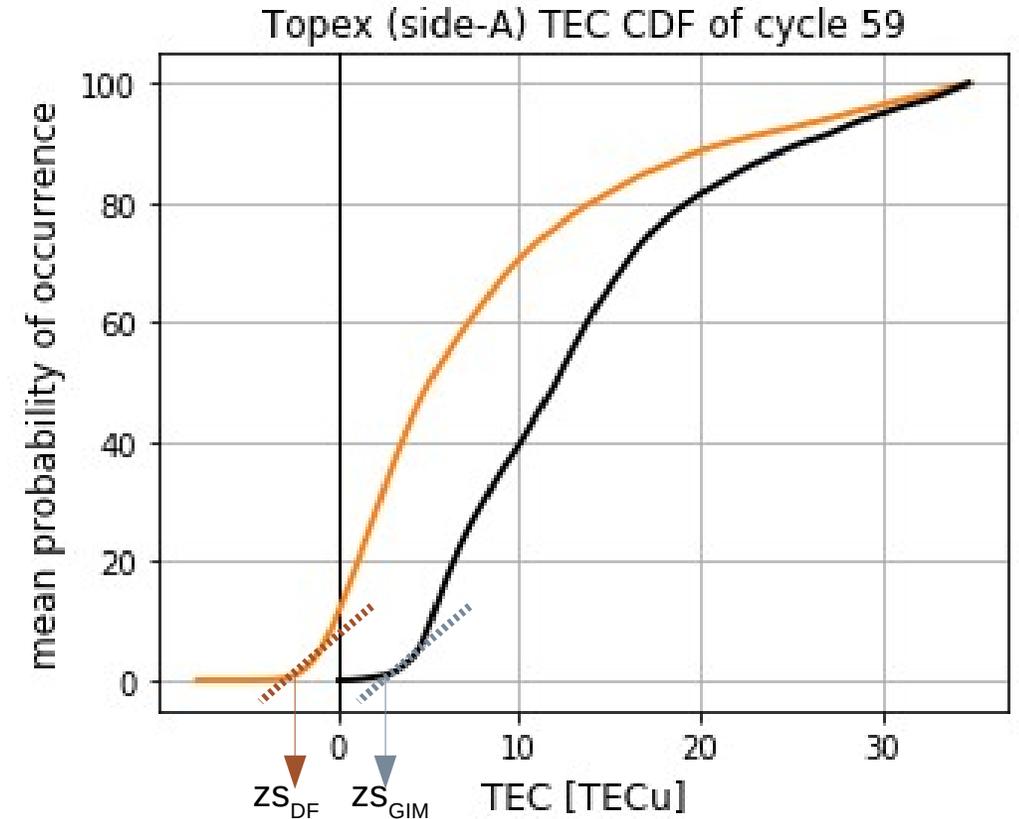
smooth the GIM and DF ionosphere correction using a 21-s running window and convert the ionosphere corrections to TECu

separately assemble both ionosphere correction datasets into CDFs with a resolution of 0.457 TECu

fit a linear polynomial between the **1%** and **10%** of both CDFs, using the CDF as the independent variable.

use **zero-significance** (zs) to evaluate each polynomial at 0-percentile.

$$\Delta\text{TEC}_{\text{DF}} = \sum(zs_{\text{GIM}} - zs_{\text{DF}})$$



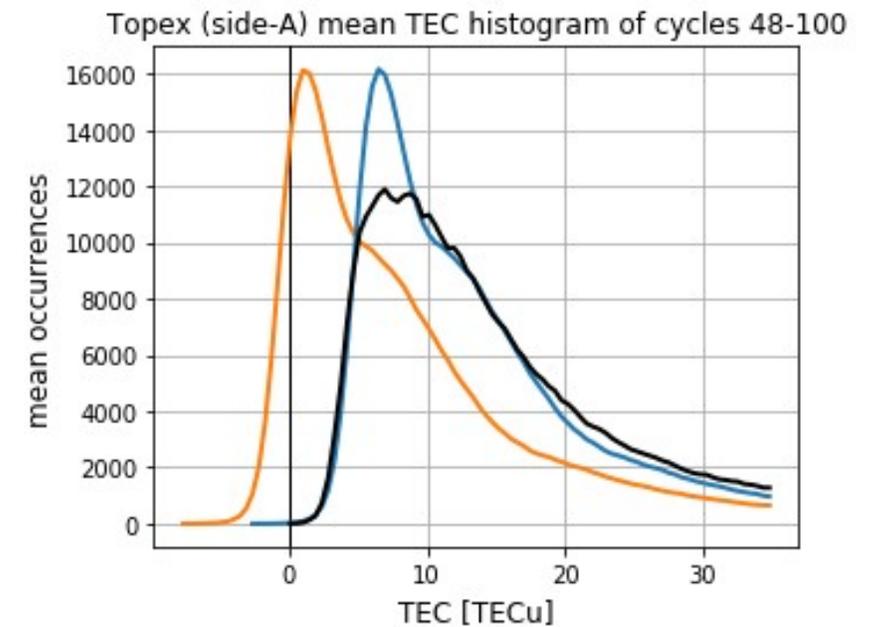
GIM is used as a reference to calibrate the DF ionosphere correction.

Calibrated ionosphere correction

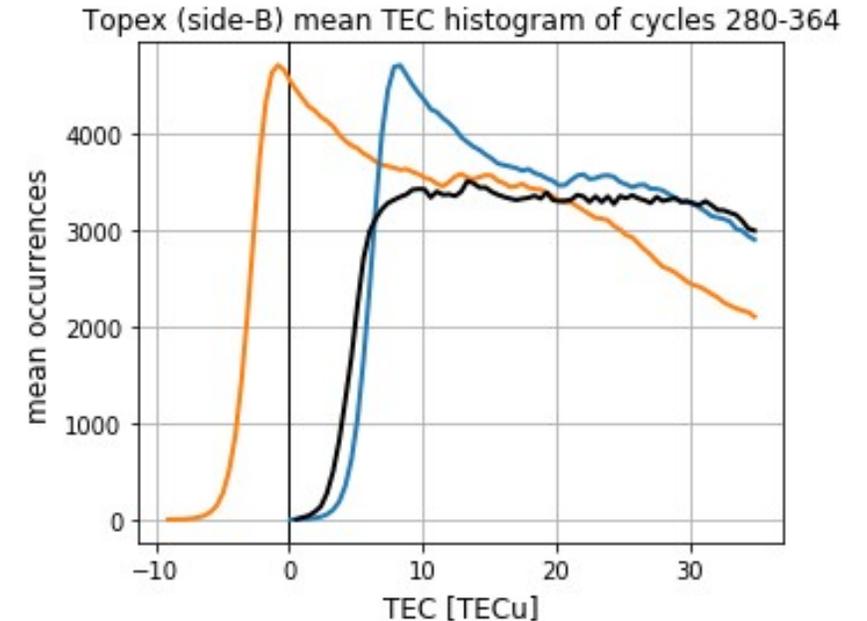
TOPEX (side A)			
$\Delta\text{TEC}_{\text{DF}} = X \text{ TECU}$	$\Delta I_{\text{rel}} = -1.216 * X$	$\Delta I_{\text{ku}} = -0.2187 * X$	$\Delta I_{\text{c}} = -1.4347 * X$
5.499 TECU	-6.687 cm	-1.203 cm	-7.889 cm
TOPEX (side B)			
$\Delta\text{TEC}_{\text{DF}} = X \text{ TECU}$	$\Delta I_{\text{rel}} = -1.216 * X$	$\Delta I_{\text{ku}} = -0.2187 * X$	$\Delta I_{\text{c}} = -1.4347 * X$
8.753 TECU	-10.644 cm	-1.914 cm	-12.558 cm

Calibration bias implementation (slides 25 and 26):

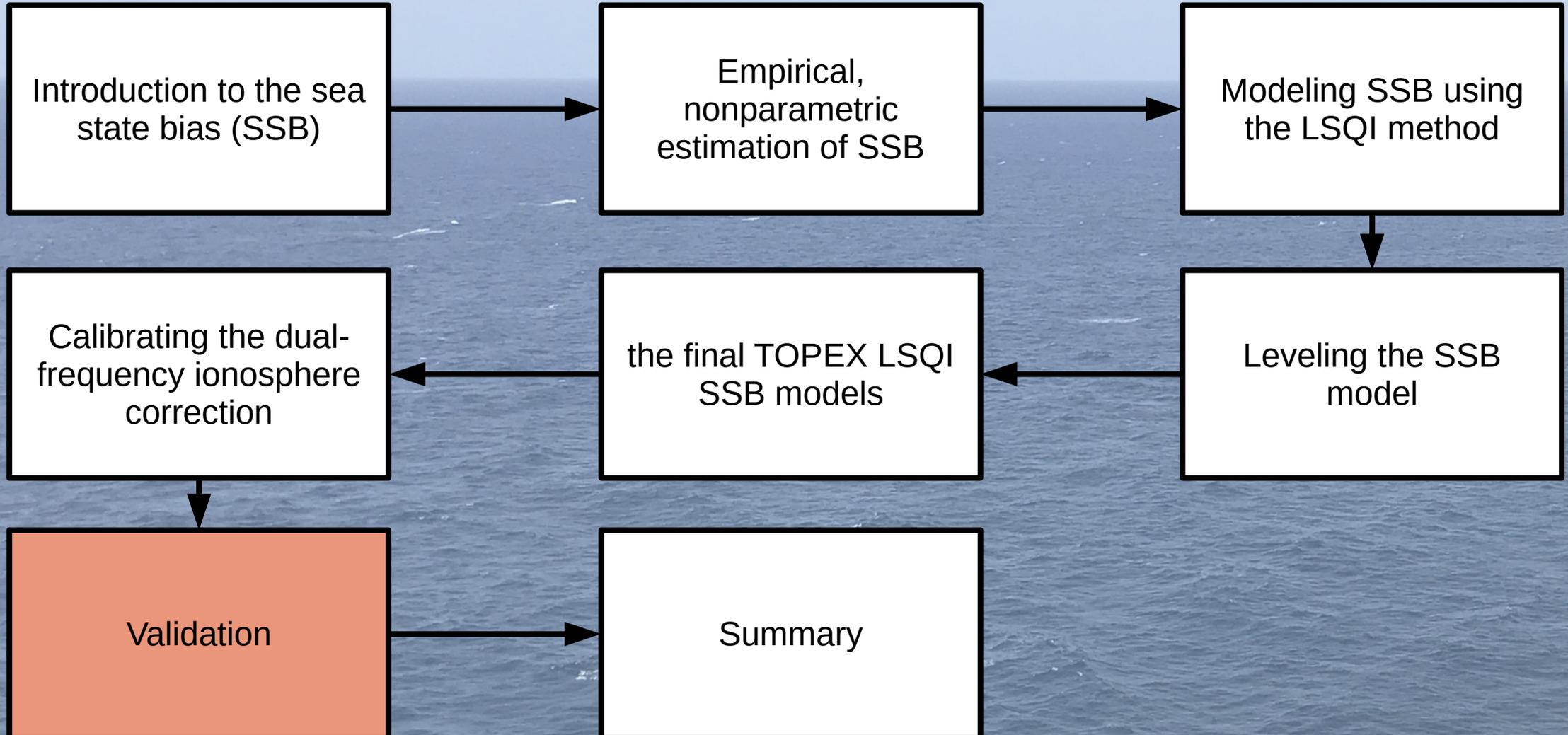
$$I_{\text{rel}} = [(R_{\text{ku}} + \text{SSB}_{\text{ku}}) - (R_{\text{c}} + \text{SSB}_{\text{c}})] + \Delta I_{\text{rel}}$$



- DF(21-s): negative TEC=8.236% (side-A), 10.781% (side-B)
- calibrated DF(21-s): negative TEC=0.01% (side-A), 0.001% (side-B)
- GIM(21-s)



Presentation Outline



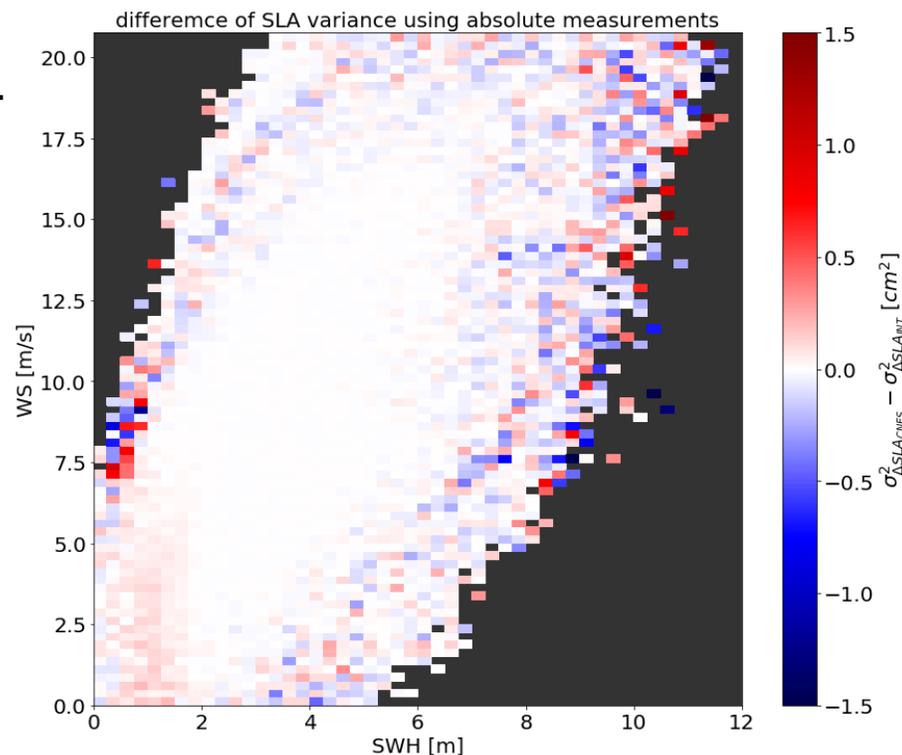
Validation using Jason-2 data

Validation approach:

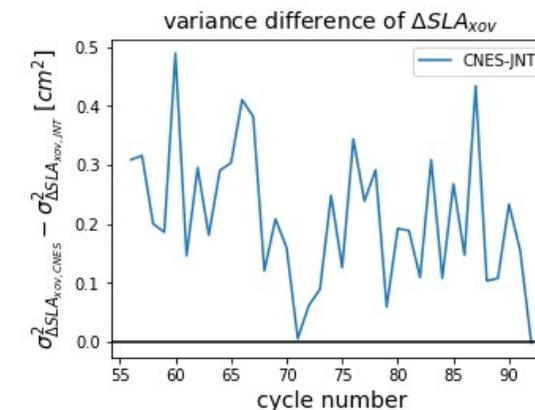
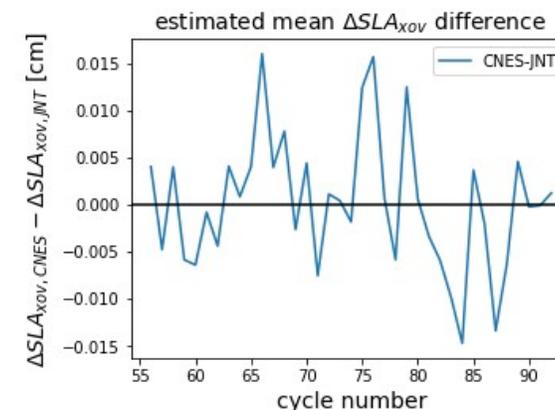
A separate LSQI model was created using the same cycles (1-36) as the CNES SSB model for Jason-2.

1. evaluate SSB for cycles 107-143 (year 2019).
2. compute the dual-frequency ionosphere correction and apply the ionosphere calibration bias
3. find SLA provided the calibrated ionosphere correction and sea state bias.
4. determine the variance of the SLA estimates for each observed node within the 2D model and per cycle.

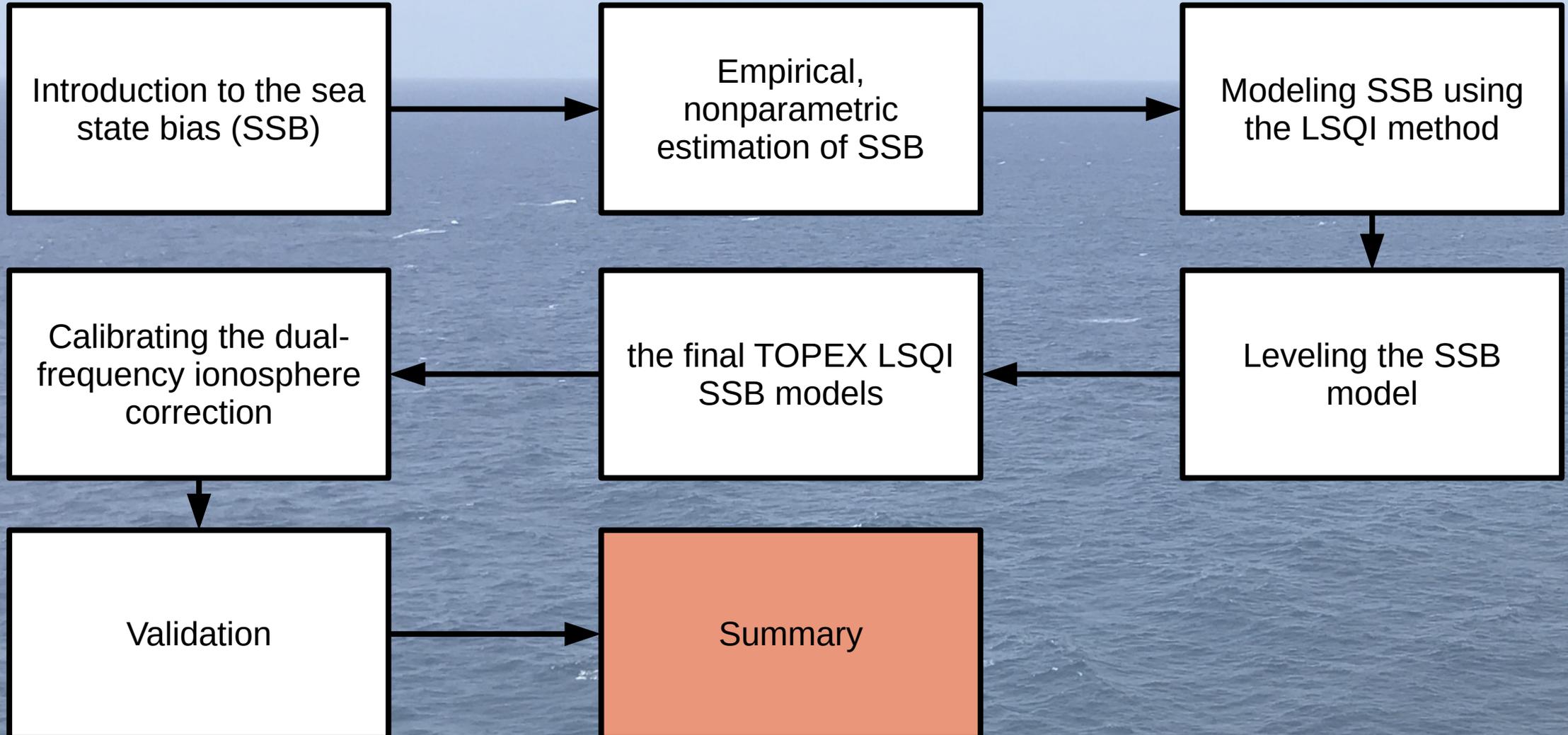
Our LSQI model has similar performance to the model provided on the Jason-2 GDR-D products.



ΔSLA_{xov} comparison using crossover measurements
CNES Jason-2 SSB model vs. joint-interpolation (JNT) Jason-2 SSB model



Presentation Outline



Summary

The LSQI method was developed as a simple and efficient approach to nonparametric SSB modeling, that also allows for a direct means to generate an SSB error budget.

The joint measurements provided as the observables to the model consist of both crossover and collinear difference measurements. The measurement combination utilizes the high temporal resolution of the crossover measurements, along with the high spatial resolution of the collinear measurements.

Post-processing of the raw LSQI model included leveling the SSB model using the zero significance of the cumulative distribution function to ensure that the majority of SSB correction values are negative.

The SSB modeling approach provides two separate solutions - (1) a raw SSB model and (2) a leveled and smoothed/extrapolated SSB model.

The final SSB models in both Ku- and C- band were then used to calibrate the dual-frequency ionosphere correction, and provide an ionosphere correction bias. The ionosphere correction bias is to be applied to the dual-frequency ionosphere correction and is specific to each pair of Ku- and C- band SSB models.

Thank you!

References

Desjonqueres, J.D., Talpe, M., Callahan, P., Desai, S., Willis, J. (2019). TOPEX Data Reprocessing using a Numerical Retracking Approach. Presented at the 2019 Ocean Surface Topography Science Team Meeting in Chicago, IL.

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