Stacking repeat cycles of 40-Hz AltiKa data resolves the geoid anomalies of very small seamounts

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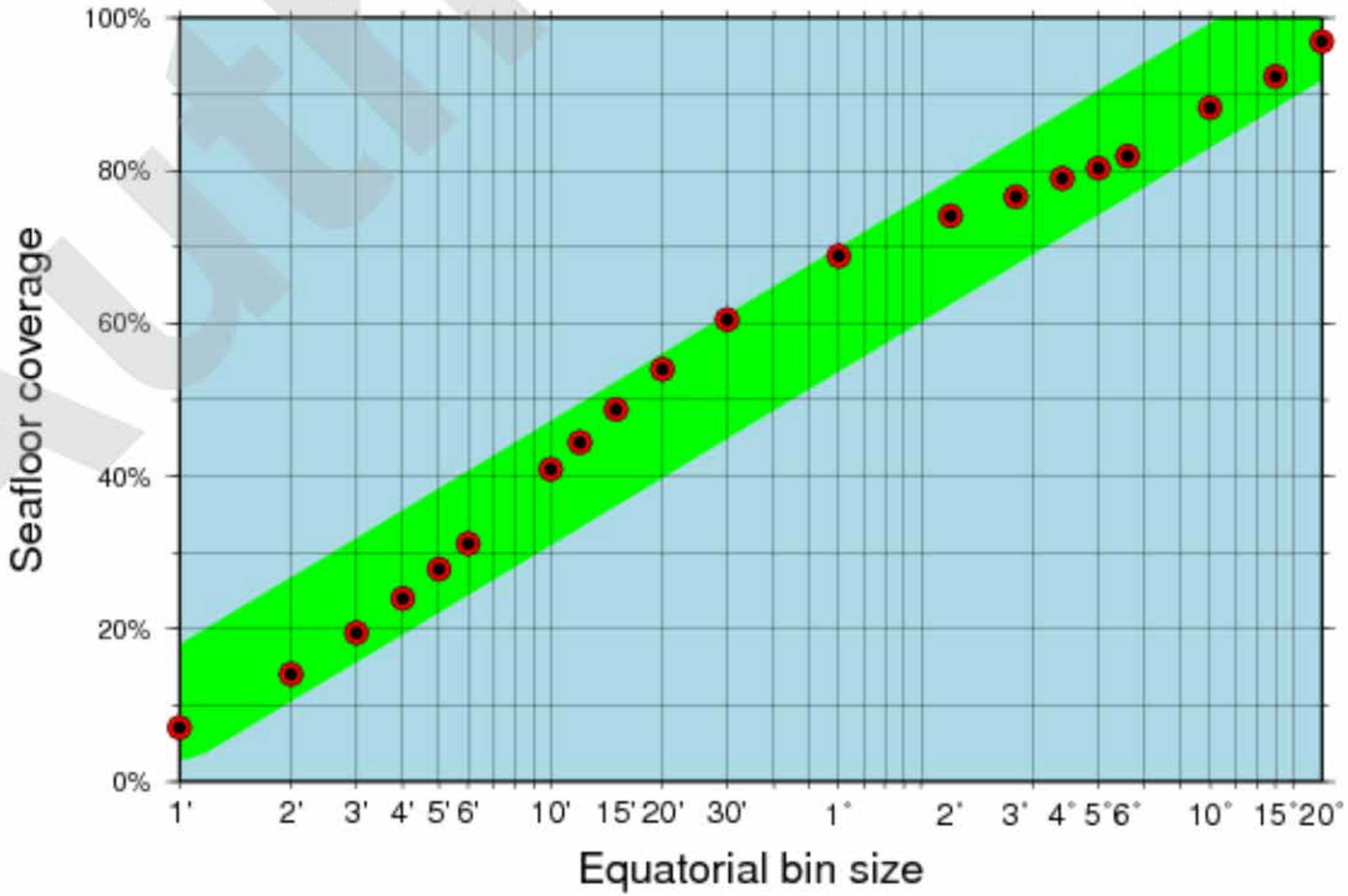
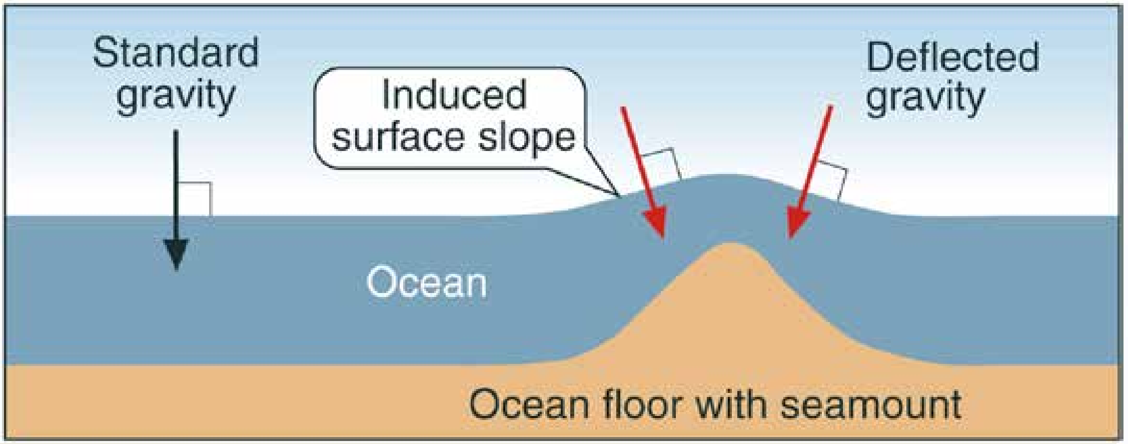
The oceans are not mapped!



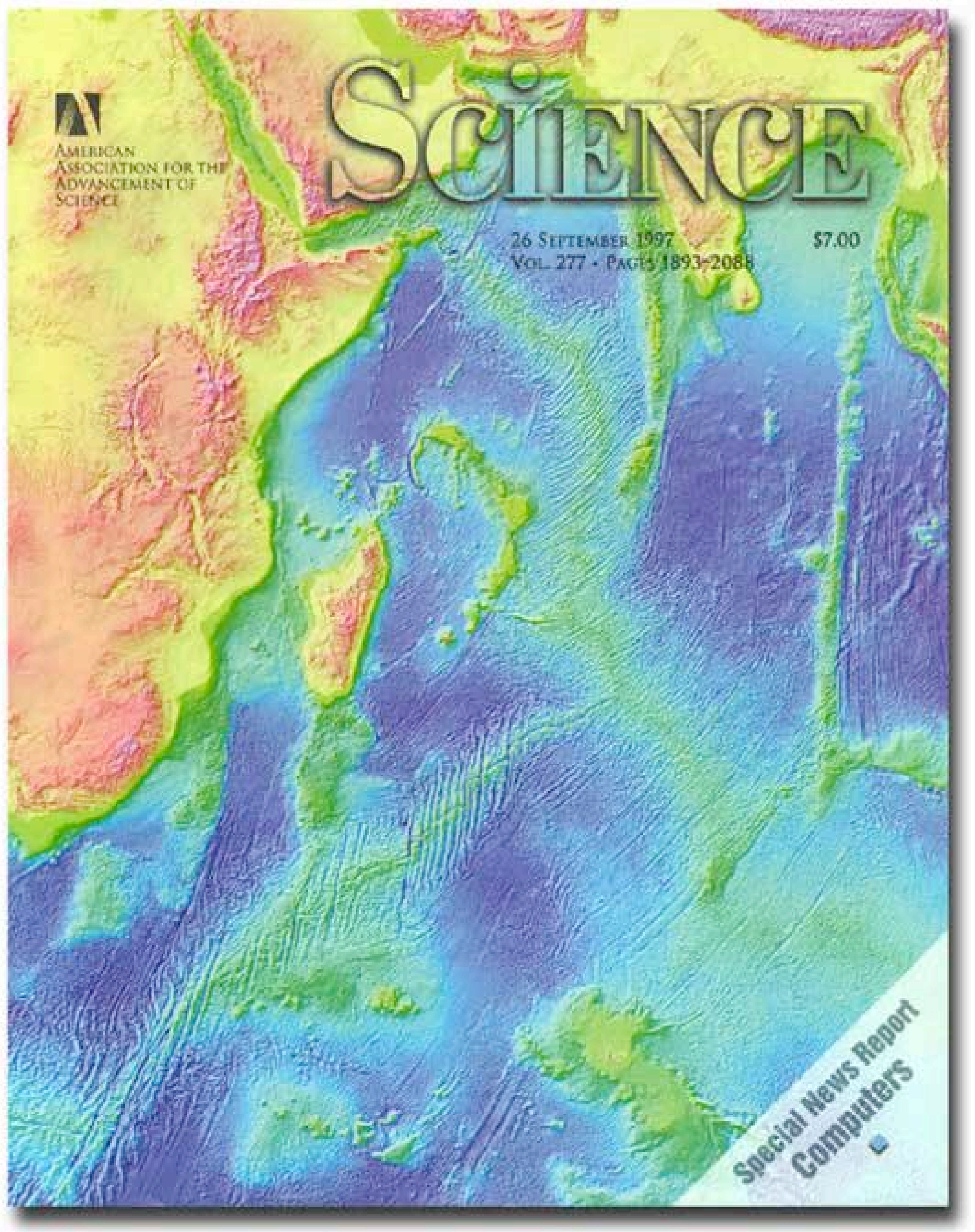
If we “tile” the global seafloor with square tiles one n.m. (1.85 km) wide, more than 90% of the tiles have NO measurements of depth in them!

Figure from Wessel & Chandler, 2011, doi:10.2478/s11600-010-0038-1
Even if we use tiles 2200 km by 2200 km, there are still some empty ones!

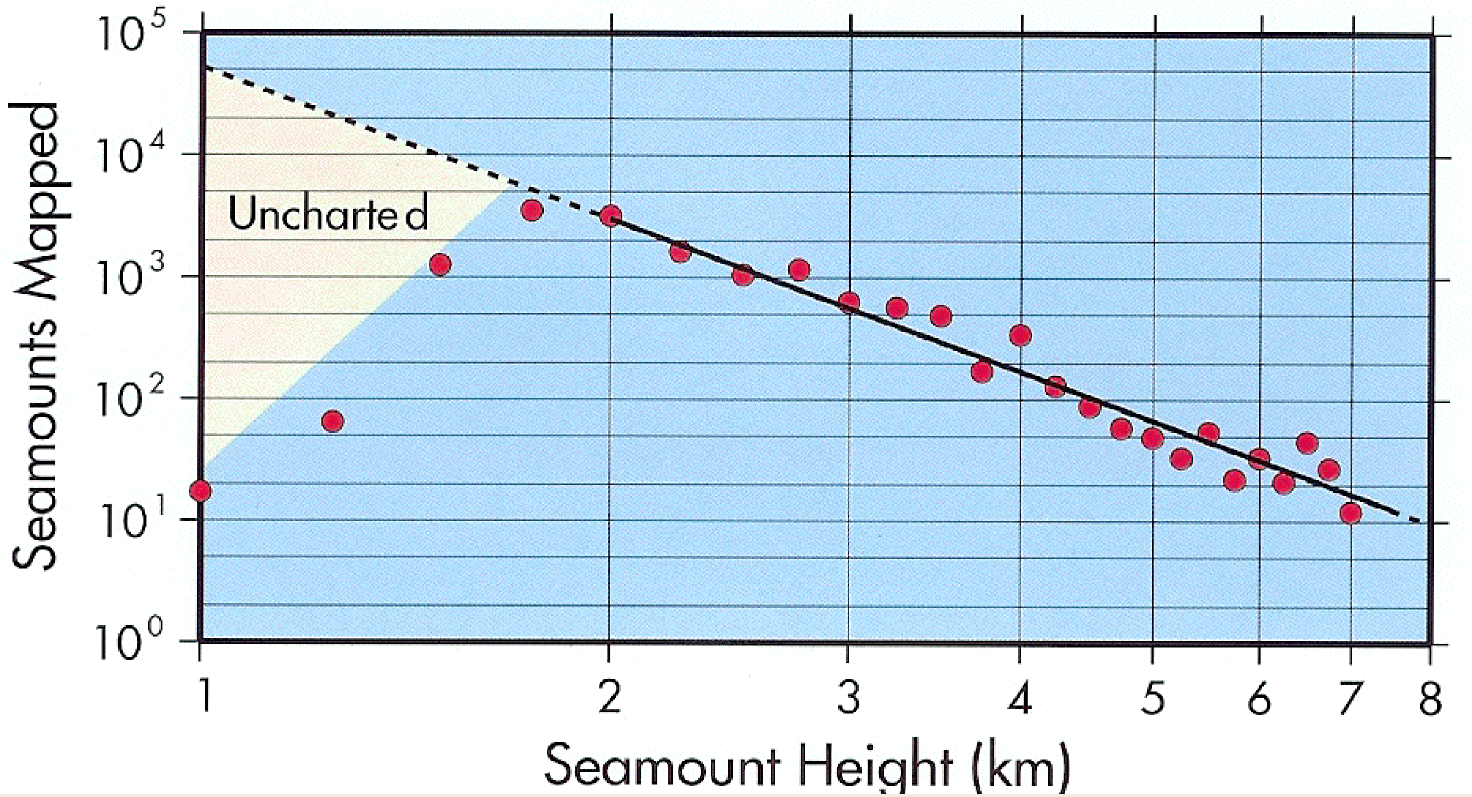
Bathymetry can be inferred from altimetry



- We use geodetic mission altimeter data to guess the shape of the seafloor.
- Altimetric sea surface slopes guide the interpolation of bathymetry to fill the gaps covering 90% of the sea floor area.



Early geodetic altimeter missions didn't find enough seamounts <2km tall (most of them)



Models of the abundance of seamounts versus their size suggests that there are 50k seamounts 1 – 2 km tall. Most of these were missed in early altimetric mapping by Geosat and ERS-1.

Figure here from a power-law ("fractal") model from Wessel [2001, doi:10.1029/2000JB000083]. One can also fit a Poisson model [Smith and Jordan, 1988, doi:10.1029/2000JB000083]. The models differ when extrapolated to extremes, but both models suggest that there are about 50,000 seamounts in the global ocean in the height range between 1 and 2 km tall.

How can altimeters find more seamounts?

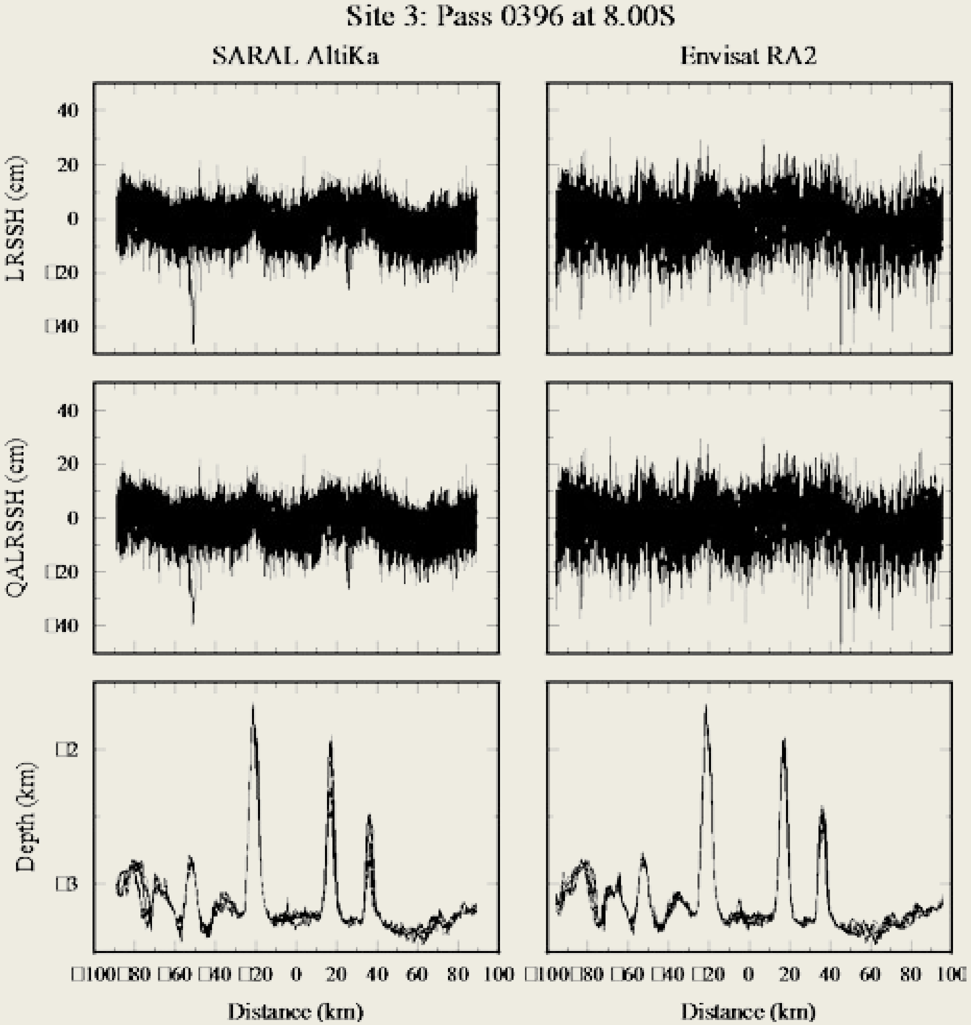
- *Finding* seamounts requires altimeters with better precision and along-track resolution.
  - Envisat, Jason-1, Jason-2 and CryoSat are better than ERS-1 and Geosat.
  - This study looks at whether SARAL/AltiKa is even better still.
- *Mapping* seamounts requires a geodetic orbit.
  - The Jason-1 EoL GM and CryoSat have helped.
  - We need a Jason-2 EoL GM (as long as possible).
  - We would love a (long) SARAL/AltiKa EoL GM also.

Pass #0396 @8˚S: 3 small seamounts

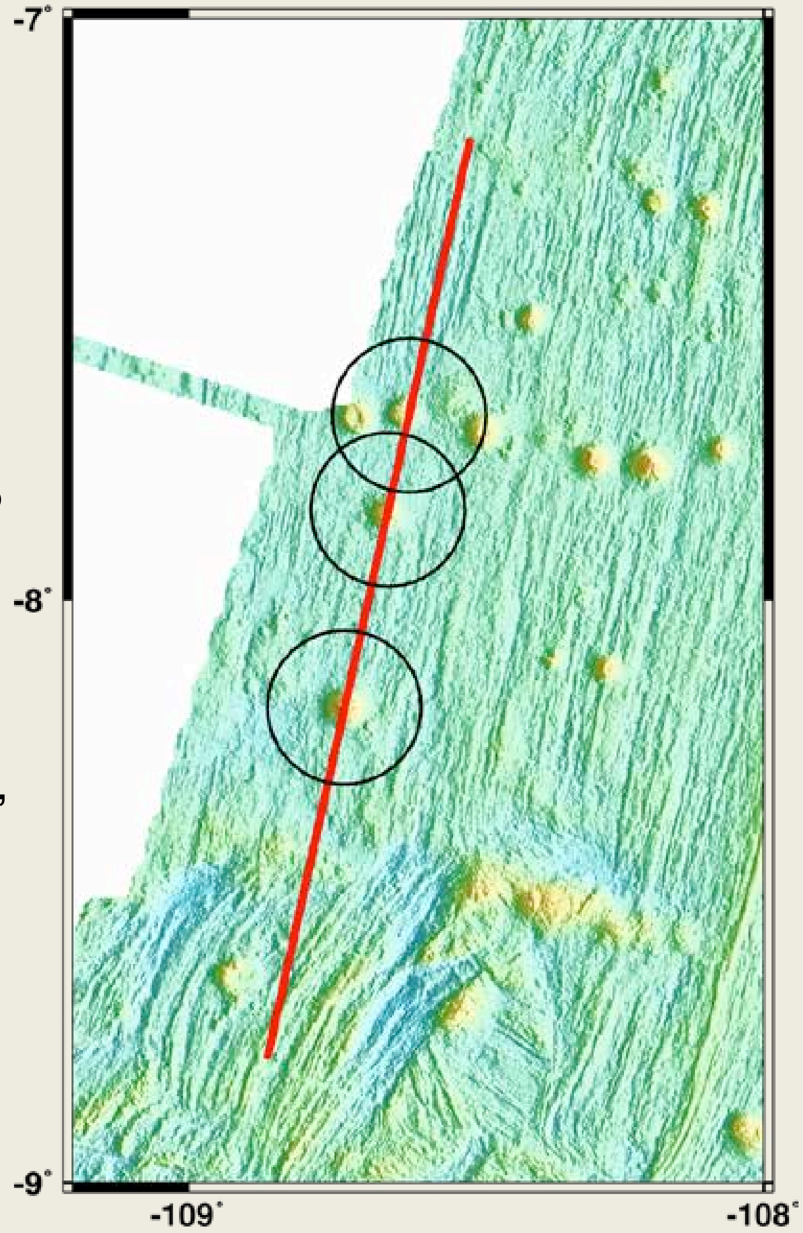
Sea level (geoid) anomaly over Seamounts. 12 repeat tracks.

AltiKa samples @ 40 Hz rate

Ku samples @ 18 Hz rate



Left: Figure from Smith, doi:10.1080/01490419.2015.1014950. Right: bathymetric survey by Cochran et al., image by Karen Marks.

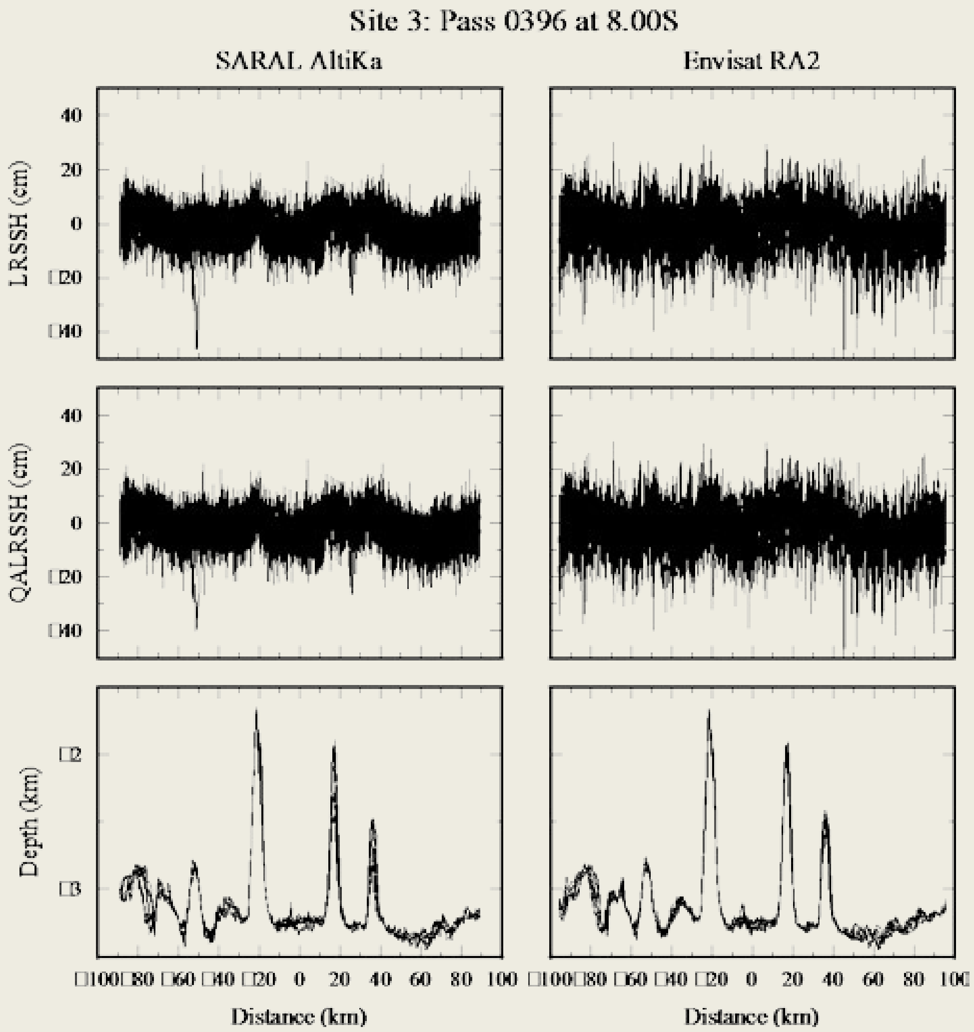


Incoherent Ka beats Incoherent Ku

Sea level (geoid) anomaly over Seamounts. 12 repeat tracks.

AltiKa samples @ 40 Hz rate

Ku samples @ 18 Hz rate



Range precision in a 1-s average of sea level: 0.79 cm for AltiKa; 1.89 cm for Envisat.

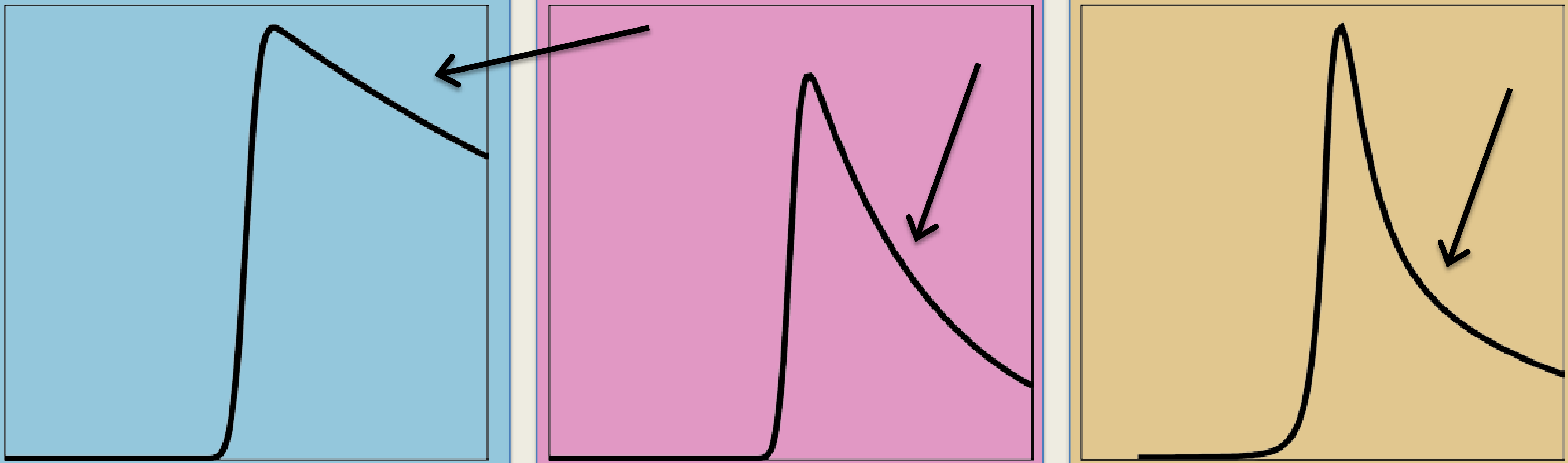
This is better by more than a factor of √(1.8/4), the ratio of their PRFs.

The extra precision comes from AltiKa's more impulse-like waveform.

For details see W H F Smith talk at December 2014 AGU meeting.

Figure from W H F Smith, AltiKa special issue, doi:10.1080/01490419.2015.1014950

Altimeter Waveforms trailing edges



Conventional

- Ku band
- Pulse limited
- Incoherent processing

SARAL AltiKa

- Ka band
- Pulse limited
- Incoherent processing

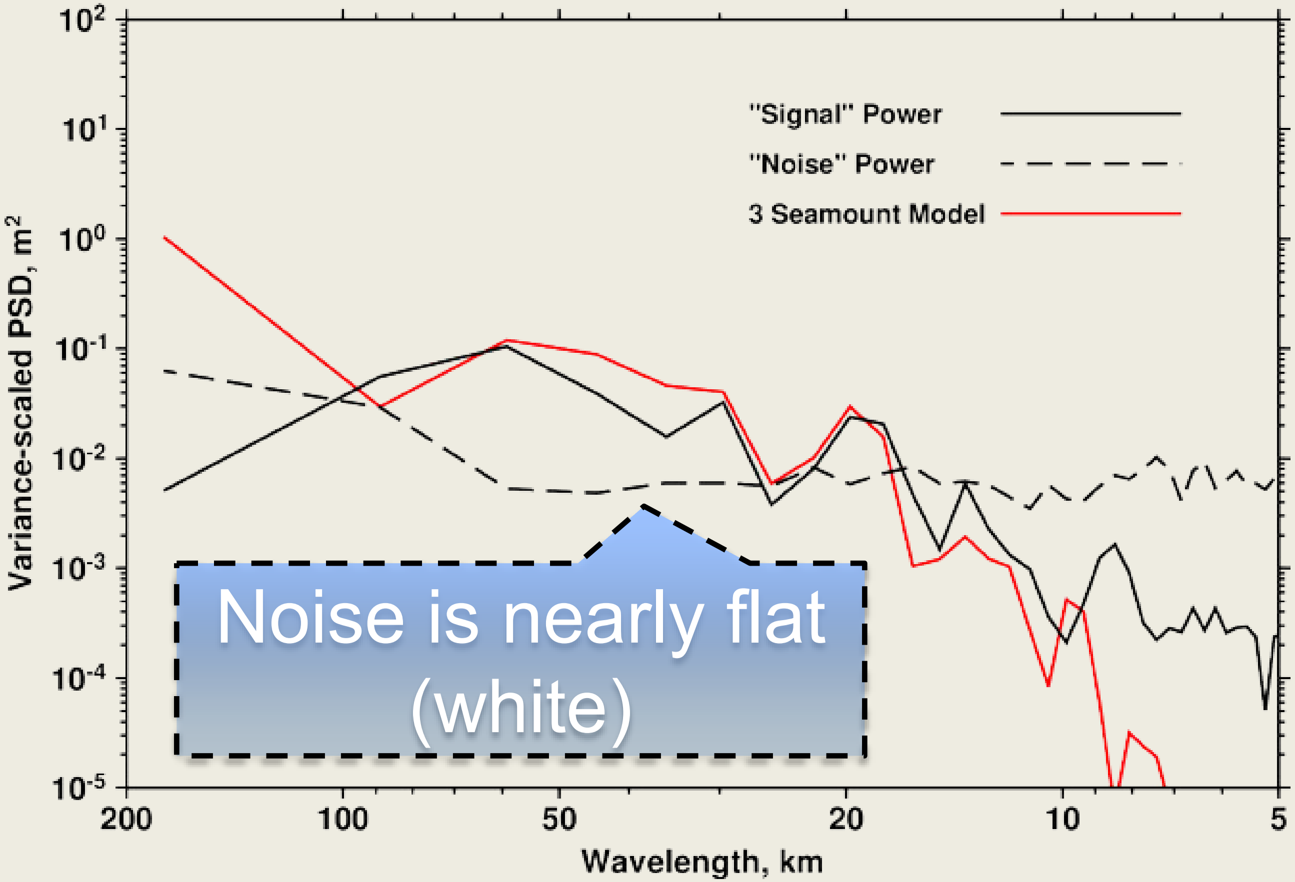
Also beam limited due to shorter wavelength

Delay-Doppler

- Ku band
- Pulse limited
- Coherent processing makes it beam limited along-track.

Ku Delay/Doppler & Ka incoherent are similar in shape. Both have the rapidly decaying tail that improves precision. AltiKa footprint is circular; D/D is not.

Small seamount signal (barely) > noise, in a single pass! (Motivation for stacking, in next few slides.)



- Treating each repeat cycle as a random realization of geoid plus "noise" (all that isn't geoid).

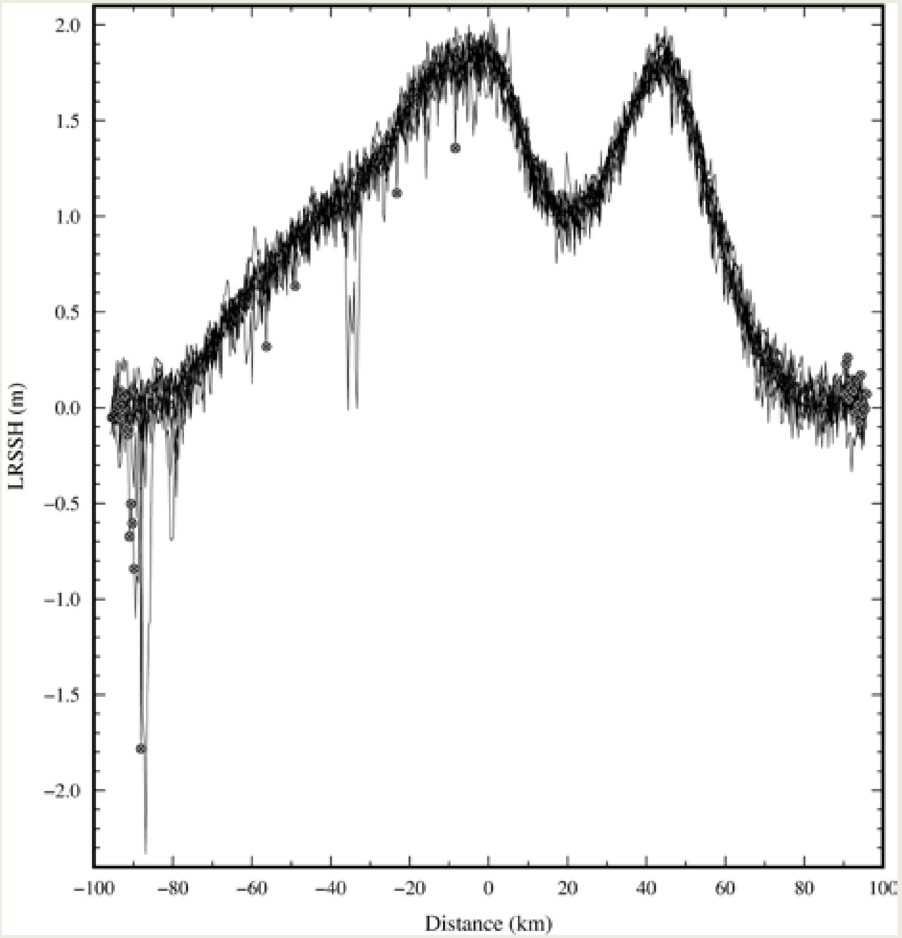
The paper in the AltiKa special issue [doi:10.1080/01490419.2015.1014950] assesses seamount "resolution" (scale bands where "signal" is above "noise" assuming that we have only one pass over any target, as would be the case if SARAL AltiKa were to do an EoL GM. For that study, repeat cycles were treated as if they were realizations of geoid ("signal") plus non-geoid ("noise"). Conclusion: An AltiKa EoL GM would be superb for mapping seamounts!

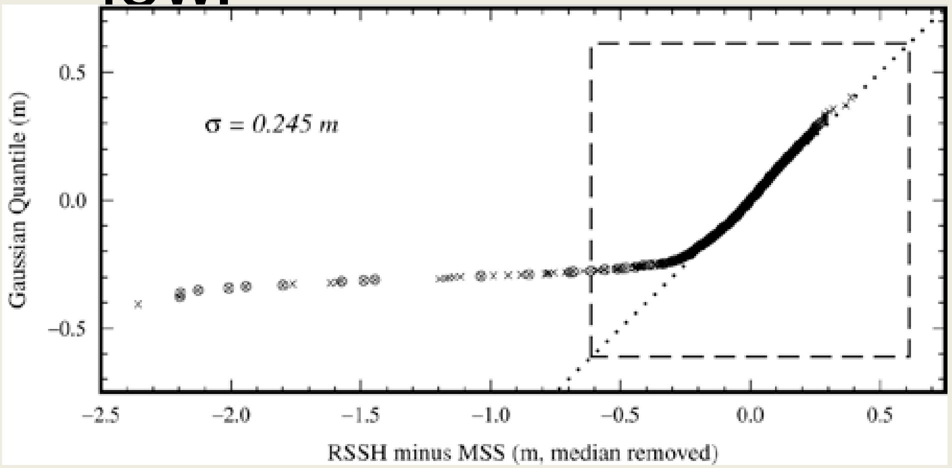
How much better can we do by stacking?

- Stacking (combining repeat cycles into a new mean [median] sea surface at 40 Hz)
- We make two stacks of a track, not one. The cross-spectrum between the two gives:
- *Geoid:geoid* -> altimeter measurement *signal:noise* (repeatable:not repeatable). [Can be done with any pass.]
- *Geoid:bathymetry* -> measurements *correlated:uncorrelated* with bathymetry. [Can be done in only a few limited areas where we have independent bathymetric survey coverage.]

Outliers -> stack median, not mean, height

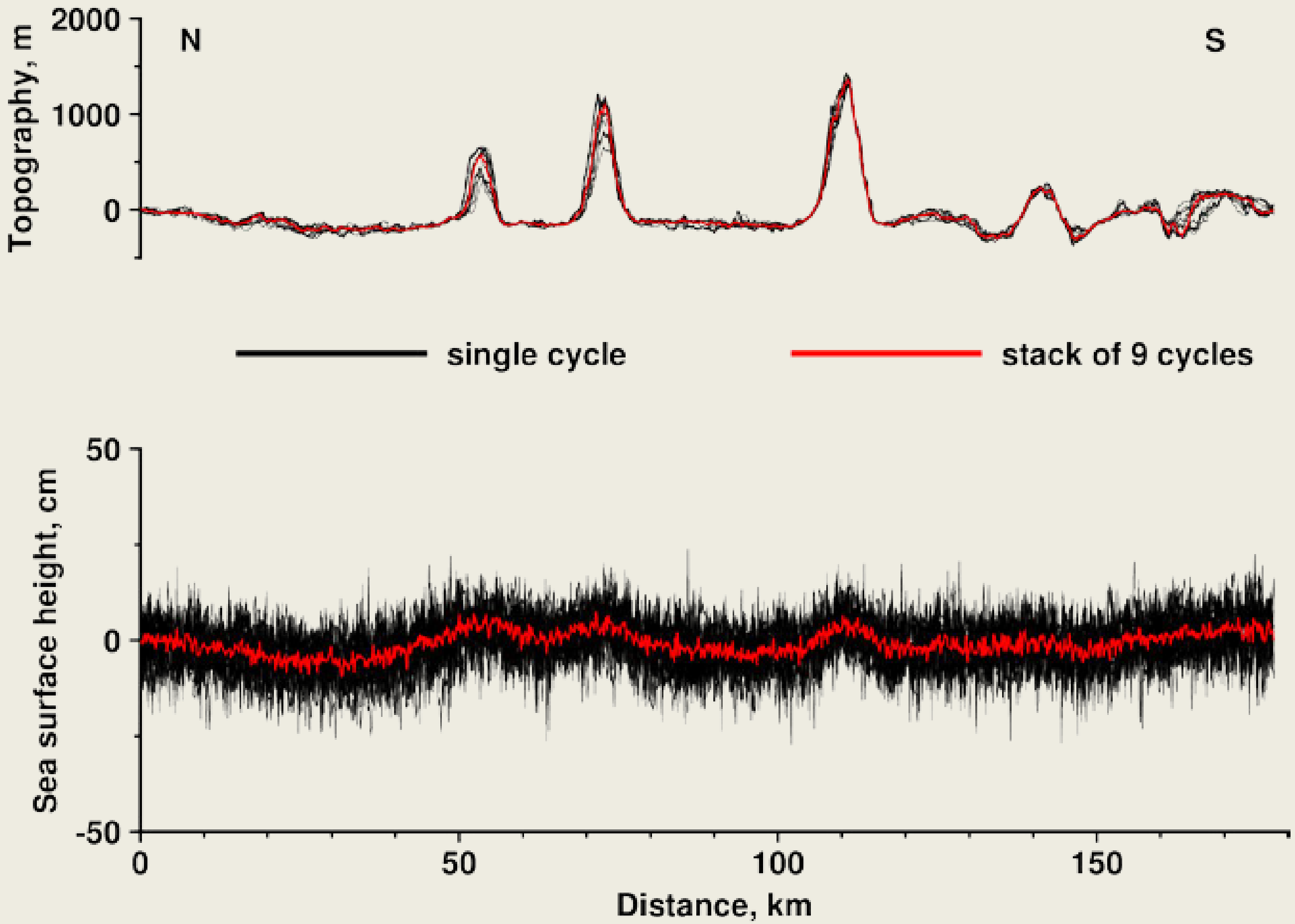
- Both SARAL AltiKa and Envisat RA2 data show outliers.
- Rain flag doesn't catch all outliers and seems to flag some apparently valid data.
- Outliers always seem to have the same sign: sea level too low.

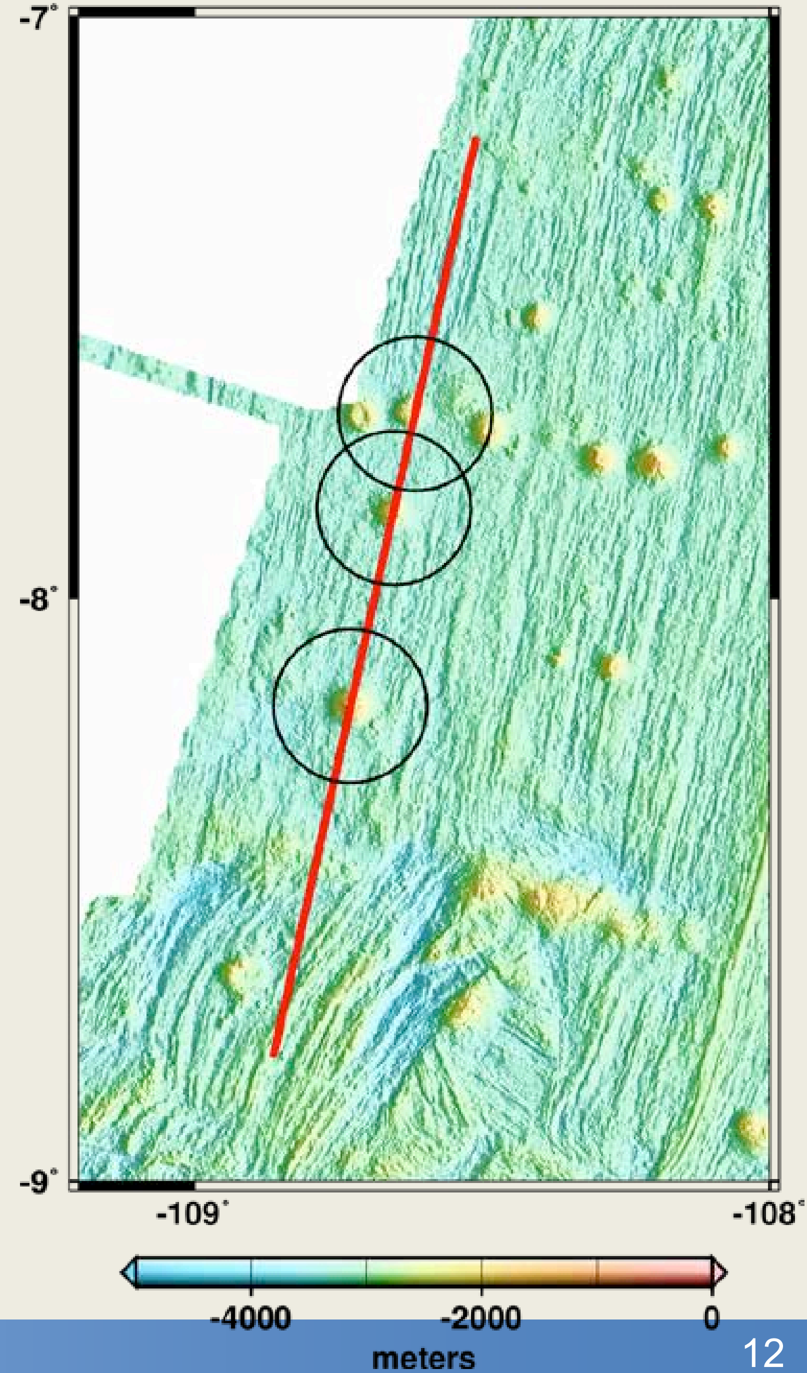




Figures from W H F Smith, AltiKa special issue, doi:10.1080/01490419.2015.1014950

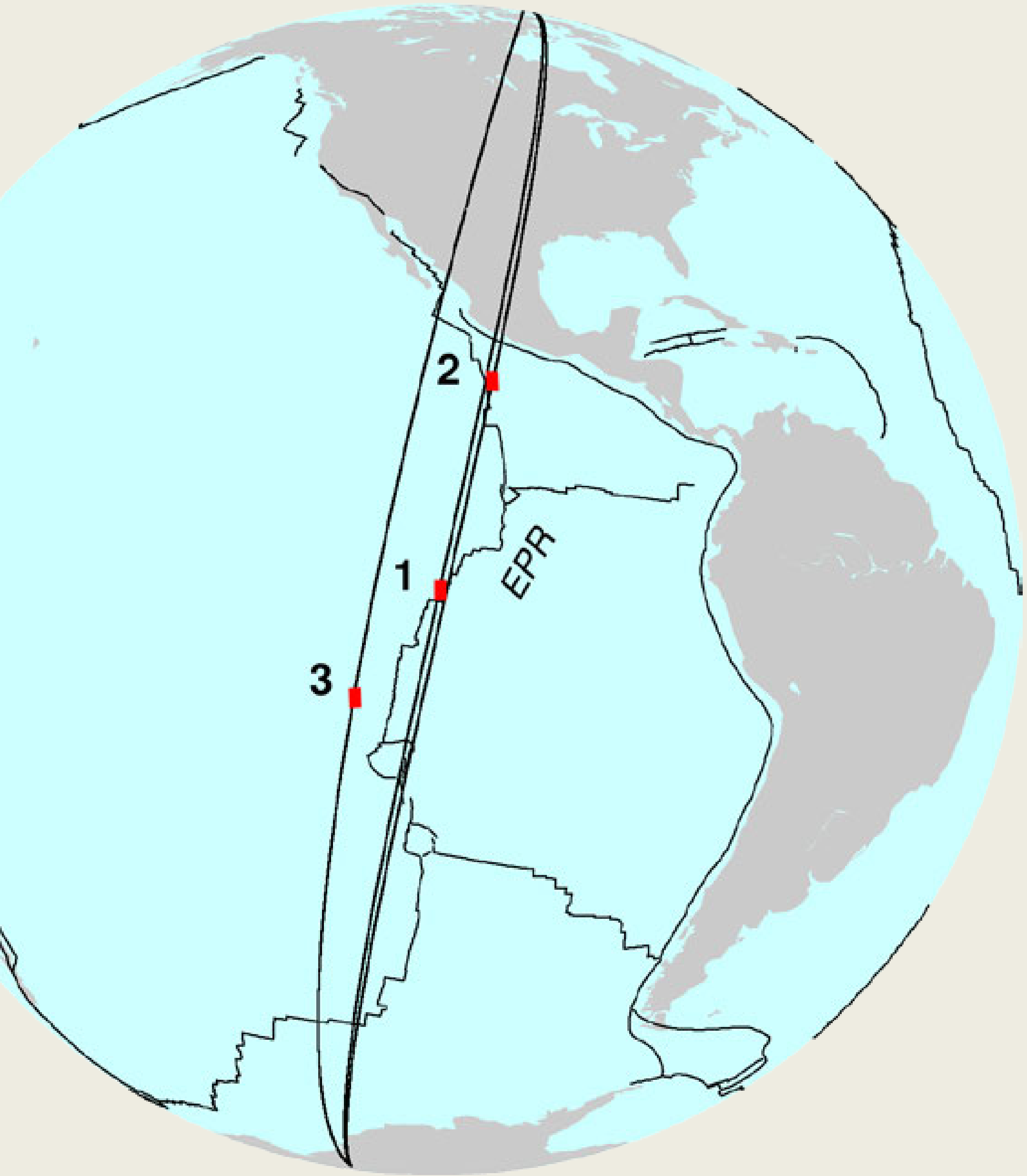
Stacking median sea level at 40 Hz





- Each pass is first “leveled” to remove long (>200 km) differences between SSH and MSS.
- Leveled passes are then “stacked” by taking the median of data closest in latitude (@ 40 Hz).

Coherency tests in 3 areas



We will look at 3 areas having multibeam bathymetric survey data with seamounts under SARAL/AltiKa 35-day repeat tracks.

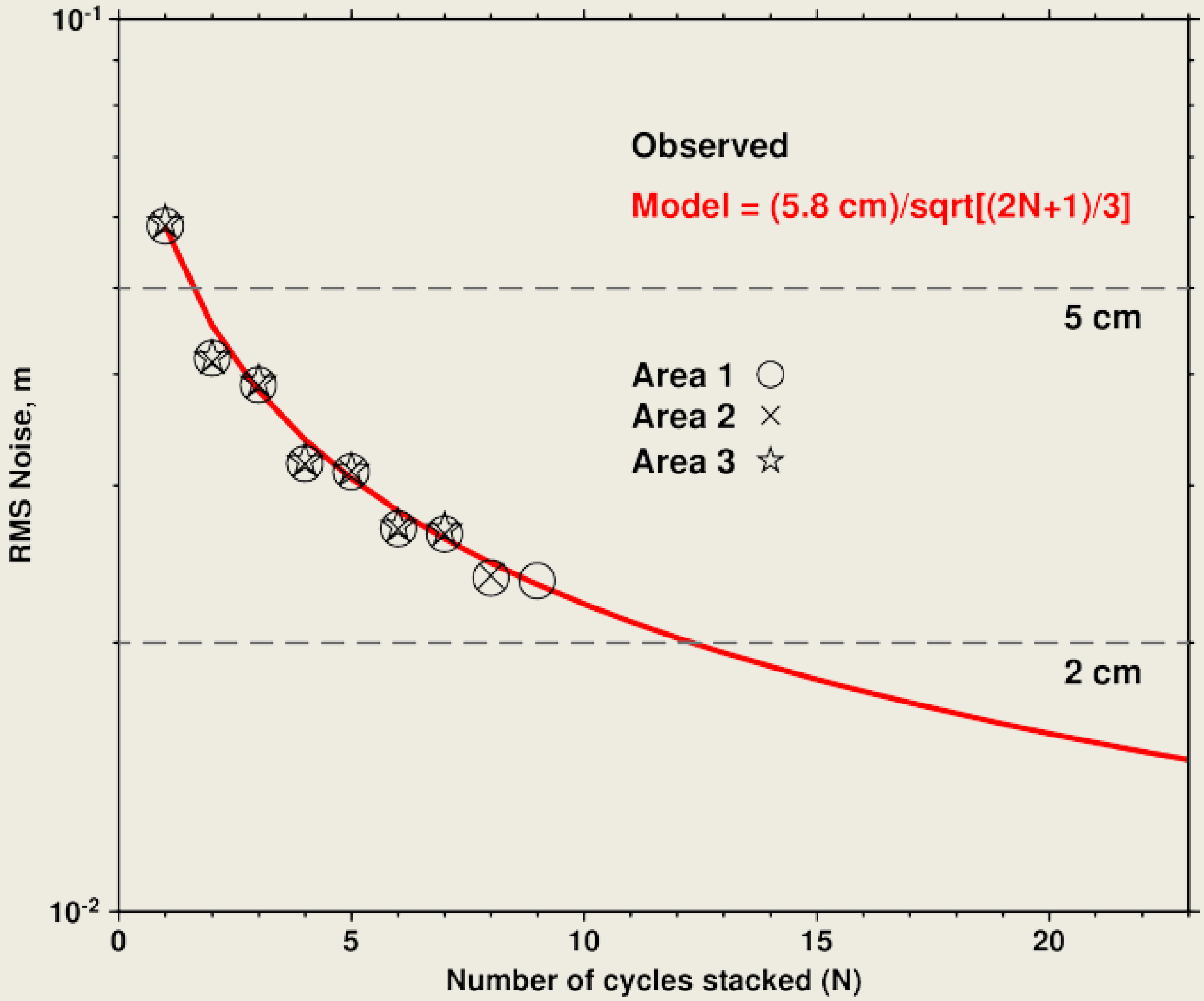
The three areas have seamounts of different sizes.

The size of the seamount is important for the bathymetry : geoid cross-spectrum, because the power in the bathymetry affects the geoid coherence.

Monte Carlo stacking

- We have used 18 (35-day) repeat cycles.
- We generate many random realizations of stacks. In each realization, a repeat cycle may be randomly assigned to either Stack_A or Stack_B, exclusively (the same data doesn't end up in both A and B). The cross-spectrum of Stack_A versus Stack_B, averaged over many realizations, estimates stack signal and noise.
- We build stacks from Nstack = 1 (no stack) to Nstack = 9, to see how signal and noise change as the number stacked increases.

Stacking reduces SSH noise



The noise in individual 40 Hz samples of sea level in any single cycle is around 5.8 cm.

This drops to 2 cm if one stacks a median of 12 cycles.

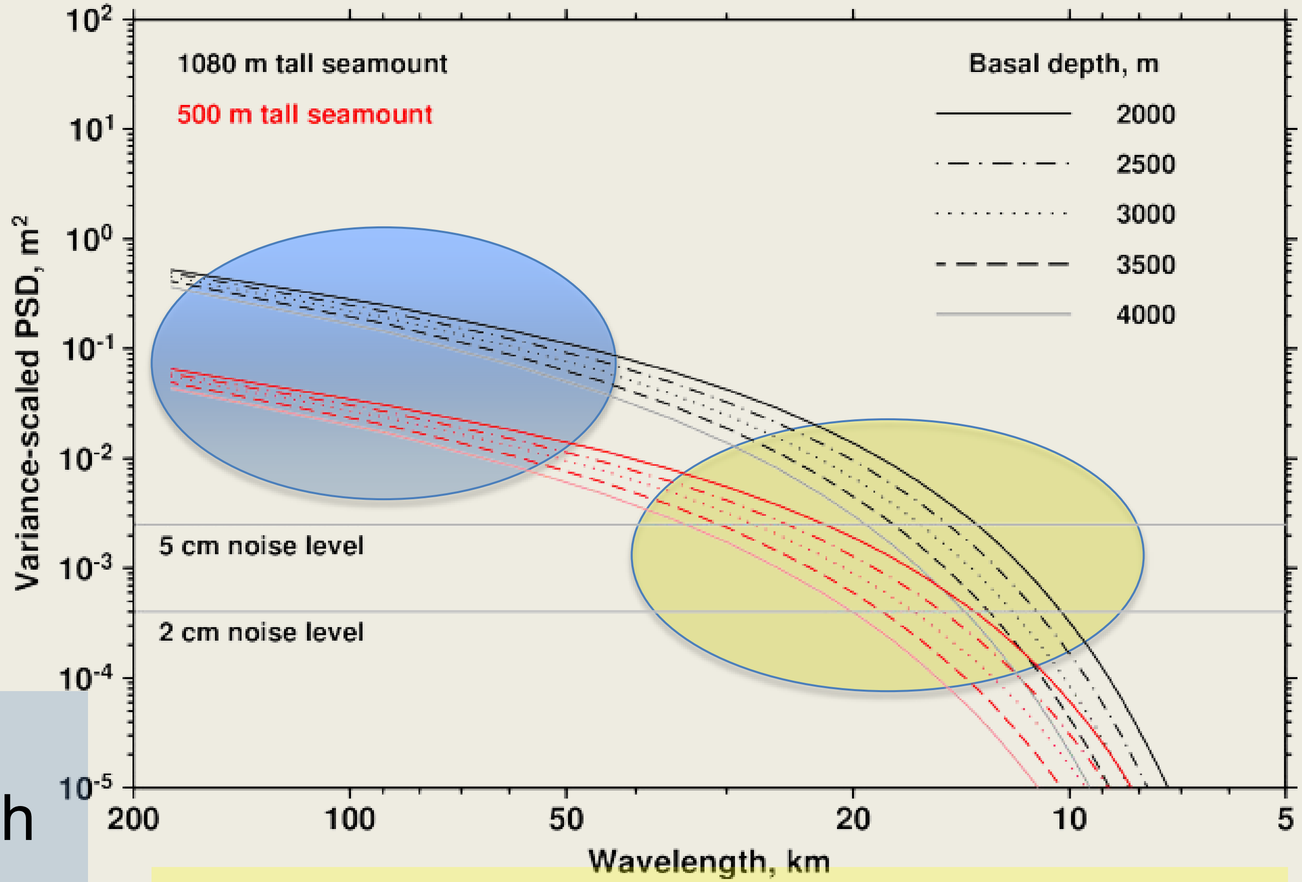
We get the same result from each of the different test areas -> measurement noise is independent of seamount size.

The results are well-described by a model curve for the expected error in a median stack, given the 5.8 cm noise in a single profile.

Variance decreases as 3/(2N+1), not 1/N, because we use medians, not means. The median is less efficient on Gaussian data but much better for data with outliers.

Need low noise to find small seamounts

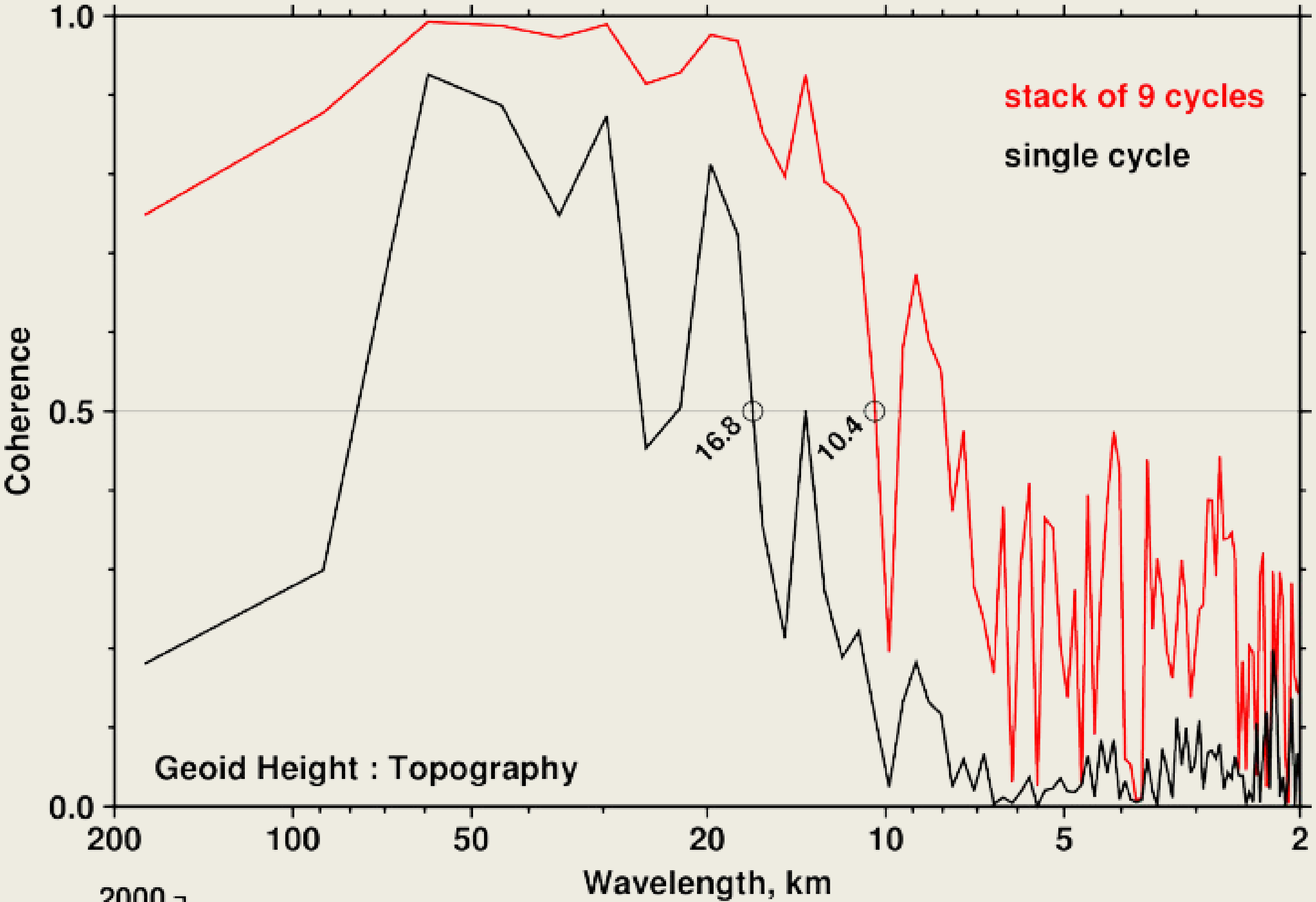
- The size of a seamount has a strong effect on the amplitude of its geoid anomaly.
- The depth of the sea floor around it has a weaker effect.

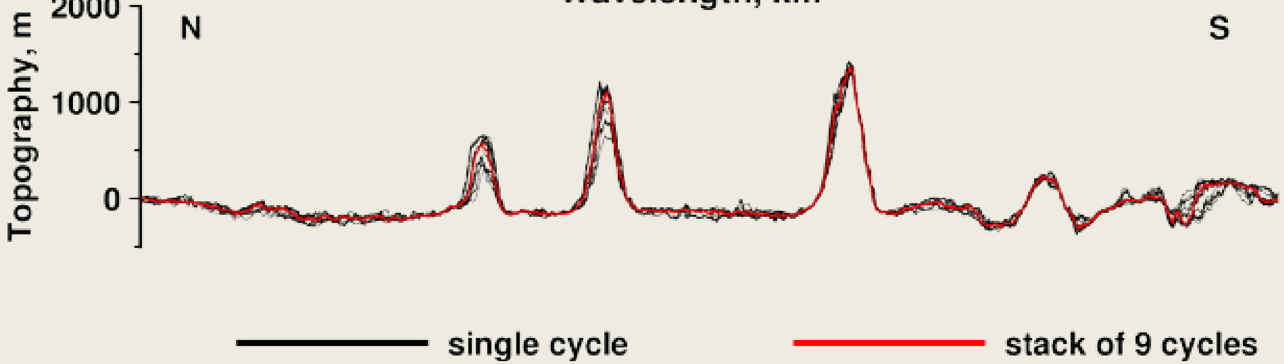


Seamounts contribute some power to the longer wavelength geoid. But these scales also have power from other sources, and anyway they have already been well mapped by previous geodetic altimeter missions.

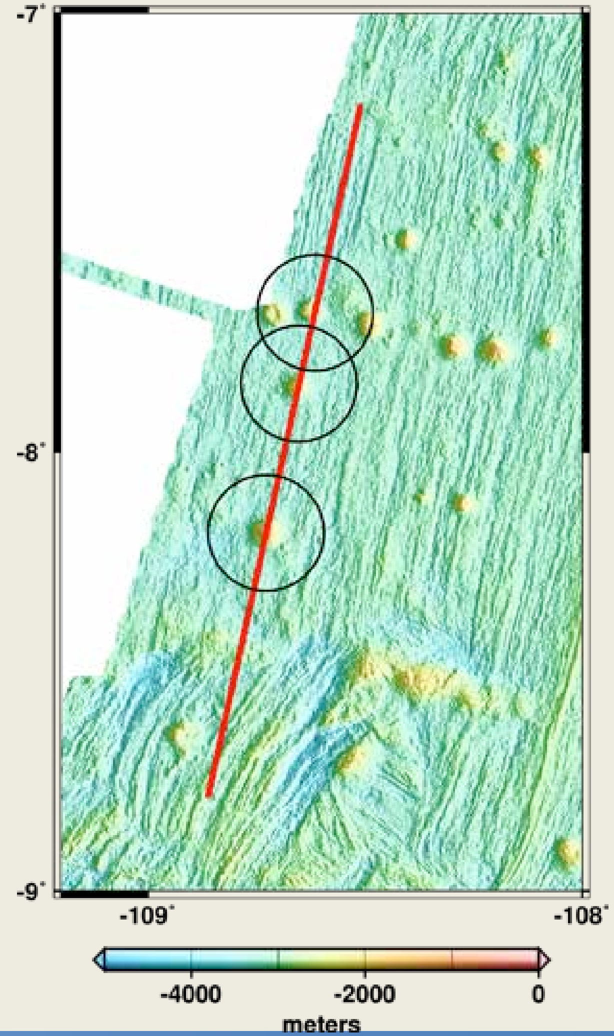
The shorter wavelengths are most important for detecting, resolving, measuring small seamounts. Lowering the noise by stacking should improve the geoid:bathymetry coherency.

Area 1 geoid:bathymetry coherency. 3 seamounts, 700 m to 1.5 km tall.

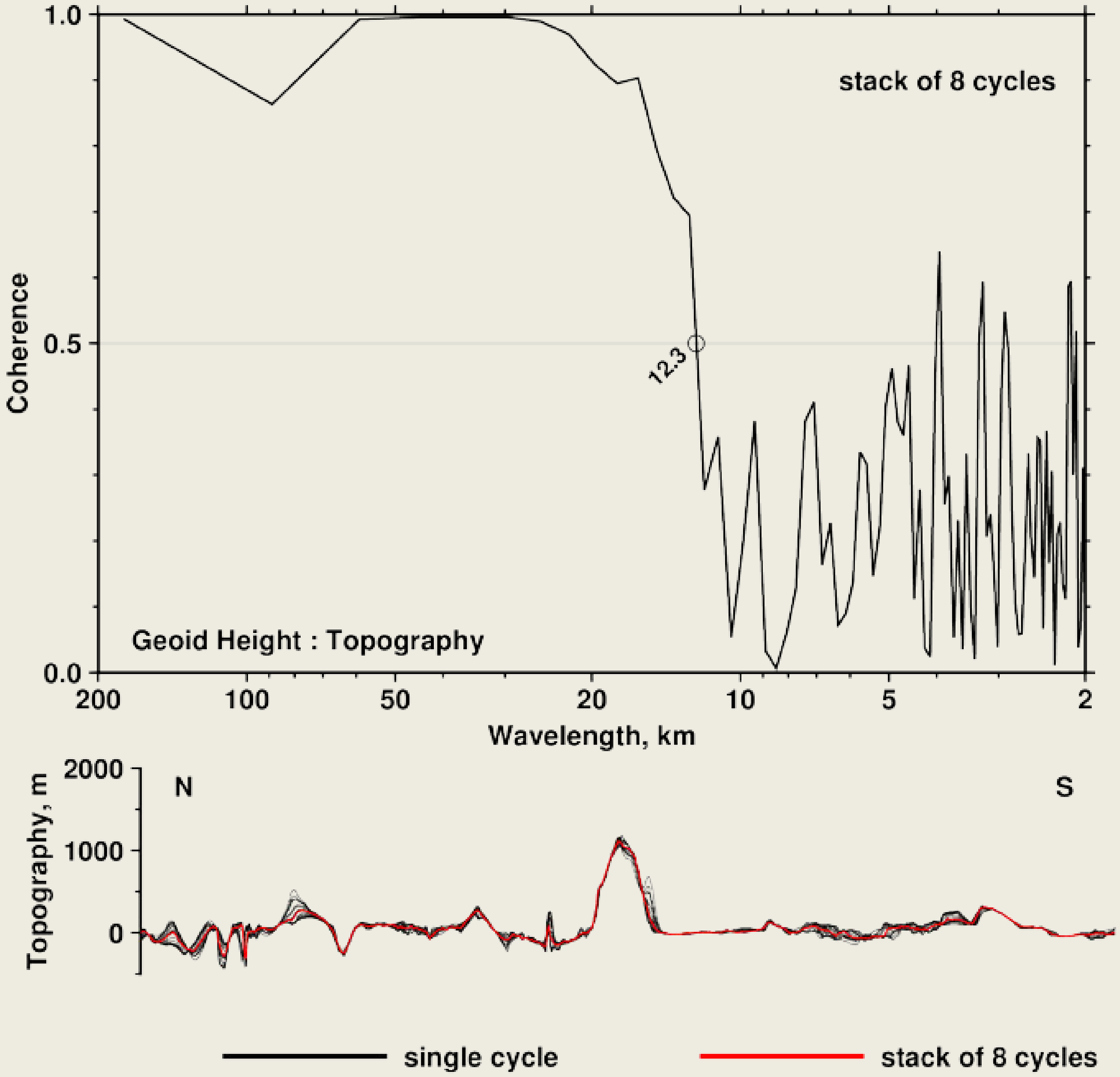




Stacking improves the coherency with bathy especially at 10-30 km.

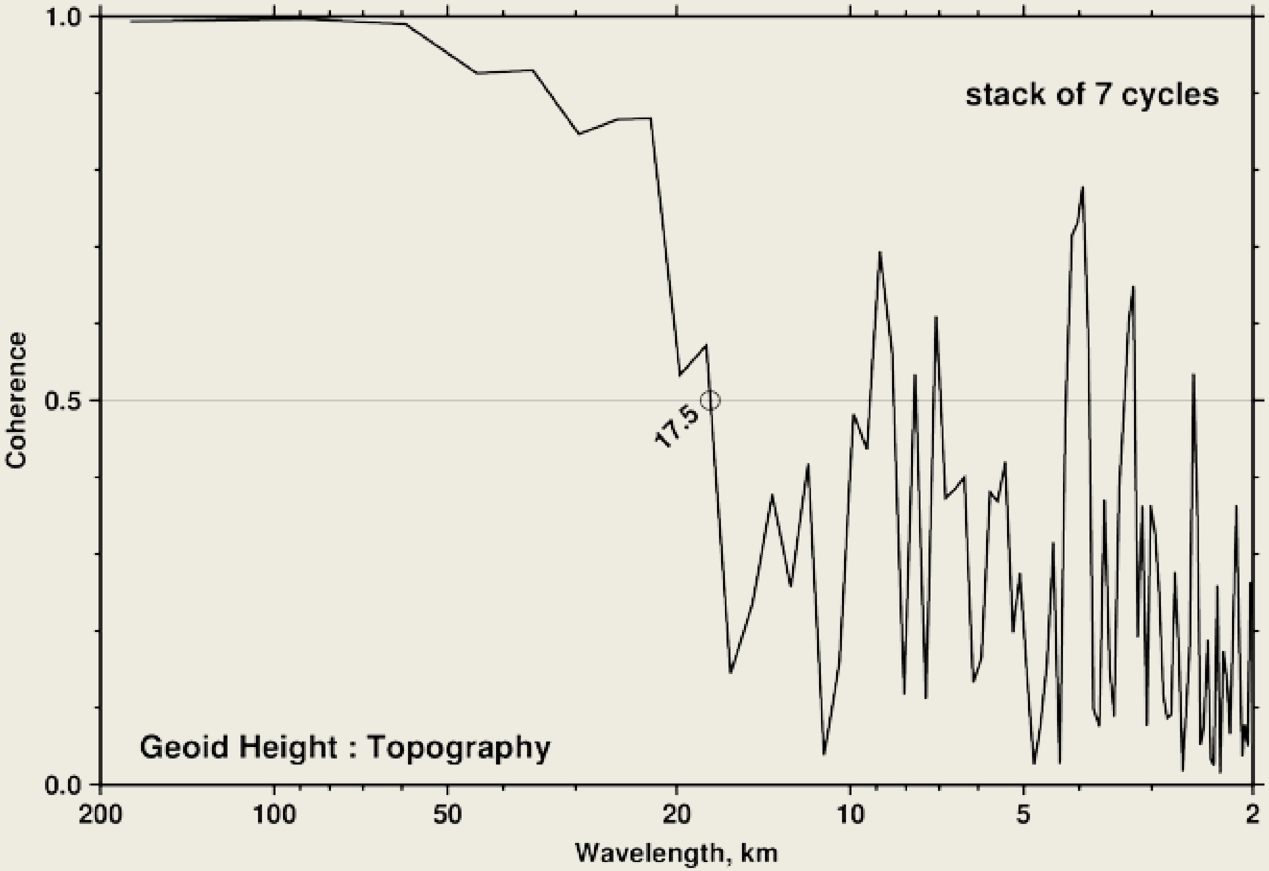


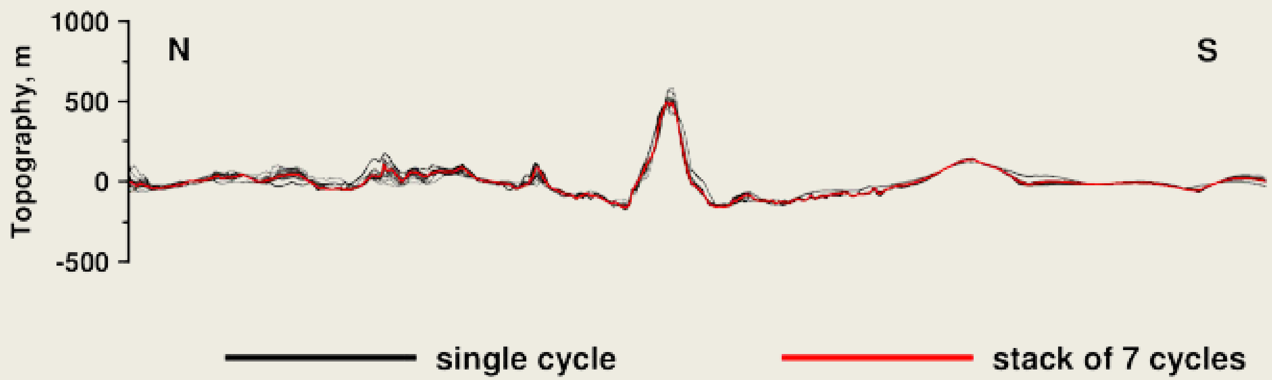
Area 2: One seamount 1 km tall



An 8-cycle stack is coherent with bathy to 12 km wavelength.

Area 3: One seamount about 600 m tall





A 7-cycle stack is coherent with bathy to 17.5 km wavelength.

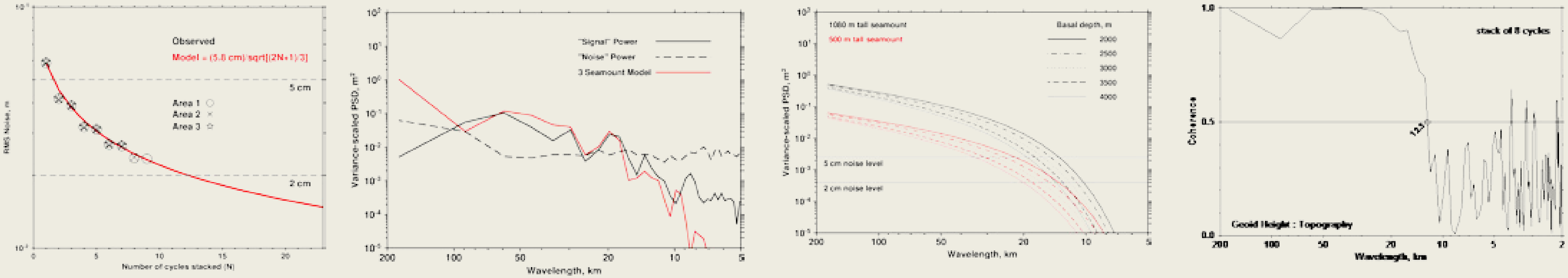
Conclusions: AltiKa SSH measurement noise

- AltiKa measurement noise at 40 Hz is nearly white and about 5.8 cm in a single cycle.
- Noise in a multi-cycle median stack decreases as the number stacked increases, exactly as expected: $[3/(2N+1)]^{1/2}$.
- Stacking one year of repeat cycles brings the 40-Hz SSH noise below 2 cm.
- Noise is independent of seamount size.

Conclusions: coherence with bathymetry

- It should be possible to find seamounts smaller than 2 km tall.
- 1.5 km tall should be easy.
- 1 km tall possible
- < 1 km tall will require stacks of very precise altimetry.

Future directions



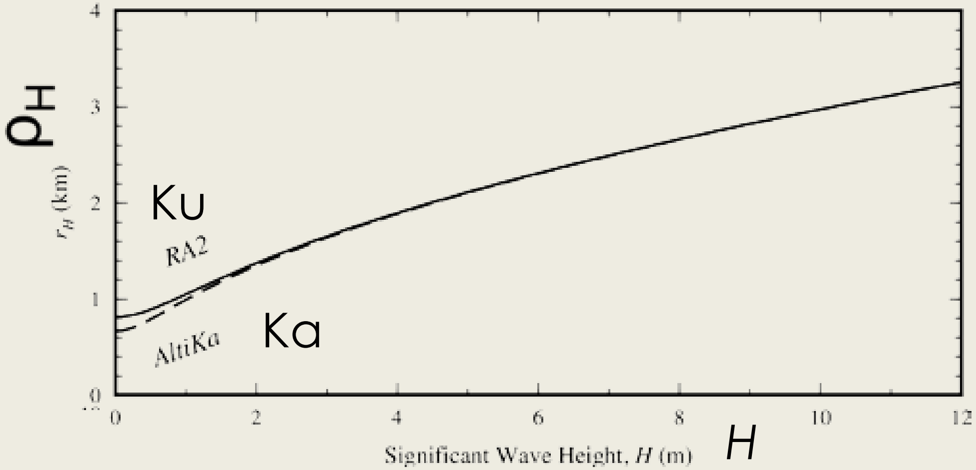
- We now have a good model for the measurement noise, the seamount signal, the range of wavelengths where signal rises above noise and where SSH is coherent with bathymetry.
- Next step is to compare SSH spectra in areas with seamounts to SSH spectra in areas without seamounts.
- With this we can build a seamount detection filter, assess trade-off between false negatives/positives, etc.
- We can then use stacks of 35-day repeat SARAL AltiKa to look for smaller seamounts.
- ***Mapping must await a geodetic mission.***

Finis

Pulse- and wave height-limited footprint radius, ρ

$\rho(e) \sim= \sqrt{(2he/\kappa)}$; $\kappa = 1 + h/R$.

Range excess $e = \Delta r_B = c/2B$ if the reflecting surface is smooth. On a rough ocean surface with significant wave height H, $e = \Delta r_H = \sqrt{[(\Delta r_B)^2 + 2\pi(H/4)^2]}$.



The pulse-limited footprint on a flat surface (H = 0) is a bit smaller at Ka than at Ku due to Ka's smaller Δr_B. The effect disappears on a rough surface (H >= 2 m). Values at left are at h = 800 km (Sentinel-3, AltiKa). For Jason, increase them by $\sqrt{[(13\kappa_{800})/(8\kappa_{1300})]}$. At H = 2 m (most common ocean condition), ρ_H ~ 1.3 km at h = 800 km and 1.6 km at h = 1300.

Figure from W H F Smith, AltiKa special issue, doi:10.1080/01490419.2015.1014950

Berger / Walsh optimum uncorrelated PRF

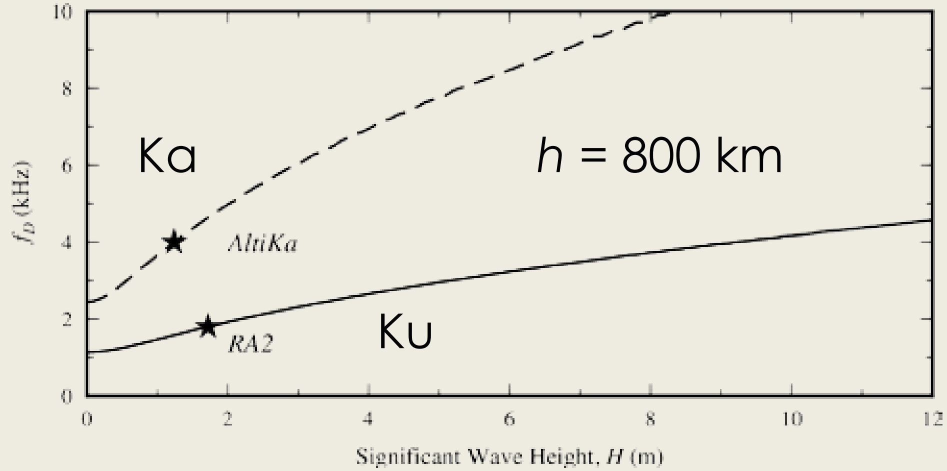
The footprint diameter ρ_H depends on *H*, *h*, and *B*.

Decorrelation distance depends on ρ_H , *h*, λ.

Velocity depends on *h*.

The decorrelation frequency, f_D, depends on all of these. It is the max rate at which we can obtain uncorrelated echoes.

Figure from W H F Smith, AltiKa special issue, doi:10.1080/01490419.2015.1014950



The curves above show f_D as a function of wave height, *H*, for AltiKa (dashed) and EnviSat RA2 (Ku, solid). Both are at the same orbit height, *h* = 800 km, but the decorrelation at Ka band is much faster than at Ku band. The PRFs of these altimeters (stars) yield uncorrelated echo sequences in nearly all sea states, and at nearly the optimum sampling rate in typical sea states.

Ka advantaged over Ku by about a factor of 3 at same altitude.

Beam limiting, 1: Ka versus Ku

- Circular antenna beam width is proportional to *λ*/*D*, where *D* is the antenna diameter.
- For the same *D*, Ka beam width is about 1/3 (8/22) of Ku beam width.
- AltiKa is beam-limited as well as pulse-limited.
- AltiKa response to an ocean surface is more like an impulse and less like a step function (Ku shape).
- Information in both leading and trailing edges, not just leading edge; more constraint on range and SWH.

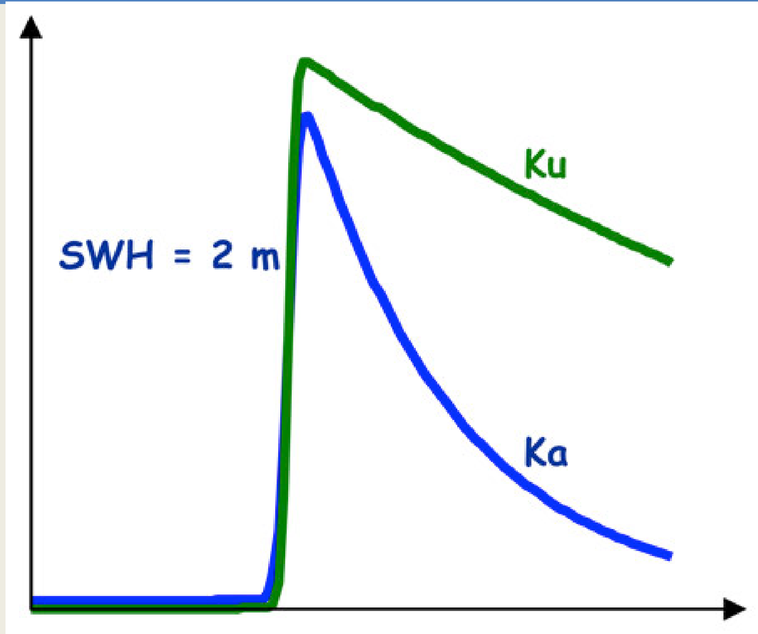


Figure from P. Vincent et al. [2006] *Sensors*,6(3):208-234.

Summary for AltiKa

- SARAL/AltiKa is a conventional altimeter (incoherent, pulse-limited circular footprint), ***but*** at Ka (λ = 8 mm) not Ku (λ = 22 mm).
- The shorter λ yields two major advantages:

(1) it can make independent measurements at a faster rate, higher PRF, more averaging;

(2) it is beam-limited, and so its impulse response is more peaked & more sensitive*.

- Result is ~2.4x higher precision sea level.
- Attenuation / Backscatter /Wind Speed different from Ku, requires new calibration/algorithms.

*Should also be less prone to “spectral bump”, but this is TBD.