Combining a Global GOCE Derived MDT with In-situ Observation for Regional Enhancement of the Mean Dynamic Topography.

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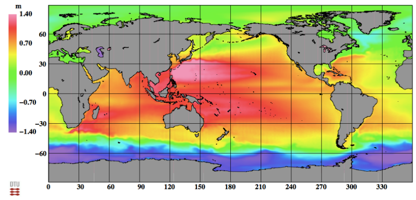
Summary
The Gravity and steady state Ocean Circulation Explorer (GOCE) satellite mission measures Earth's gravity field with an unprecedented accuracy at short spatial scales. Previous results have demonstrated a significant advance in our ability to determine the ocean's general circulation. In this study, a global mean dynamic topography (MDT) derived using a gravity models from GOCE combined with the DTU13MSS mean sea surface is used as a reference model. Then regional analyses are carried out using in-situ observations of the geostrophic surface currents. Subsequently, the in-situ observations are used in a regional enhancement of the estimated MDT and its associated currents. The data are combined using an optimal estimation technique such as least squares collocation, that is based on the functional relationship between the MDT as well as their a-priori statistical characteristics. The methodology and preliminary results will be presented.



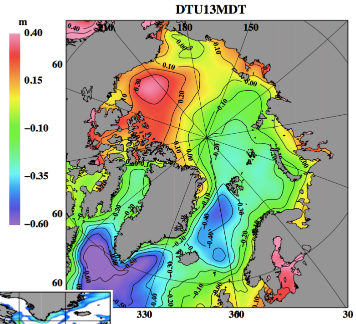


Figure 1. The DTU13MDT mean dynamic ocean topography model.

Computation of the Mean Dynamic Topography
The geodetic approach for deriving the mean dynamic topography (MDT) is to use a mean sea surface (MSS) obtained from satellite altimetry and a Geoid model (*N*) derived using GOCE data. The MDT is obtained as MDT = MSS – *N*.
Subsequently, a proper filtering of the differences is required to eliminate the short scale geoid signals that are not recovered by the gravity model, to obtain a useful estimate of the MDT. Usually, the filtering is carried out using the isotropic truncated Gaussian filter with a half-width at half-maximum around 1.0 spherical degree.
In this study the DTU13MSS mean sea surface is used. The geoid is computed using EIGEN-6C3 gravity model (Förste et al, 2011) for which the unmodeled parts of the geoid is much smaller because EIGEN-6C3 is a combination model where, e.g., GOCE, GRACE and surface gravity based on satellite altimetry have been used. In addition, the shorter wavelength part of the geoid were removed using the EGM2008 geopotential model (Pavlis et al, 2008). Naturally, the use of altimetric gravity over the oceans will not improve the estimation of the MDT but less filtering is required. In this computation an isotropic truncated Gaussian filter with a half-width at half-maximum of 0.75 spherical degrees was used. Approaching the Equator an an-isotropic filter was used to overcome problems with stripes. Furthermore, the computation of geostrophic current components, especially the North-south velocity, was regularised at the Equator. The DTU13MDT is shown in Figure 1. The geostrophic surface flows are shown in Figure 2.

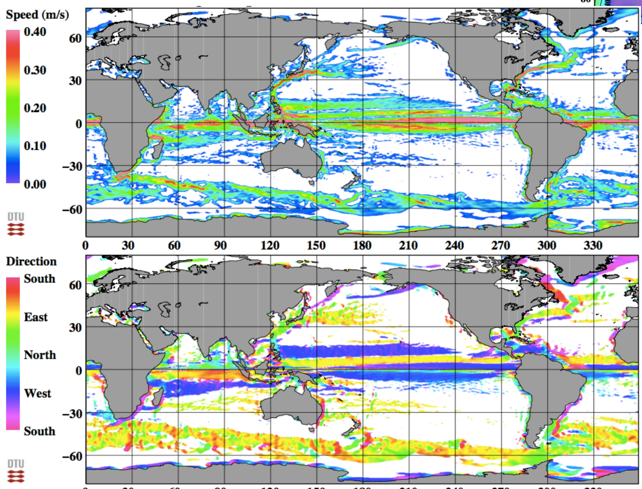


Figure 2. Geostrophic surface current speed and direction from DTU13MDT for speeds > 5cm/s

Regional enhancement using drifter data
To improve the MDT estimation – both the filtering and to combine with the in-situ data – an optimal estimation technique is applied.
We use the technique called Least Squares Collocation: Rigorous estimation of a quantity *x* from a set of observations *y*:

$$x = c_x^T \left(C + D\right)^{-1} y$$

with rigorous error/-covariance estimates:

$$\hat{\sigma}_x = \sigma_x - c_x^T \left(C + D\right)^{-1} c_x$$

Actual signal and error covariances are taken into account.

The covariance functions are modeled in a spherical approximation using a sum of a series of Legendre's Polynomials.
Linear functionals are applied on the covariance function associated with the gravity potential field C_{TT} to obtain, e.g.:

$$C_{NN} = \sum_{i=2}^{\infty} \left(\tfrac{1}{\gamma}\right)^2 \sigma_i^{TT} P_i(\cos\psi) \quad C_{\Delta g \Delta g} = \sum_{i=2}^{\infty} \left(\tfrac{i-1}{R}\right)^2 \sigma_i^{TT} P_i(\cos\psi)$$

and the use of a reference field can be taken into account

$$\sigma_i^{TT} = \begin{cases} \varepsilon_i & i = 2,\ldots,N \\ \dfrac{A}{(i-1)(i-2)(i+4)} \left(\dfrac{R_B^2}{R^2}\right)^{i+1} & i = N+1,\ldots \end{cases}$$

Where ε_i are the error degree variance associated with the reference field.
Note that $i \to \infty$ and that $\sigma_i \to 0$ faster than i^{-3} (smoothness)

For the Mean Dynamic Topography we may (Knudsen '91):

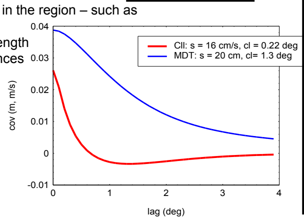
$$C_{SS} = \sum_{i=2}^{\infty} \sigma_i^{SS} P_i(\cos\psi)$$

with the degree variance model:

$$\sigma_i^{SS} = b\left(\frac{k_2^3}{k_2^3 + i^3} - \frac{k_1^3}{k_1^3 + i^3}\right) s^{i+1}$$

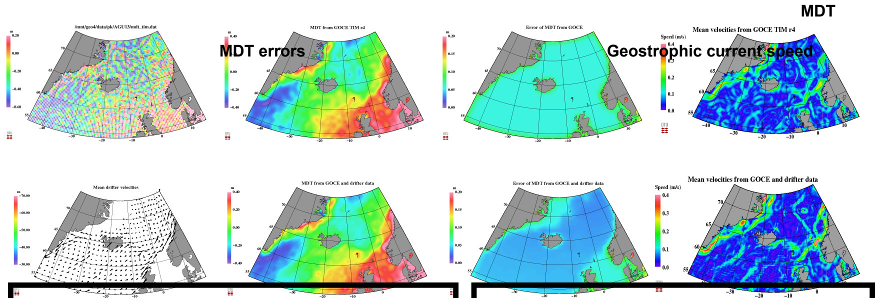
Again, $i \to \infty$ and that $\sigma_i \to 0$ faster than i^{-3} (smoothness)

where *b*, k_1, k_2, and *s* are determined so that the expression fits the empirical values - in the region – such as
1. MDT variance
2. MDT correlation length
3. X-dot, y-dot variances



GOCE

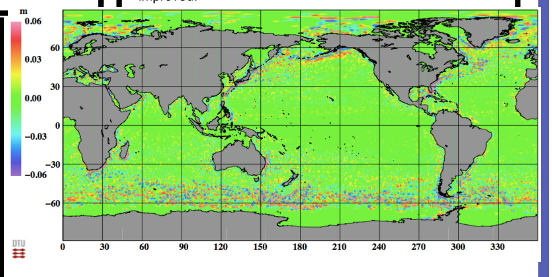
Data



Results
The preliminary results show that
1. The optimal technique is successful in filtering the MDT estimates based on GOCE and the MSS alone. Note that the results are not smoothed. In addition, rigorous errors estimated of the MDT have been calculated (~6 cm) as well as geostrophic surface currents.
(Plots are showed in the upper row)
2. By combining with 0.25x 0.5 deg mean drifter velocities the filtering of the MSS-GOCE data was improved (less shorter wavelength features are seen), the errors have dropped to ~4 cm, and the estimated currents show a much more consistent features. (Plots are shown in the lower row)

Perspectives
The GOCE MDT display the well known features related to the major ocean current systems. In addition, the GOCE gravity model has enhanced the resolution and sharpened the geometry of those features. A computation of the geostrophic surface current speeds clearly display the improvements in the description of the current systems. Sub-current systems and their different branches and flow paths are revealed. The results of this analysis using an optimal technique to combine GOCE MDT with drifter data show that the estimation of the MDT and the associated geostrophic surface currents in particular, may be further improved.

On-going research.
Based on the positive results obtained in the Faeroes Islands/GOCINA region a series of R&D activities have been initiated to further develop the methodology. The aim is to use the optimal estimation technique (called collocation in this presentation) to perform a global integration of mean drifter velocities in the MDT estimation.
One of the main challenges is associated wit the modelling of the covariance functions associated with the MDT and the geostrophic surface currents. In the current approach both the MDT heights and their horizontal derivatives are modelled using isotropic and homogeneous statistical properties. This works fine at mid-high latitudes, but in the equatorial regions this assumption results in unrealistically high apriori variances of the geostrophic currents (infinite at the Equator). At the Equator the beta-plane approximation involving the second order horizontal derivatives as described by Lagerloef et al. should be implemented. In addition, alternative latitude dependent covariance function models should be evaluated.
Another major challenge is the processing of the drifter data to obtain mean drifter velocities and error estimated associated with those. Both corrections for Ekman flow and for meso-scale variability needs to be considered.



The plot above show some preliminary results of a computation of MDT height residuals relative to DTU13MDT from drifter velocities. In this test the covariance functions were modelled so that the apriori variances of the current components are homogeneous globally. Hence the apriori MDT height variance goes to zero at the equator.
The preliminary results show interesting features associated with ocean circulation, i.e. the short-scale parts not recovered by the DTU13MDT as described above, as well as features associated with the errors in that model originating from errors in the geoid model.