Particle diffusion against mean flow as a new framework for local estimate of mixing coefficient

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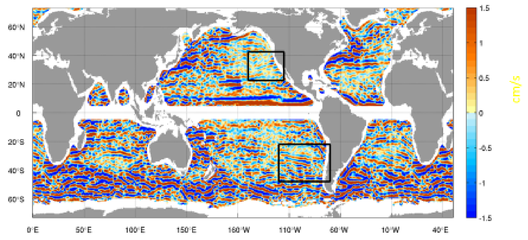
Abstract
One of classical methods, used to estimate mixing coefficient, utilizes the rate of dispersion of particles from a source or from initial locations. This method is based on the assumption that on times significantly larger than Lagrangian time scales steady asymptotic statistic exists. The "cloud" of particles expands in time that limits the ability of the method to resolve complex structures. In addition, calculations are performed in a coordinate system, moving with the mean flow, so that, when the flow is strong, it may be difficult to translate Lagrangian information in to Eulerian grid needed for practical applications.
Such method implies the existence of sufficient spectral gap between scales of resolved (large-scale, time-averaged or slowly changing) currents and anomalies (traditionally referred as "eddies"). At the same time, recent studies reveal a tremendously complex structure of the mean ocean circulation, in which fronts remain sharp and narrow even on long-time averages and long-living, coherent mesoscale eddies are often organized along preferred paths, producing a weak but persistent "texture" of the mean ocean circulation.
Our study proposes a new technique, allowing to increase spatial resolution of the mixing coefficient estimates, by converting Lagrangian trajectories into Eulerian probability density function near the source. The method is based on the analytical solution of the diffusion equation for a tracer released from a singular source in the presence of a mean flow and generalized for two-dimensional case and for particles with a finite life span. The method characterizes excursions of the particles against the mean flow and uses them to estimate mixing. For the same mixing coefficient, the stronger the flow the weaker are the excursions. We show that in most areas of the real ocean excursions of satellite-tracked drifting buoys against the mean flow are relatively small that justifies our method. Results of the analysis of drifting buoys and simulated particle trajectories, derived from satellite data of altimetry and scatterometry and consistent with the actual drifter "spaghettis" (work in progress), are used to study the factors controlling the intensity of the multi-scale horizontal mixing near the ocean surface.

- Eddy fluxes are significant but very hard to quantify within a simple advection/diffusion framework:

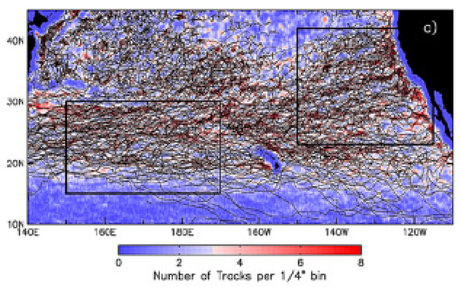
$\partial C/\partial t + \mathbf{V}\cdot\nabla C = \nabla(K\cdot\nabla V) + \ldots$

- Climate models do not resolve eddies and, therefore require their parameterization.
- Eddy parameterization is difficult because:
  - eddy time and space scales are not too different from the scales of the "mean" flow;
  - eddy dynamics are complex;
  - eddy-mean interaction is strong;
  - etc.

Striations have width comparable with the eddy size in long-time mean zonal geostrophic velocity at the ocean surface, high-pass filtered horizontally with two-dimensional 4° filter.

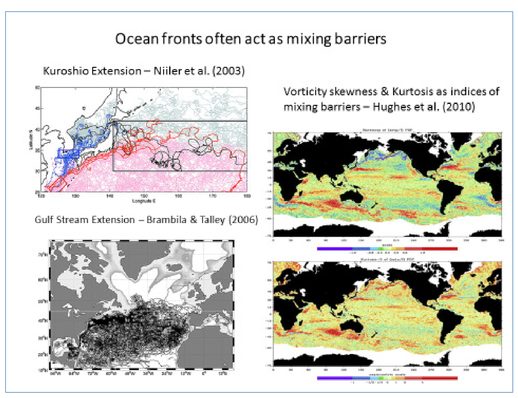


Eddy distributions are highly heterogeneous and very long. Preferred eddy paths are common



Schlax and Chelton (2008)

Ocean fronts often act as mixing barriers

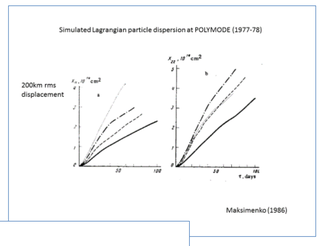


One of most common approaches of deriving mixing coefficient from trajectories of Lagrangian drifters is based on Taylor (1921):

$$K_{jk} = \frac{1}{2}\frac{\partial}{\partial t}\langle d'_j(t-t_0\,|\,\mathbf{x}_0,t_0)d'_k(t-t_0\,|\,\mathbf{x}_0,t_0)\rangle$$

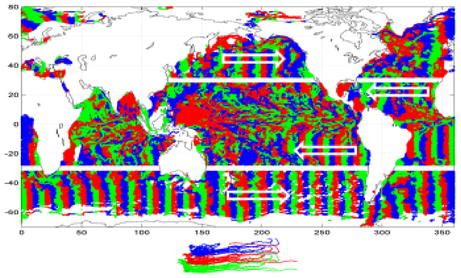
where d'_j is particle displacement due to velocity anomaly.

In many cases, by the time when $\frac{\partial}{\partial t} < d'_j d'_k > \to const$, d'_j is already large. So that the method is essentially non-local and may not resolve internal scales of the mixing.



Analysis of trajectories of drifting buoys, drogued at 15m, showed that mean flow significantly suppresses upstream excursions of the drifters. This dominance of advection over mixing leads to a new method for the quantification of the mixing

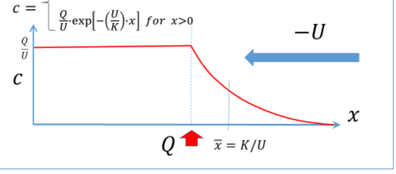
Overlaid fragments of drogued drifter trajectories sorted according to their start points within 20-degree longitude bands.



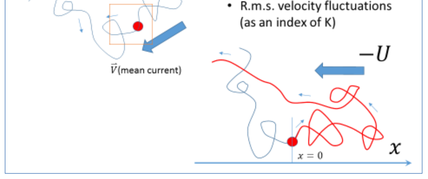
Exact analytical solution for one-dimensional tracer advection/mixing equation from a singular source in a steady flow

$$\partial_t c = U\cdot\partial_x c + K\cdot\partial_{xx} c = 0$$

$$c = \begin{cases} \frac{Q}{U} & for\ x \le 0 \\ \frac{Q}{U}\cdot\exp\left[-\left(\frac{U}{K}\right)\cdot x\right] & for\ x>0 \end{cases}$$

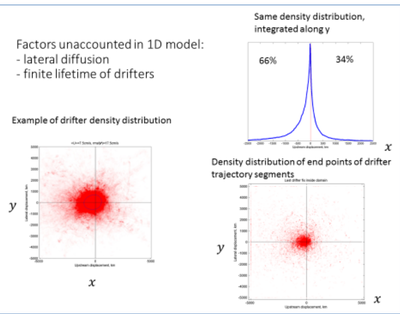


Interpretation of Eulerian tracer concentration as a probability density of locations of Lagrangian particles



- Rotate according to mean flow direction
- Adjust start point coordinates
- Sort according to local values of
  - Magnitude of mean current
  - R.m.s. velocity fluctuations (as an index of K)

Factors unaccounted in 1D model:
- lateral diffusion
- finite lifetime of drifters



2D case with a constant death rate

$$\partial_t c = -\frac{c}{T} + U\cdot\partial_x c + K\cdot(\partial_{xx} c + \partial_{yy} c) = 0$$

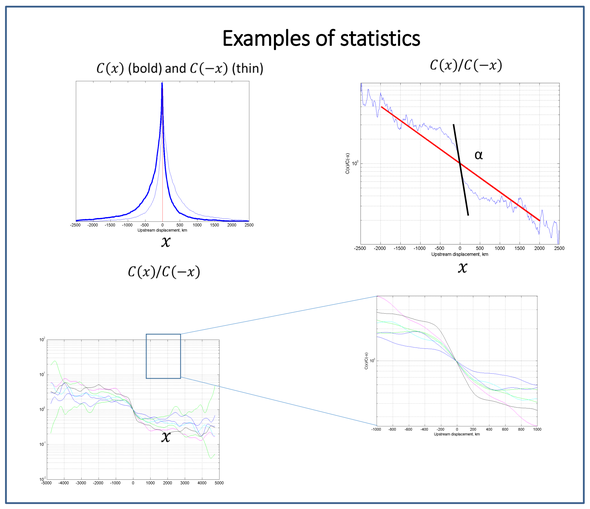
By introducing $C(x) = \int_{y_1}^{y_2} c(x,y)\cdot dy$

$$C(x) = \begin{cases} \frac{Q}{U}\cdot\exp\left\{\left[-U+sqrt\left(U^2+4\cdot\frac{K}{T}\right)\right]/(2\cdot K)\cdot x\right\} & for\ x \le 0 \\ \frac{Q}{U}\cdot\exp\left\{\left[-U-sqrt\left(U^2+4\cdot\frac{K}{T}\right)\right]/(2\cdot K)\cdot x\right\} & for\ x>0 \end{cases}$$

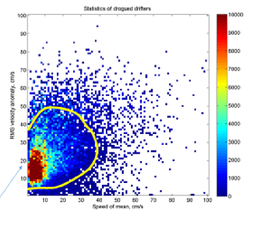
Important property of the solution (**same as for 1D case**):

$$\frac{C(x)}{C(-x)} = \exp\left[-\left(\frac{U}{K}\right)\cdot x\right]$$

Examples of statistics

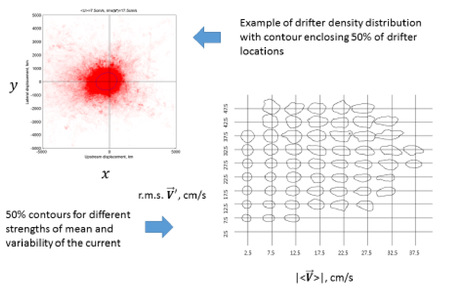


Sorting according to $|\langle\vec{V}\rangle|$ and r.m.s. $\vec{V}'$

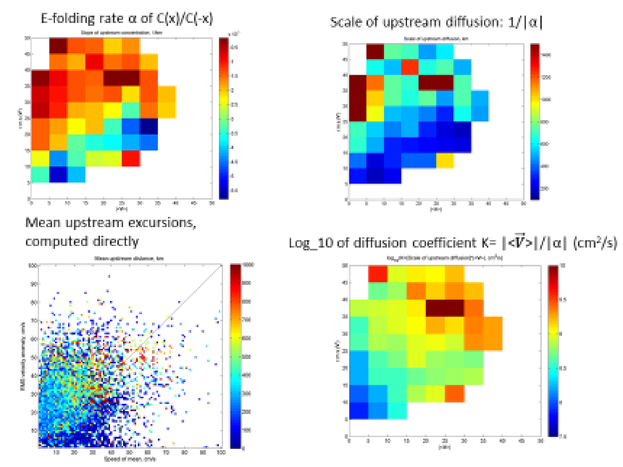


Parameter ranged used to compute statistics

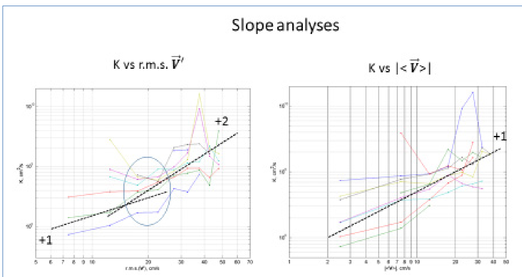
Examples of statistics



Dependence of mixing on mean current and eddies strengths



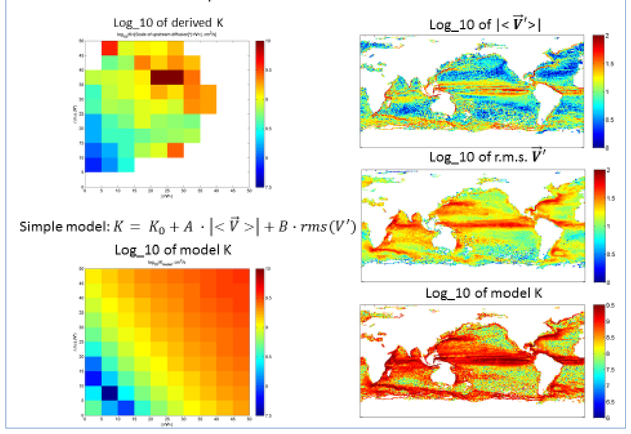
Slope analyses



+1 slope: $K \sim V' \cdot L_K$

+2 slope: $K \sim V'^2 \cdot T_K$

Simple model of diffusion coefficient



Simple model: $K = K_0 + A\cdot|\langle\vec{V}\rangle| + B\cdot rms(V')$

Summary

- Mean flow limits upstream diffusion of passive tracer and upstream-downstream differences can be used to estimate mixing coefficients on smaller scales.
- Trajectories of Lagrangian drifters were used to estimate upstream excursions and diffusion coefficient.
- Diffusion coefficient is found depending both on EKE and on the magnitude of the mean flow (although the latter two are not strictly correlated).
- Simple diffusion parameterization is introduced.