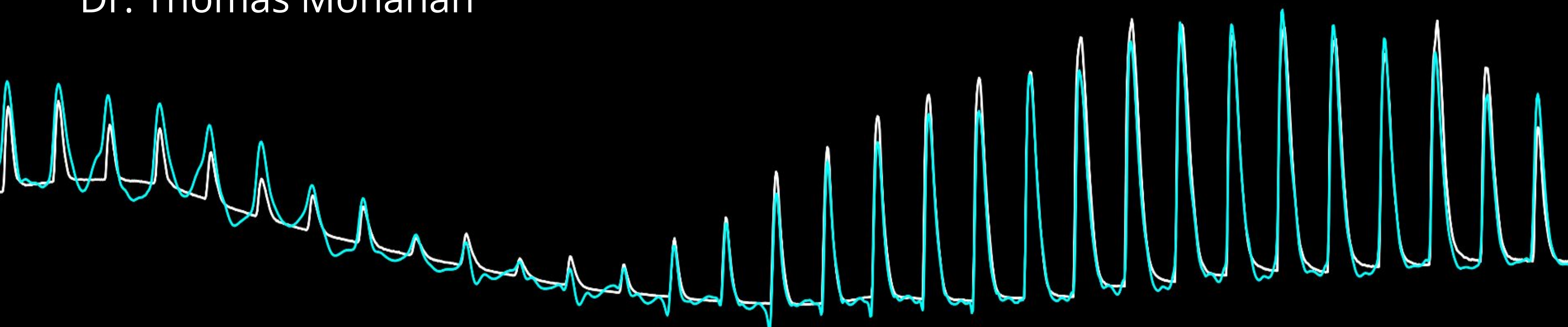
Progress towards a general altimetry derived tidal process model

Dr. Thomas Monahan



In collaboration with: Michael Hart-Davis, Jeffrey Polton, Stephen Roberts, and Thomas Adcock

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National Oceanography Centre
Ministry of Infrastructure and Water Management
Eric and Wendy Schmidt AI in Science Postdoctoral Fellowship, a program of SCHMIDT FUTURES
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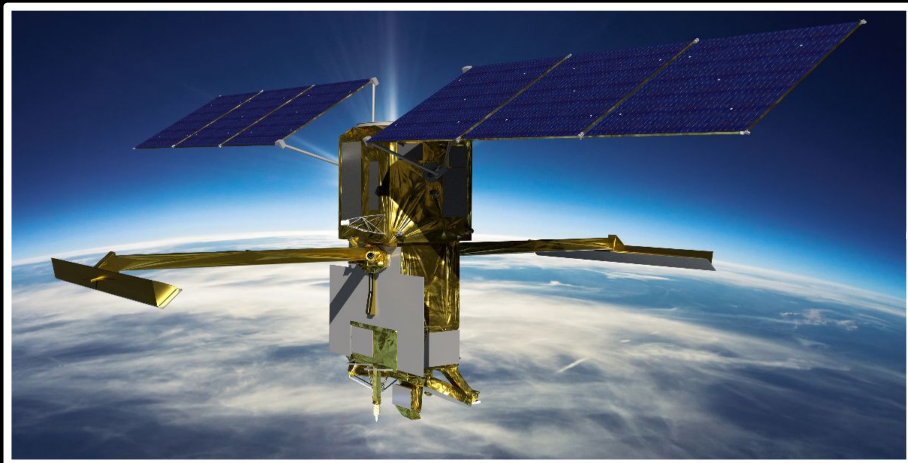
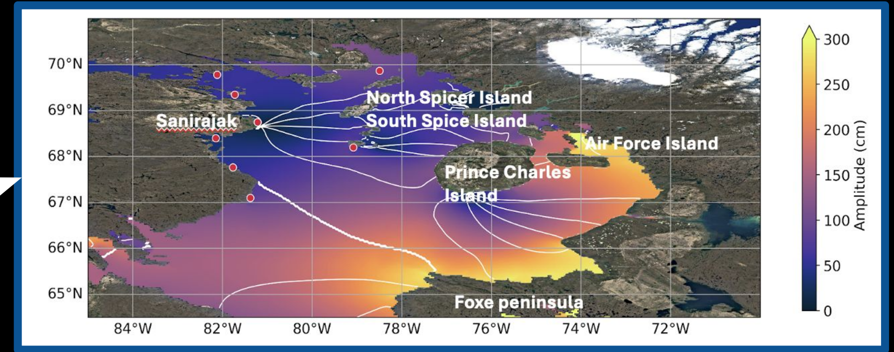
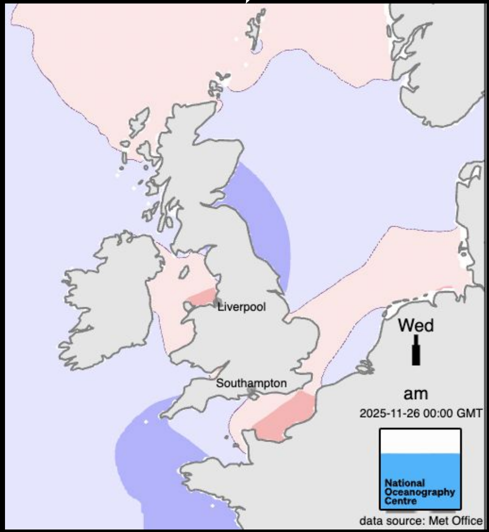
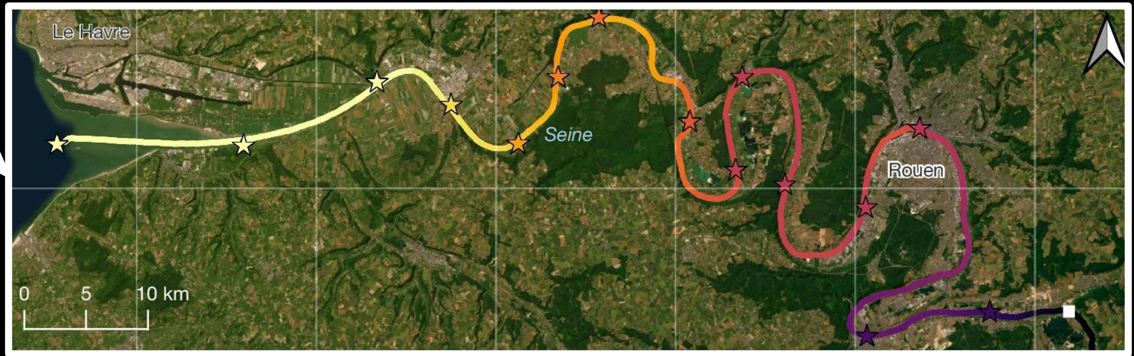




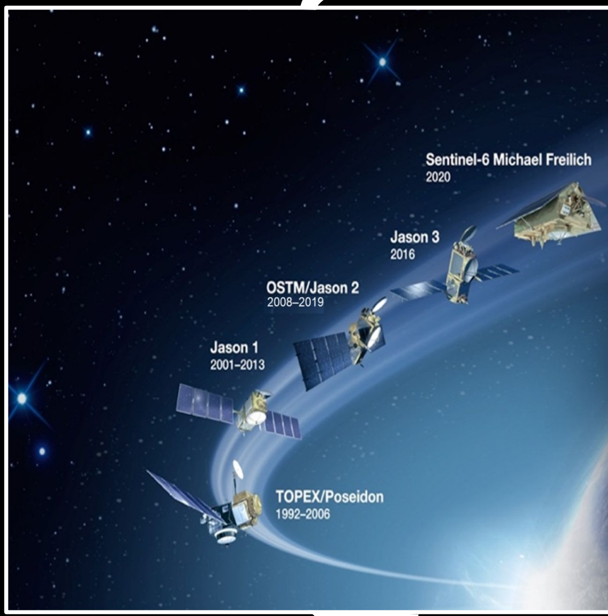
See our poster!





Hart-Davis, Michael G., et al. "Observing the tidal pulse of rivers from wide-swath satellite altimetry." *Nature* (2026): 1-8.

How do we use altimetry to improve our representation of high-frequency processes?



GOT

⋮

FES/DAC

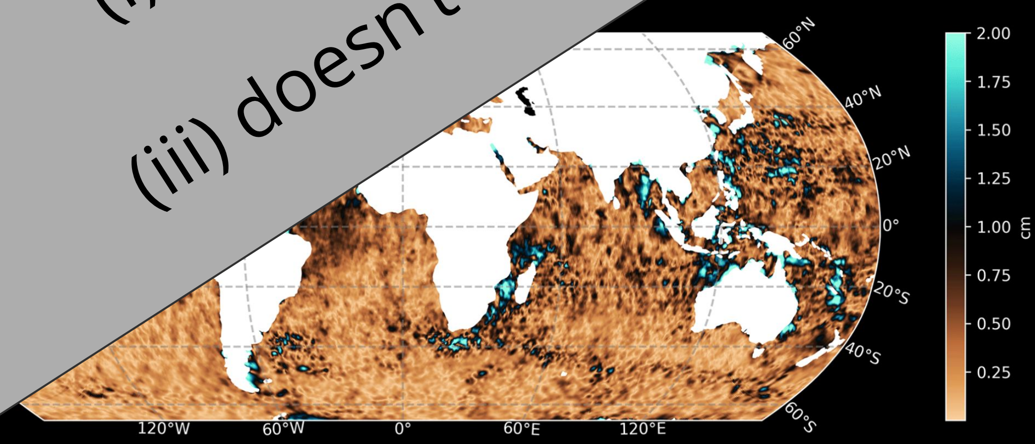
EOT

Residual analysis

Existing Model - Observations =

Final Forecast = Exist

Challenges:
(i) Just applies to tides
(ii) Aliasing
(iii) doesn't use the model

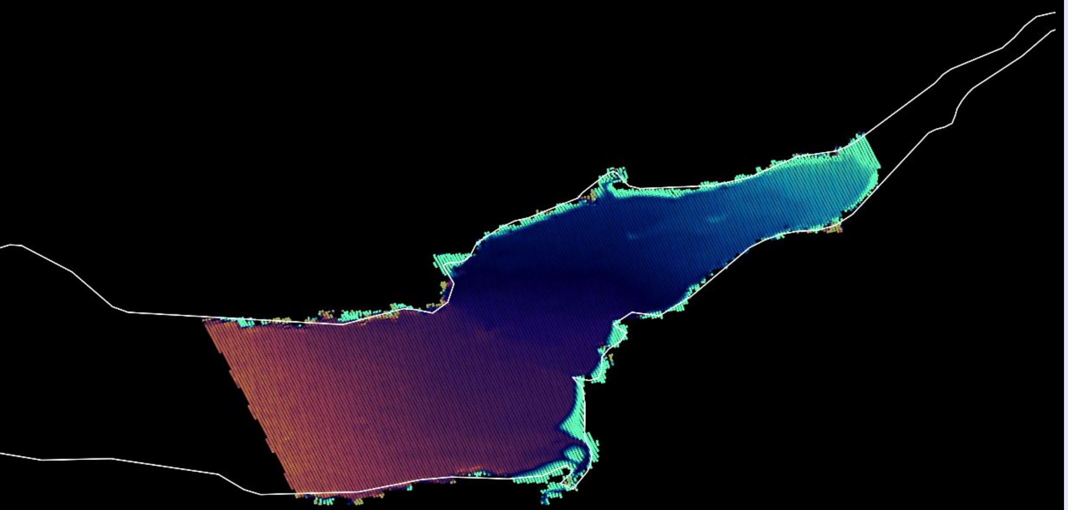


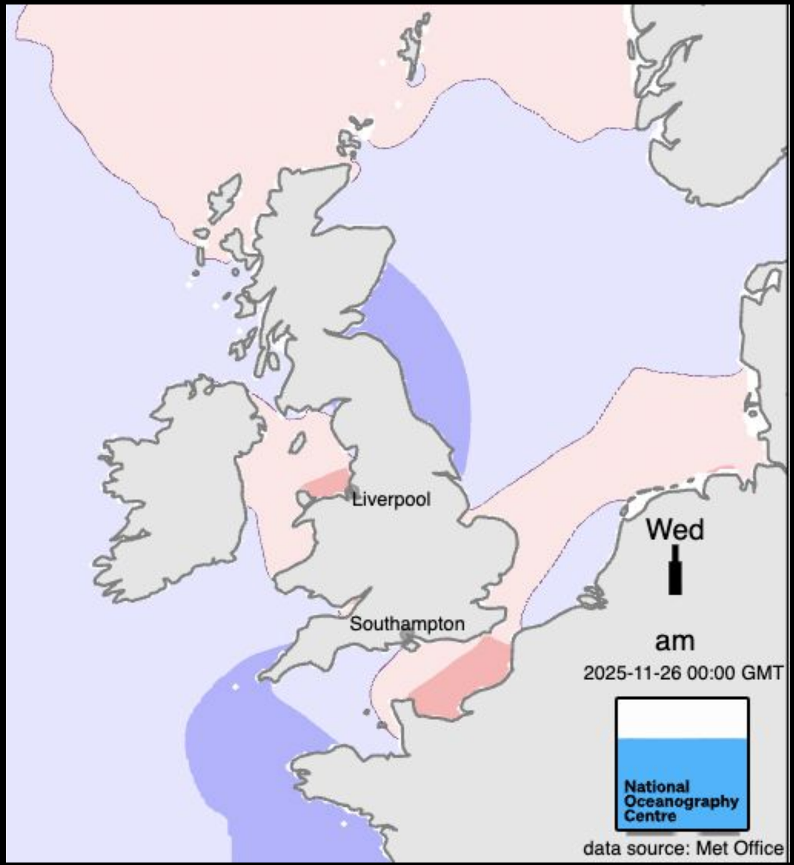
How can we use data to improve these models in challenging regions?

Observation:

Sea-levels smoothly vary between nearby locations.

$$\text{As } A \to B\text{: } \zeta_A \to \zeta_B$$





*ignoring hydraulic jumps

Claim:

IF, no exogenous forcing exists between stations, the sea-level at station **A** can be related to that of station **B** by delays and distortions of station A.

$$\zeta(x_B, t_n) \approx \sum_{\ell=-L}^{L} \alpha_\ell \, \zeta(x_A, t_n - \ell \Delta t)$$

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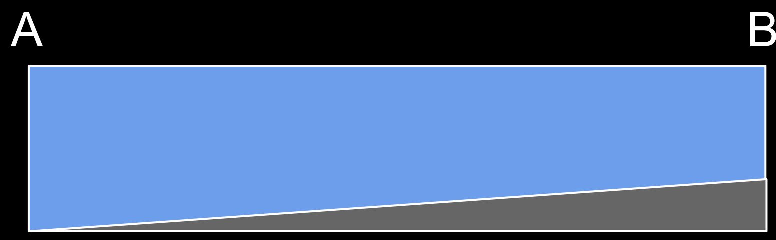

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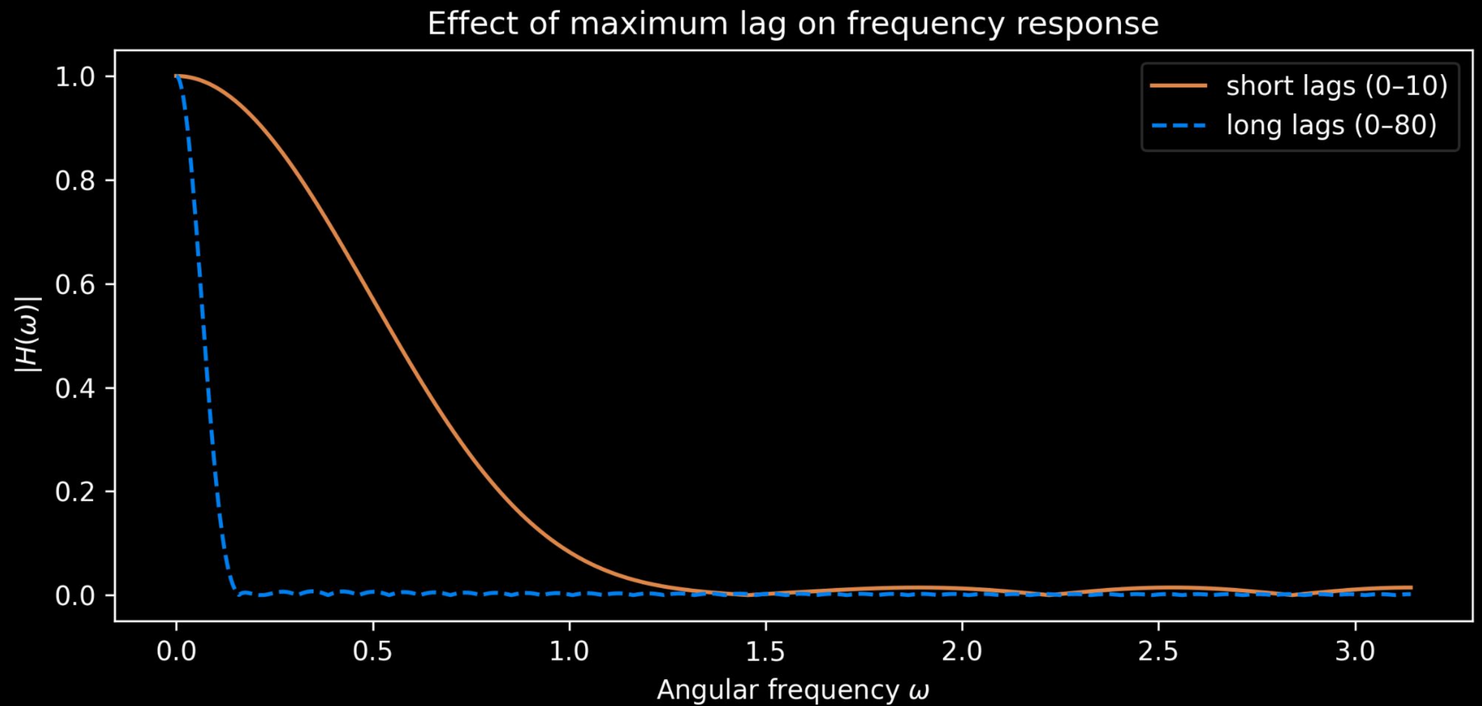
Matte, Pascal, and Silvia Innocenti. "Filling gaps and extending inland tide gauge records to support accurate long-term and extreme water level analyses." *Continental Shelf Research* (2026): 105668.

A frequency interpretation

$$H(\omega) = \sum_{\ell \in \mathcal{L}} \alpha_\ell \, e^{-i\omega\ell\,\Delta t}$$



But linear kernels aren't always enough...

Bed friction

$$\tau_b = C_d\,|u|\,u \;\sim\; u^2$$

Primary source of overtides in rivers and estuaries

Depth–transport coupling

$$\frac{\partial}{\partial x}(\eta\, u)$$

Elevation $\times$ velocity: tide–surge interaction

Depth–wind modulation

$$\frac{\tau_w}{\rho\,(h+\eta)} \;\approx\; \frac{\tau_w}{\rho\,h}\left(1-\frac{\eta}{h}\right)$$

Surge modulates the effective wind response

A **linear** kernel can only redistribute existing frequencies.

The **bilinear** kernel

$$\sum_{\tau_1,\tau_2} w_2(\tau_1,\tau_2) \quad \hat{\zeta}(t\,-\,\tau_1)\,\hat{\zeta}(t\,-\,\tau_2)$$

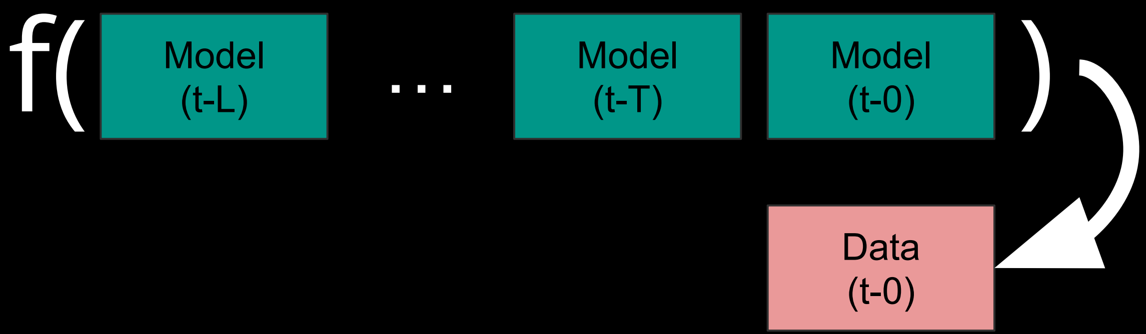
generates new harmonics

— it is the data-driven analogue of weakly nonlinear shallow-water theory.

A time-invariant framework for post-processing shallow water models

$$\zeta(t) \approx b + \sum_{\tau=0}^{L} w_1(\tau)\,\hat{\zeta}(t-\tau) + \sum_{\tau_1=0}^{L}\sum_{\tau_2=0}^{L} w_2(\tau_1,\tau_2)\,\hat{\zeta}(t-\tau_1)\hat{\zeta}(t-\tau_2),$$

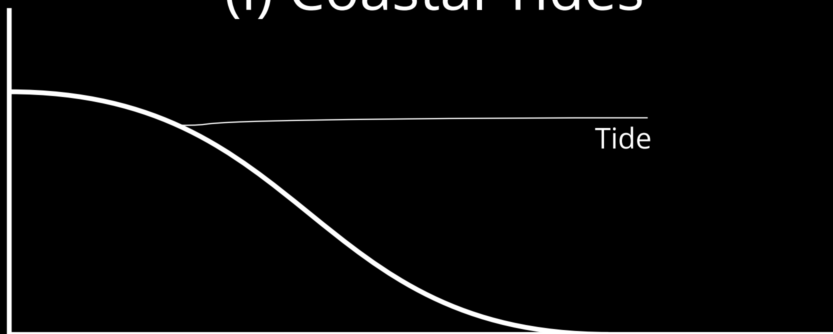
Recipe:



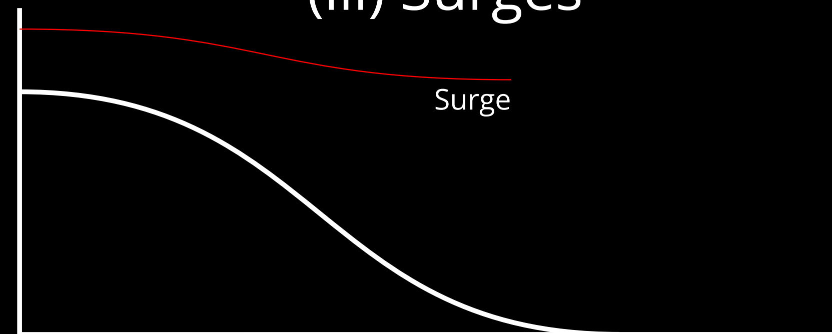
Time-invariant postprocessing

3 contexts.

(i) Coastal Tides



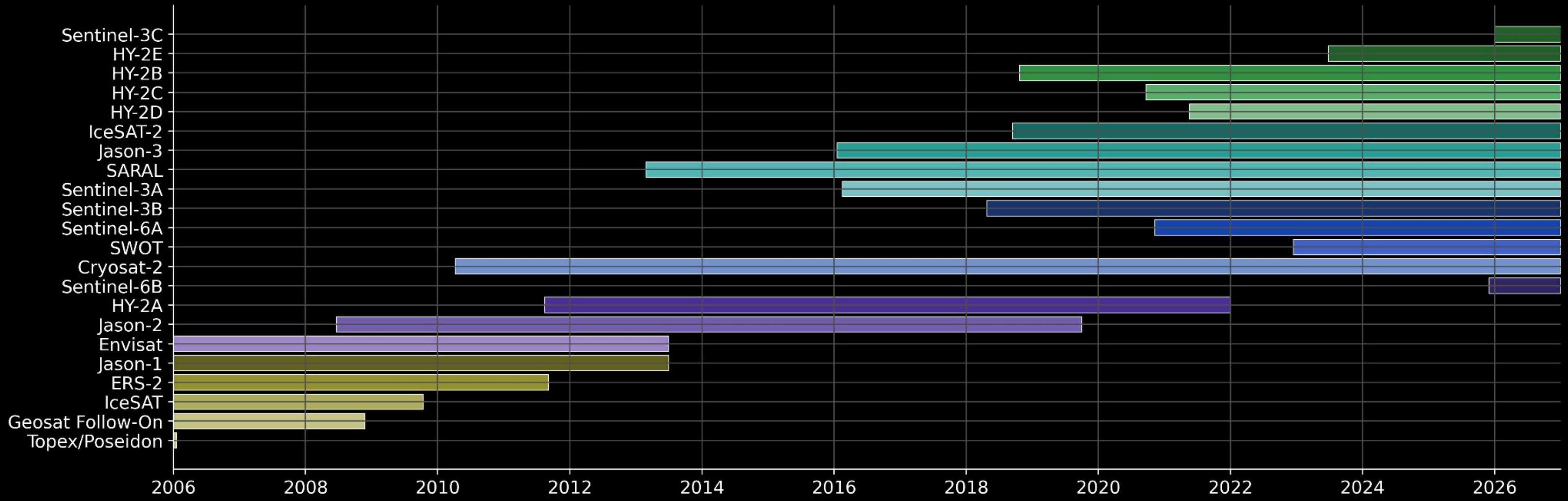
(iii) Surges



(ii) Tidal Rivers

OpenADB

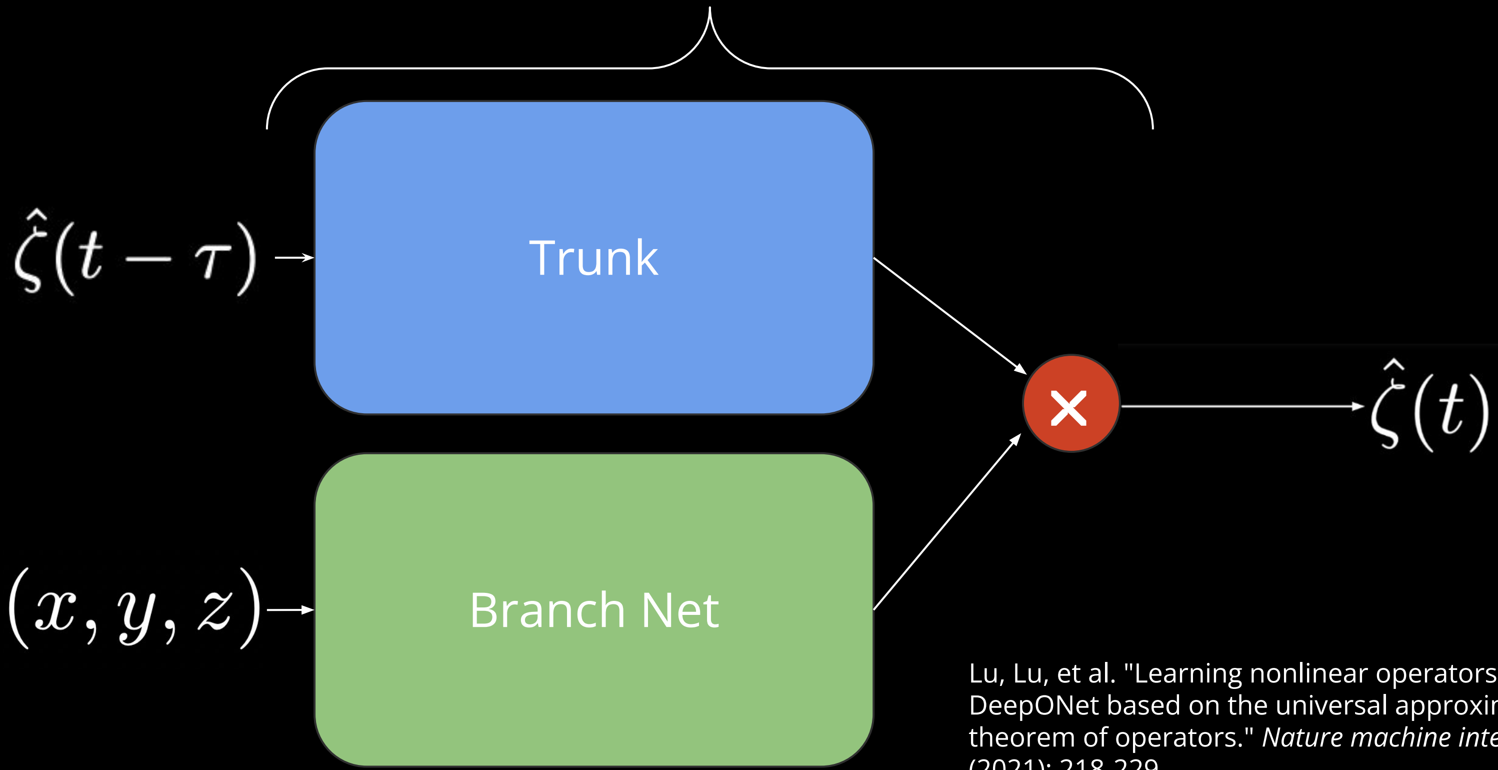
Questions (Ask Mike!)



$$\zeta(x,y,t) \approx b + \sum_{\tau=0}^{L^{(1)}} w_1(x,y,\tau)\,\hat{\zeta}(t-\tau) + \sum_{\tau_1=0}^{L^{(2)}} \sum_{\tau_2=\tau_1}^{L^{(2)}} w_2(x,y,\tau_1,\tau_2), \hat{\zeta}(t-\tau_1)\hat{\zeta}(t-\tau_2)$$

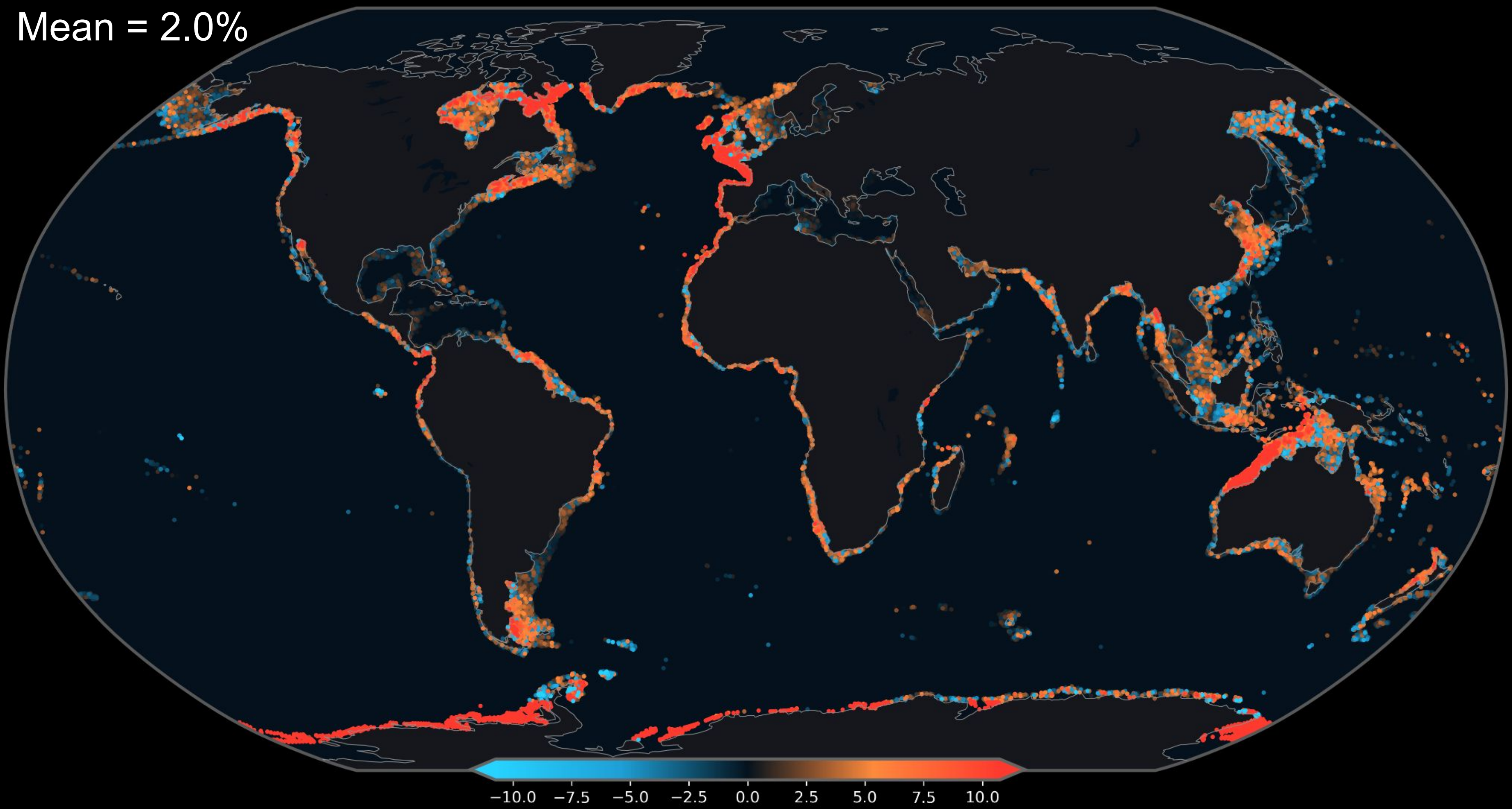
A spatially coherent response operator

$$\zeta(x,y,t) \approx b + \sum_{\tau=0}^{L^{(1)}} w_1(x,y,\tau)\,\hat{\zeta}(t-\tau) + \sum_{\tau_1=0}^{L^{(2)}} \sum_{\tau_2=\tau_1}^{L^{(2)}} w_2(x,y,\tau_1,\tau_2), \hat{\zeta}(t-\tau_1)\hat{\zeta}(t-\tau_2)$$



Lu, Lu, et al. "Learning nonlinear operators via DeepONet based on the universal approximation theorem of operators." *Nature machine intelligence* 3.3 (2021): 218-229.

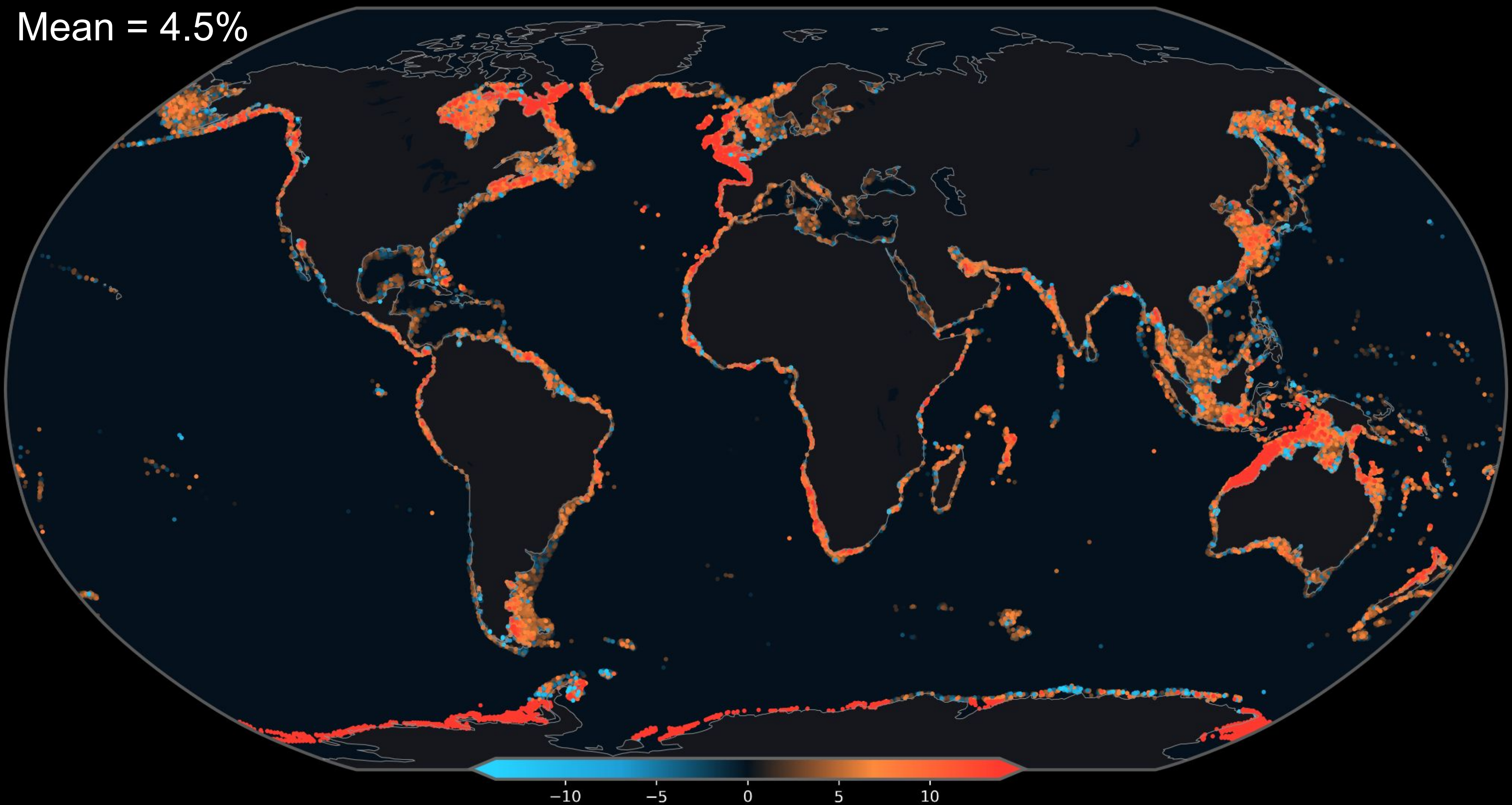
GOAT(FES) Variance Reduction



% Variance Reduction: red = better, blue = worse.

GOAT(FES +EOT20) Variance Reduction

Mean = 4.5%



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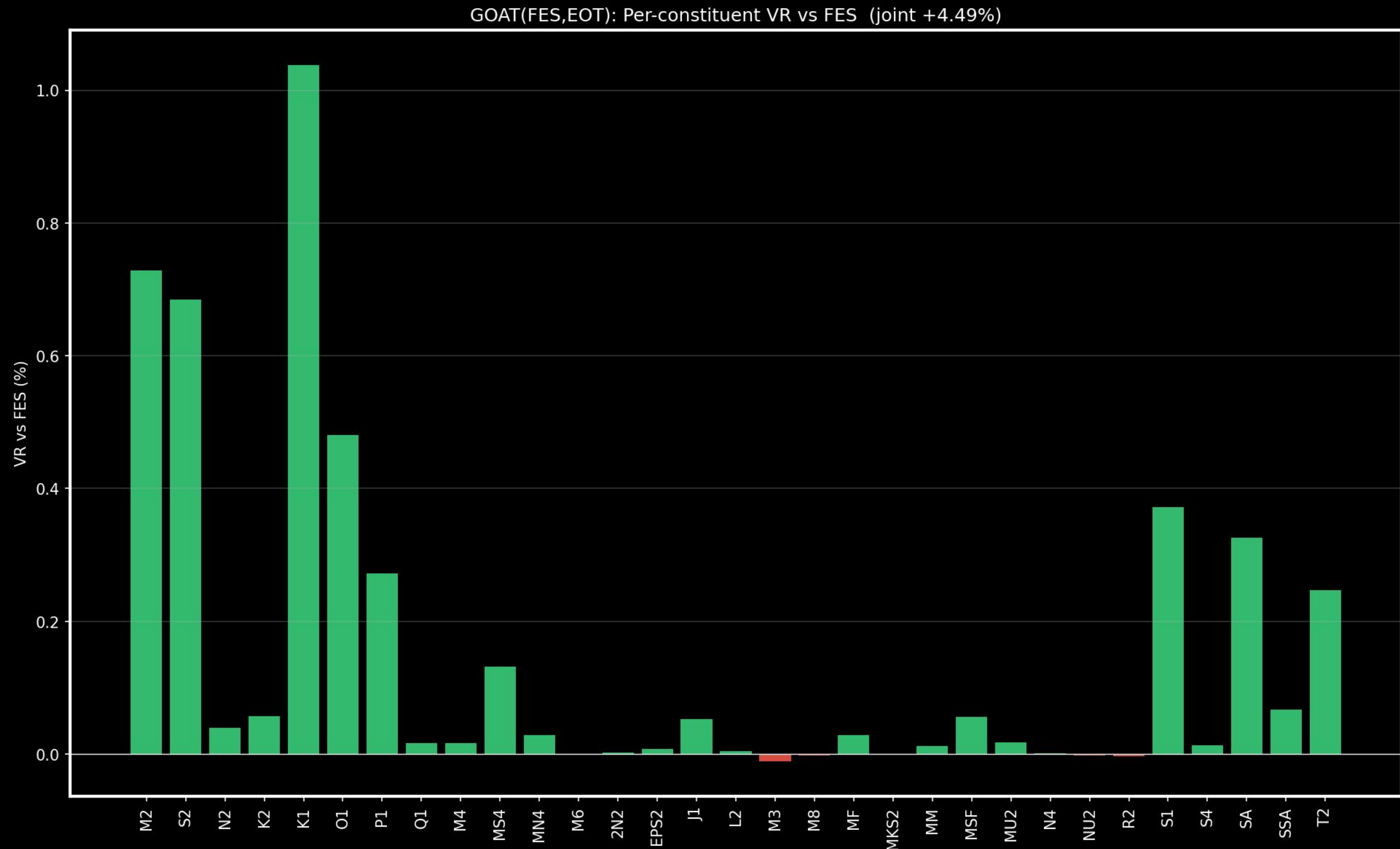
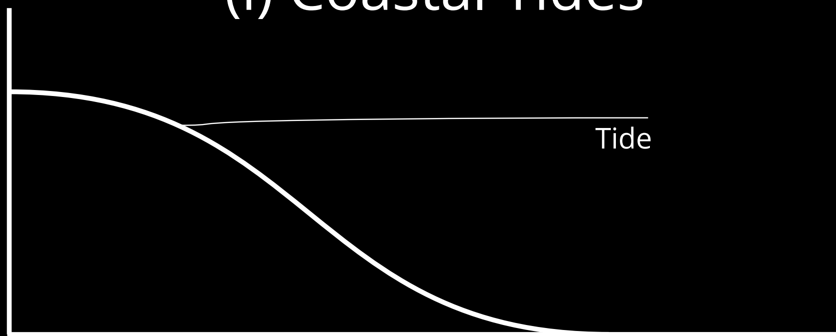
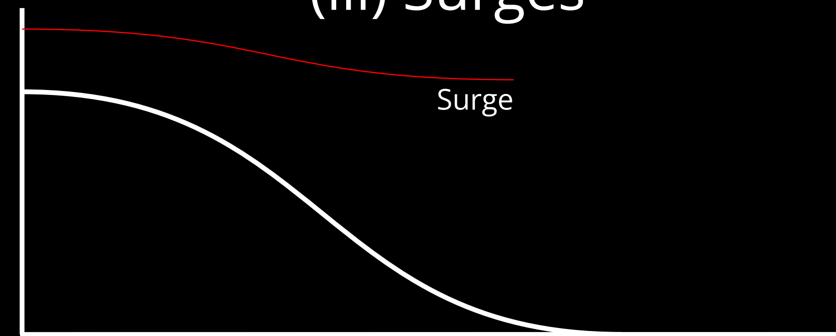

GOAT(FES,EOT): Per-constituent VR vs FES (joint +4.49%)
VR vs FES (%)
1.0
0.8
0.6
0.4
0.2
0.0
M2
S2
N2
K2
K1
O1
P1
Q1
M4
MS4
MN4
M6
2N2
EPS2
J1
L2
M3
M8
MF
MKS2
MM
MSF
MU2
N4
NU2
R2
S1
S4
SA
SSA
T2

3 contexts.

(i) Coastal Tides

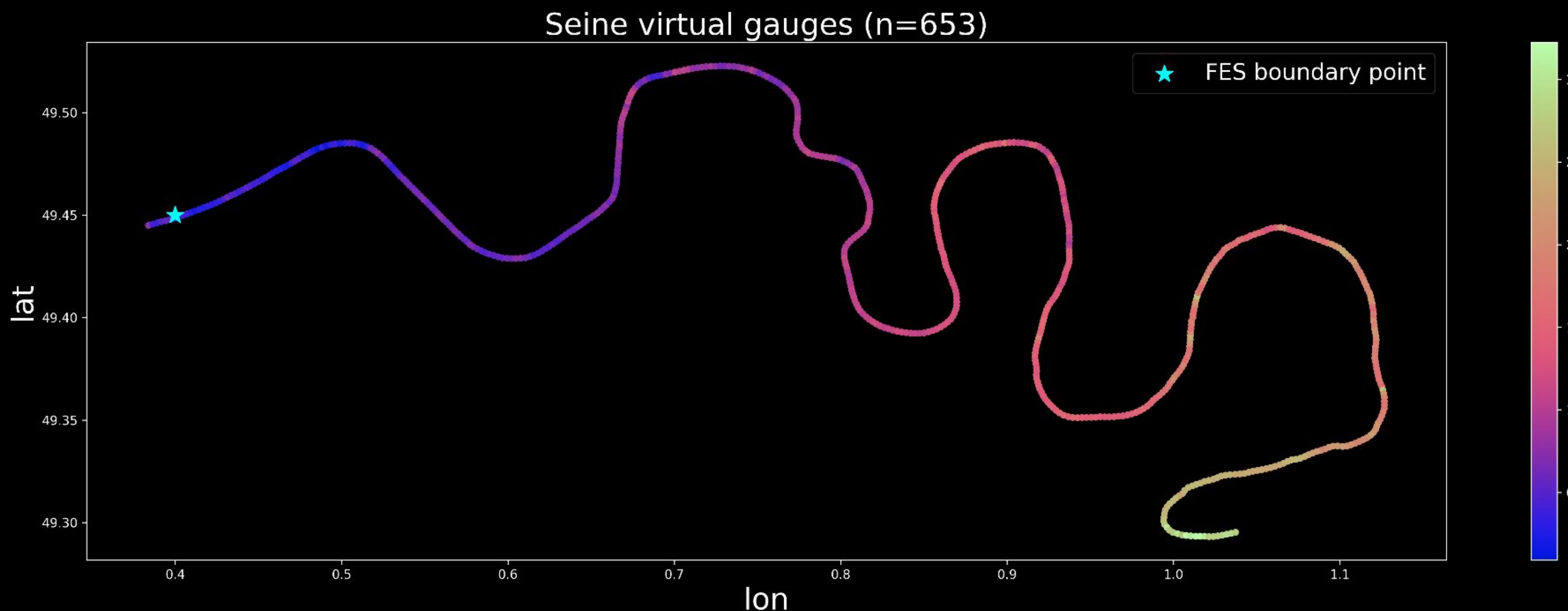


(iii) Surges



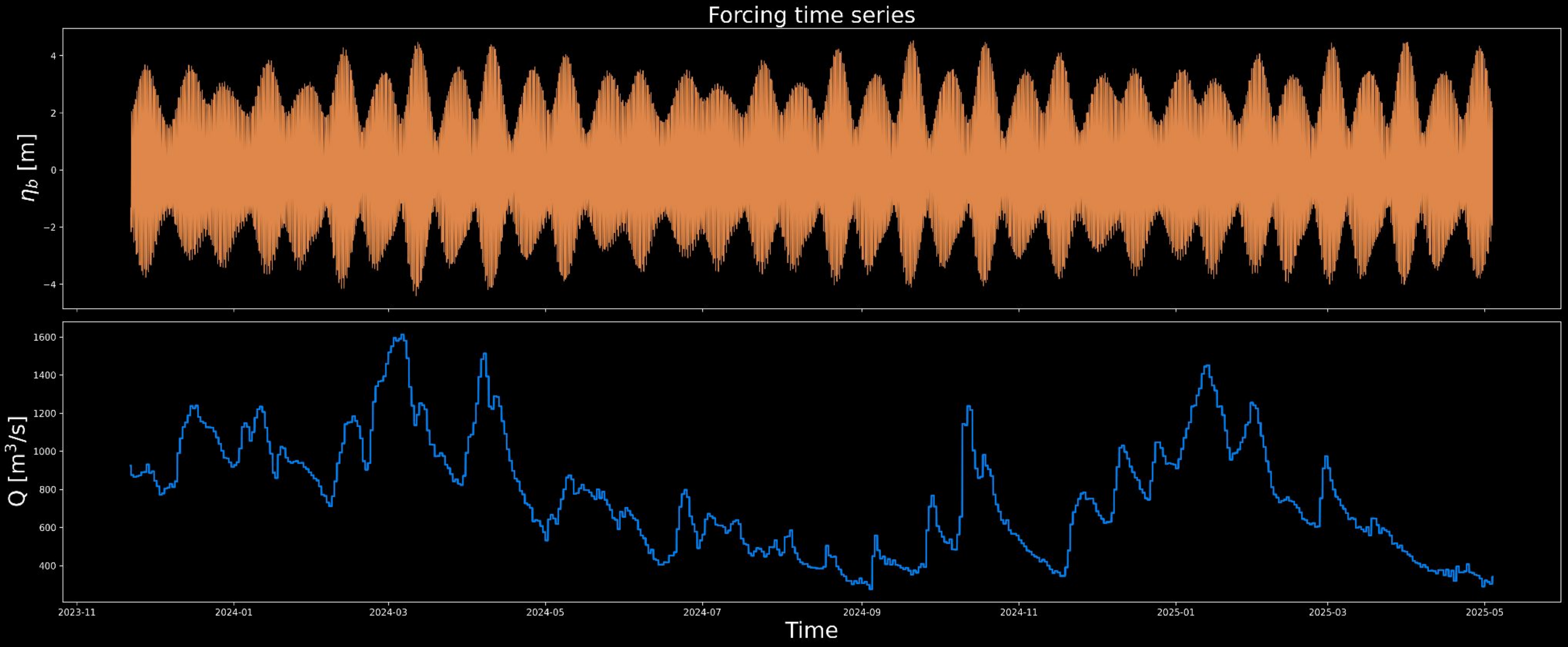
(ii) Tidal Rivers

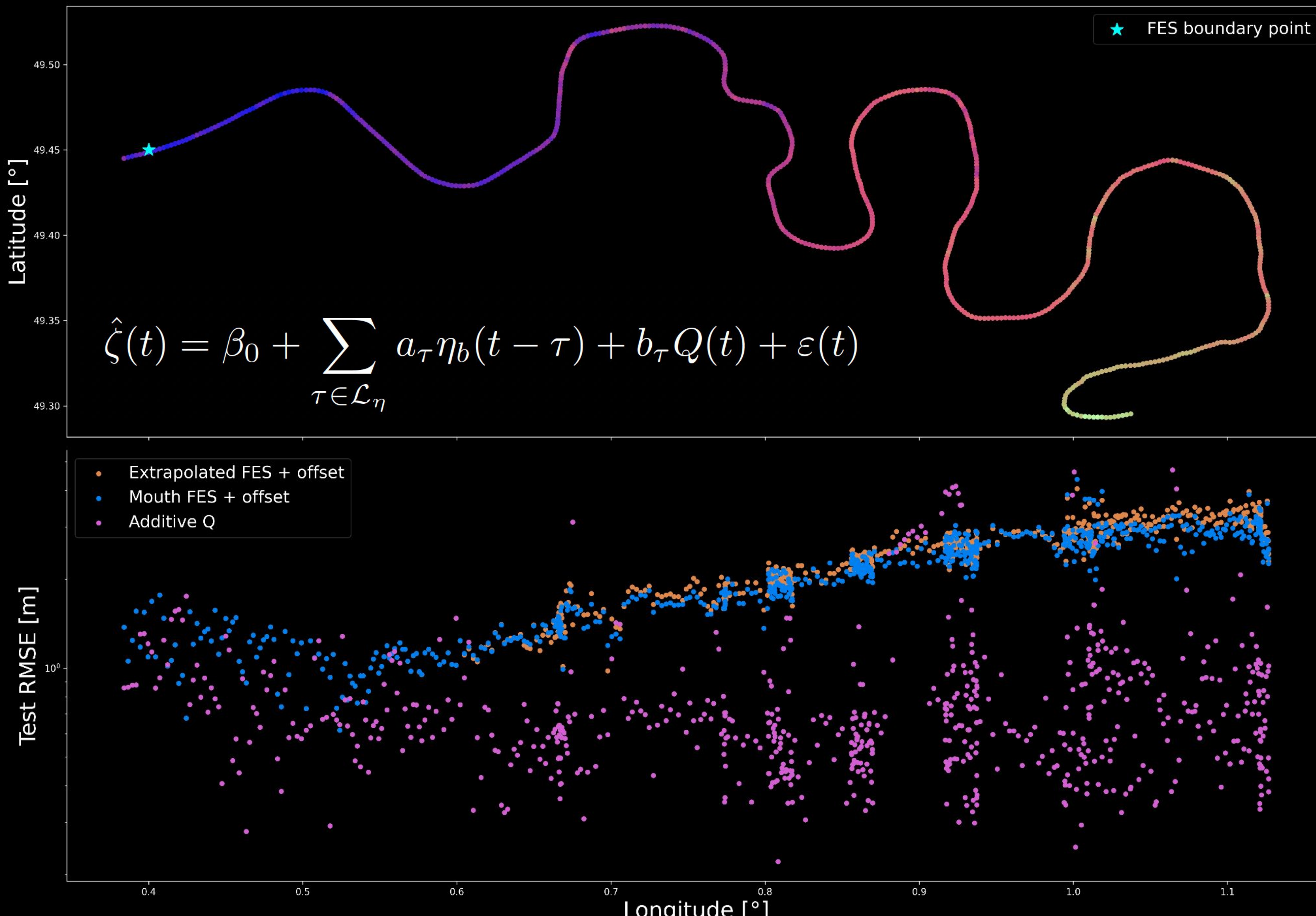
Extending global tide models up tidal rivers



$$\hat{\zeta}(t) = \beta_0 + \sum_{\tau \in \mathcal{L}_\eta} a_\tau \eta_b(t-\tau) + b_\tau Q(t) + \varepsilon(t)$$

Tidal Rivers

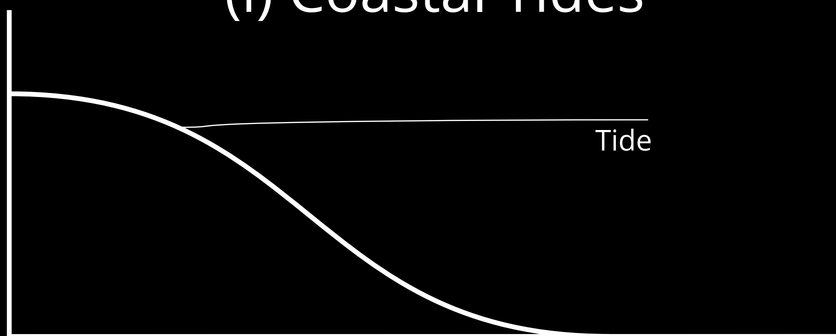




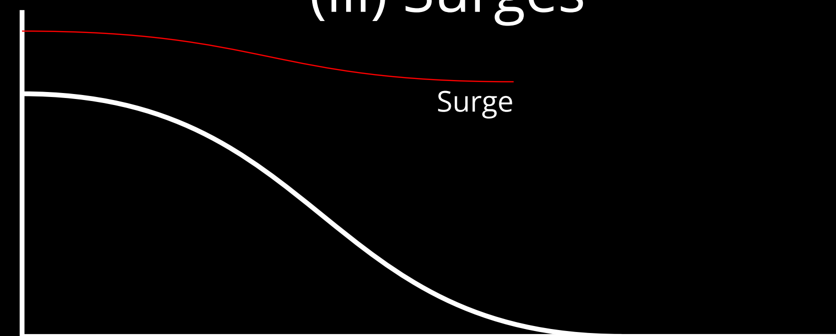
FES boundary point
Latitude [°]
49.50
49.45
49.40
49.35
49.30
$\hat{\zeta}(t) = \beta_0 + \sum_{\tau \in \mathcal{L}_\eta} a_\tau \eta_b(t-\tau) + b_\tau Q(t) + \varepsilon(t)$
Extrapolated FES + offset
Mouth FES + offset
Additive Q
Test RMSE [m]
10^0
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
Longitude [°]

3 contexts.

(i) Coastal Tides

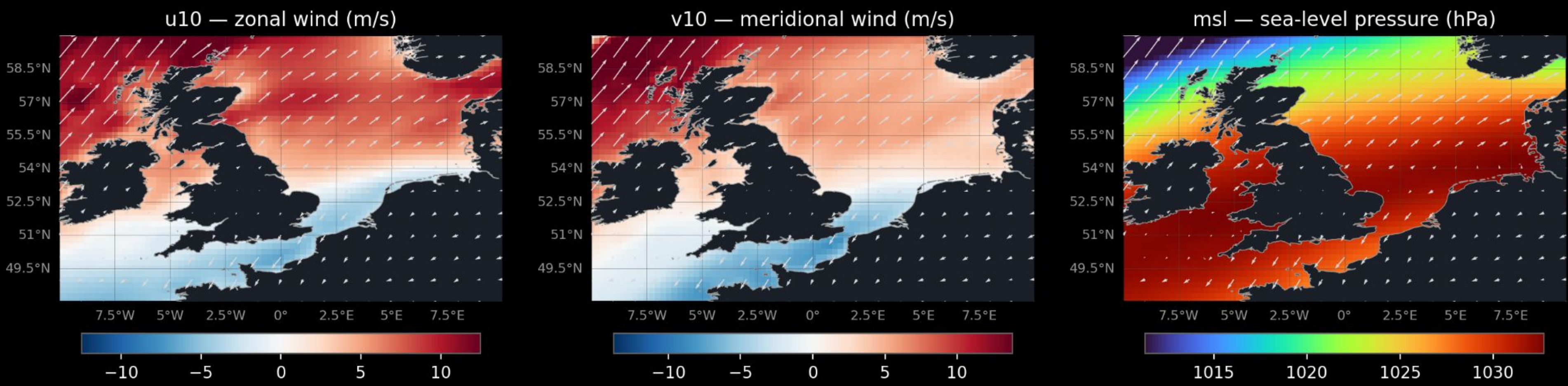


(iii) Surges

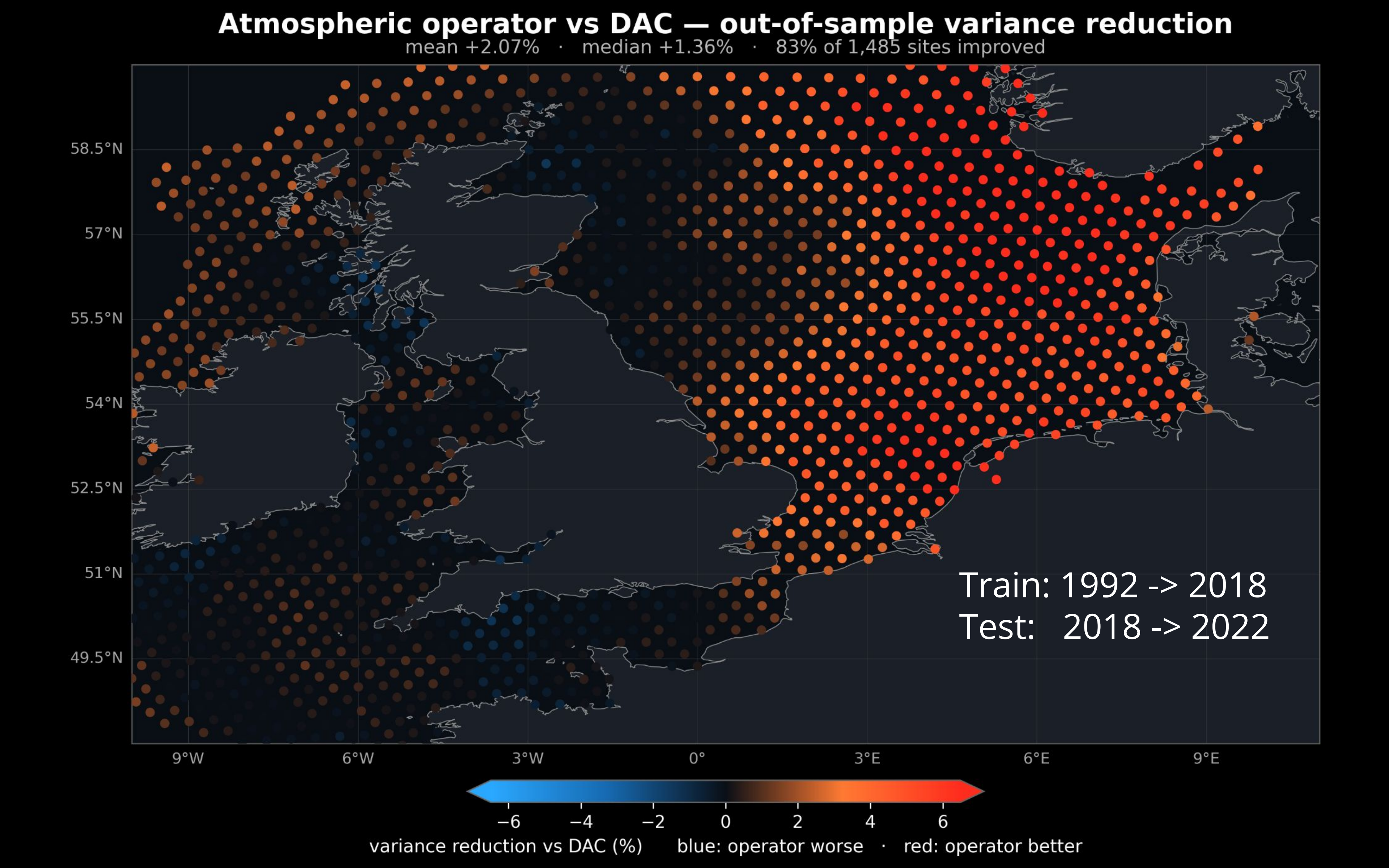


(ii) Tidal Rivers

DAC



$$
\begin{aligned}
\hat{\zeta}(t) = {} & \sum_{\tau} \alpha_u(\tau) \cdot u(t-\tau) + \sum_{\tau} \alpha_v(\tau) \cdot v(t-\tau) + \sum_{\tau} \alpha_{\mathrm{msl}}(\tau) \cdot \mathrm{MSL}(t-\tau) \\
& + \sum_{\tau} \beta_u(\tau) \cdot \bar{u}_{5^\circ}(t-\tau) + \sum_{\tau} \beta_v(\tau) \cdot \bar{v}_{5^\circ}(t-\tau) + \sum_{\tau} \beta_{\mathrm{msl}}(\tau) \cdot \bar{\mathrm{MSL}}_{5^\circ}(t-\tau)
\end{aligned}
$$

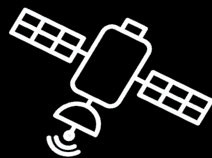

Atmospheric operator vs DAC — out-of-sample variance reduction
mean +2.07% · median +1.36% · 83% of 1,485 sites improved
58.5°N
57°N
55.5°N
54°N
52.5°N
51°N
49.5°N
9°W
6°W
3°W
0°
3°E
6°E
9°E
Train: 1992 -> 2018
Test: 2018 -> 2022
−6
−4
−2
0
2
4
6
variance reduction vs DAC (%)
blue: operator worse · red: operator better

Learned admittances are interpretable.

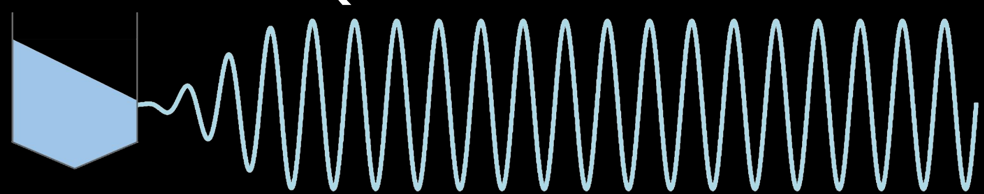
Conclusions.

- Broadband response operators provide an interesting way forward in **augmenting** existing global tide models for hard problems.
- Optimal combinations of global tide models should be considered further
- These operators when modified to allow for river modulation can **enable existing global tide models to extend up tidal rivers.**
    - The broadband operators can be learned from sparse SWOT data.
- Response type approaches may be useful for **identifying**, and in some case **learning**, deficiencies of surge models.

Special thanks to collaborators, partners, and funders… Especially High-freq!



Questions?



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NOVELTIS
LEGOS
CLS
cnes
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How do we estimate w?

$$\zeta = \mathbf{w}^{\mathsf{T}} X + \epsilon$$

Linear Regression!

$$\zeta = [\zeta(t_1), \zeta(t_2), \ldots, \zeta(t_N)]$$

$$X = [\hat{\zeta}(t_i - \tau), \ldots, \hat{\zeta}(t - L), \hat{\zeta}(t_i - \tau_1)\hat{\zeta}(t_i - \tau_2), \ldots]^{\mathsf{T}}$$

Monahan, T., Tang, T., Roberts, S., & Adcock, T. A. A. (2025). Tidal corrections from and for SWOT using a spatially coherent variational Bayesian harmonic analysis. Journal of Geophysical Research: Oceans, 130, e2024JC021533. https://doi.org/10.1029/2024JC021533

Why not higher order?

$$\zeta(t) \approx b + \sum_{n=1}^{P} \sum_{\tau_1,\ldots,\tau_n=0}^{L} w_n(\tau_1,\ldots,\tau_n) \prod_{j=1}^{n} \hat{\zeta}(t-\tau_j),$$

Classical response method:

Constructs narrow-band filters over tidal species.

M6 = trilinear product M2 X M2 X M2

Lag-based approach

Constructs broad-band frequencies across entire tidal spectrum.

M6 = product of M4 X M2? Or even redistribution of M6 already in model.