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Optimal Retracking Solutions for Enhanced Ocean Altimetry

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Outline

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- Context and motivation
- Waveform noise and estimators
- Optimality and unbiasedness for enhanced parameters estimation over Ocean: Sentinel-6MF Ku band data
- Beyond the standard approach: a Bayesian perspective
- Beyond Sentinel-6: Preliminary results for Sentinel-3 PLRM
- Conclusions

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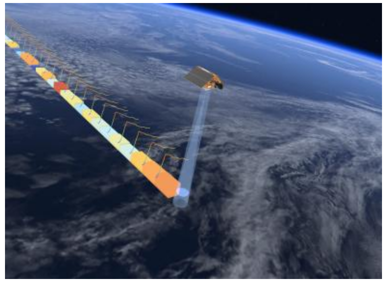
Optimal Estimation of the Geophysical Parameters Over Ocean for the Sentinel-6MF Reference Mission

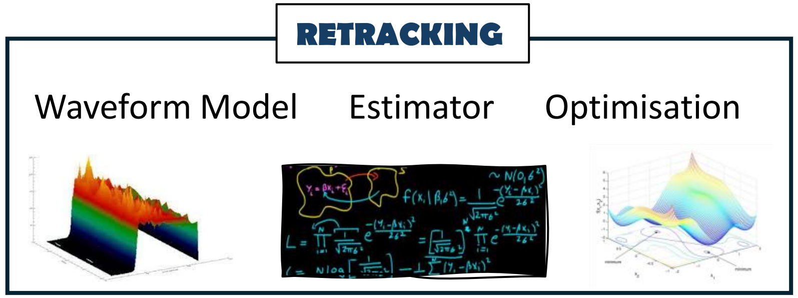
Anna Mangilli, Thomas Moreau, Claire Maraldi, Marta Alves, Oriane Gassot, Alejandro Egido, Laiba Amarouche, Fanny Piras, Pierre Thibaut, François Boy, Nicolas Picot, and Franck Borde

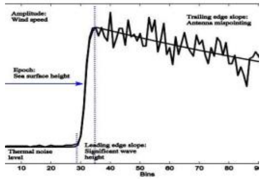
[Mangilli et al. IEEE 2025]

Context and motivation

- **Goal of retracking analysis: accurate and efficient estimation of the geophysical parameters.**
- **Key step to build robust records of Essential Climate Variables** from radar Altimetry for a rigorous assessment of the climate change (MSL, SWH, ...)

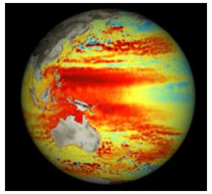


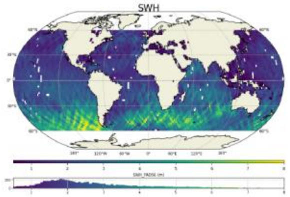




Corrections

Sea Level, waves, σ_0, ...





- For long term stability, precision and unbiasedness, **retracking algorithms should account for:**
    - **The real time evolution of instrumental properties**, as the Point Target Response (**PTR**)
    - a **realistic waveform noise characterization**
- Operational retrackers do not (MLE4) or only partially (NUM RTK) account for these features

An **accurate noise characterization is necessary to have optimal and unbiased estimation of the geophysical parameters**

Waveform noise and estimators

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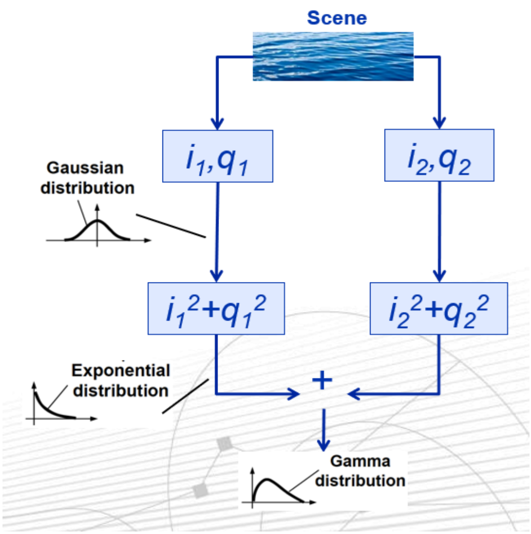
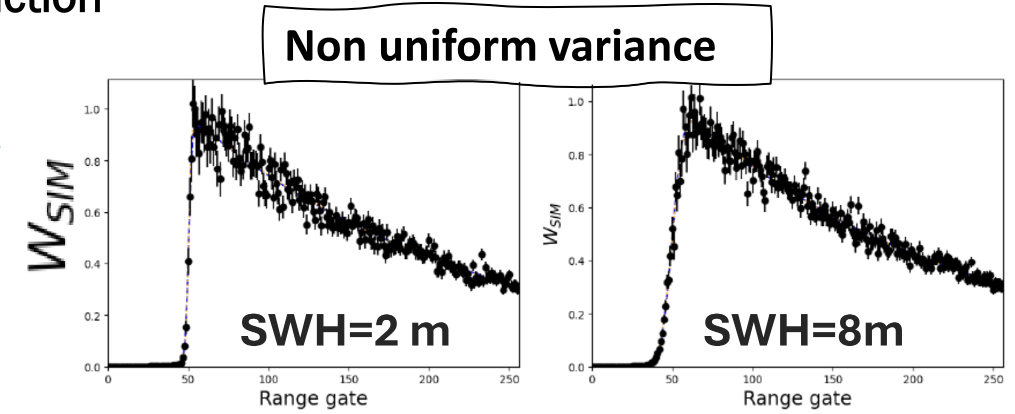


The observed radar waveform is a backscattered radar wave signal corrupted by a **multiplicative speckle noise** whose distribution is described by a **Gamma function**

Theoretical signal =**waveform model**

Gamma distributed multiplicative noise

$$S(\boldsymbol{k},\boldsymbol{\theta}) = \boxed{s_t(\boldsymbol{k},\boldsymbol{\theta})} + \boxed{n'(\boldsymbol{k},\boldsymbol{\theta})}$$



- **Maximum Likelihood Estimator** (**MLE**): **non-Gaussian Likelihood** [Halimi+2015], expansive numerical optimization (ex. `Adpative` retracker [Tourain+2021, Thibaut+2021] e.g. Jason2&Jason3)

Data

$$-ln\mathcal{L} = \sum_n L_n \frac{y(k_n)}{s_t(k_n,\theta)} - \sum_n (L_n - 1)\, ln(y(k_n)) + \sum_n L_n\, ln(s_t(k_n,\theta)) \qquad L_n = ENL(k_n,\theta)$$

Model

- **Unweighted Ordinary Least Square estimator** used in **operational retrackers (MLE4, Numerical_RTK)**: assume uniform variance

Cost function : $(\boldsymbol{y} - \boldsymbol{s}_t)^T (\boldsymbol{y} - \boldsymbol{s}_t)$

Waveform noise and estimators

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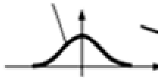
Theoretical signal
=**waveform model**

$$S(k,\theta) = \boxed{s_t(k,\theta)} + \boxed{n'(k,\theta)}$$

Gamma distributed multiplicative noise

On board averaging of N echos. **For N >>1**, the Gamma distribution converge to a Gaussian distribution (central limit theorem)

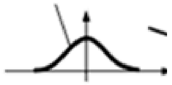
Gaussian distribution



$$n'(k,\theta) \sim \mathcal{N}\left(0, \frac{s_t^2(k,\theta)}{ENL}\right)$$

- **MLE: likelihood function** can be approximated to a **multi variate Gaussian distribution** [Rodriguez 1998, Halimi et al 2015]

Gaussian distribution



$$-2ln\mathcal{L} \underset{L>>1}{\sim} (\boldsymbol{y} - \boldsymbol{s_t})^T \boldsymbol{C}^{-1} (\boldsymbol{y} - \boldsymbol{s_t}) + \text{const}$$

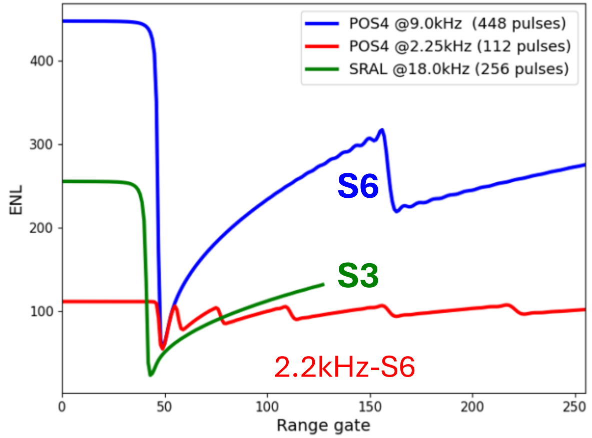
$$C_{k_n k_n} = \frac{s_t^2(k_n,\theta)}{\mathrm{ENL}(\mathrm{k_n},\theta)}$$

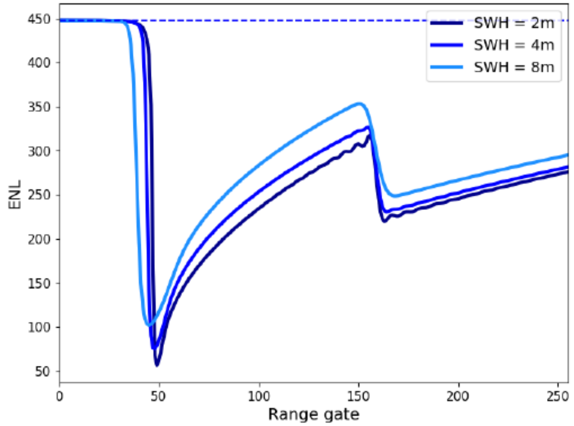
- MLE equivalent to **Weighted Least Squares (WLS)** estimator:

$$\hat{\theta}_{MLE} = \text{argmin}_{\theta}(-\ln\mathcal{L}) \sim \text{argmin}[(\boldsymbol{y} - \boldsymbol{s}_\mathrm{t})^\mathrm{T} \boldsymbol{C}^{-1} (\boldsymbol{y} - \boldsymbol{s}_\mathrm{t})] = \hat{\boldsymbol{\theta}}_{WLS}$$

- Advantage of WLS approximation: efficient numerical computation with **LS optimisation** (not the case with the full non-Gaussian likelihood)

Effective Number of Looks:





Sentinel-6 retracking solutions

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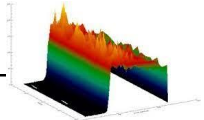
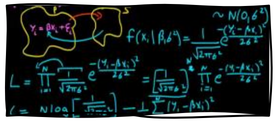
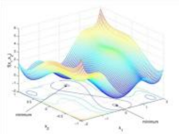
Weighted solutions:

- Include model-dependent noise variance directly in the fit as weights
- Main computational cost arises from PTR convolution, not the optimization step
- Iterative noise estimation increases computation time by ~2–3× compared to unweighted numerical solutions

Retracker name	Model	Estimator	Optimization
FastAdaptiveS6	Adaptive (numerical PTR)	Weighted Least Square (equiv. MLE)	Levenberg–Marquardt
RTK_NUMLE_S6	Brown with numerical PTR	Weighted Least Square (equiv. MLE)	Levenberg–Marquardt
NUM_OLS_AD_S6	Adaptive (numerical PTR)	Unweighted Ordinary Least Square (OLS)	Levenberg–Marquardt
NUM_OLS_BH_S6	Brown with numerical PTR	Unweighted Ordinary Least Square (OLS)	Levenberg–Marquardt
Bayes6	Adaptive / Brown (numerical PTR)	Bayesian	MCMC

WEIGHTED solutions

UNWEIGHTED solutions

Comparison with **operational numerical retracker PDAP NR**

NUM_RTK S6 PDAP	Brown with numerical PTR	Unweighted Ordinary Least Square (OLS)	Levenberg–Marquardt	**UNWEIGHTED**

Validation on simulations

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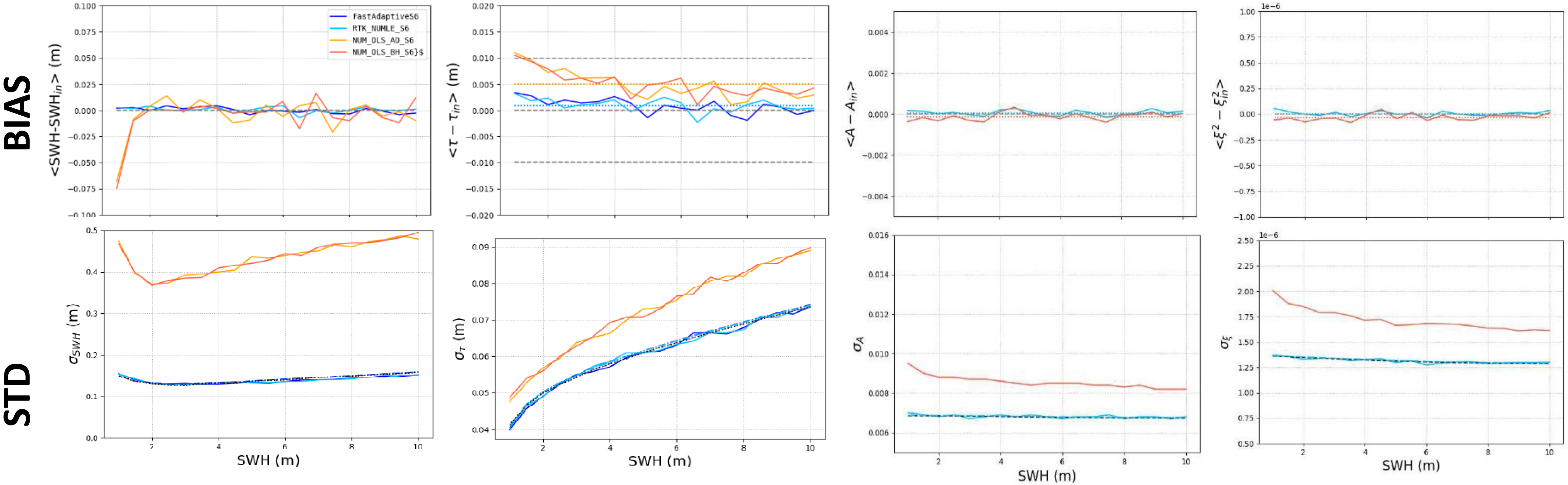
Comparison between the **WEIGHTED** and UNWEIGHTED (=same model, no weights)

SWH **EPOCH** **Ampl** **Mispointing**



----- Theory: Cramer-Rao (CR) bounds of minimum variance

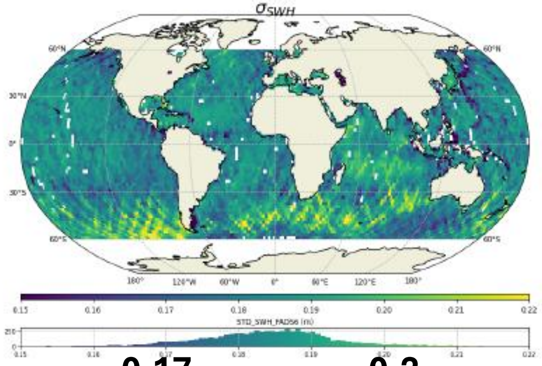
WEIGHTED solutions: **unbiased and optimal=**parameters std **compatible with Cramer-Rao bounds**. Significative noise reduction for all the parameters wrt unweighted OLS.

UNWEIGHTED: **suboptimal=parameters variance significantly higher than CR bounds,** epoch bias up to ~1cm and SWH bias of ~6 cm for small SWH=1m

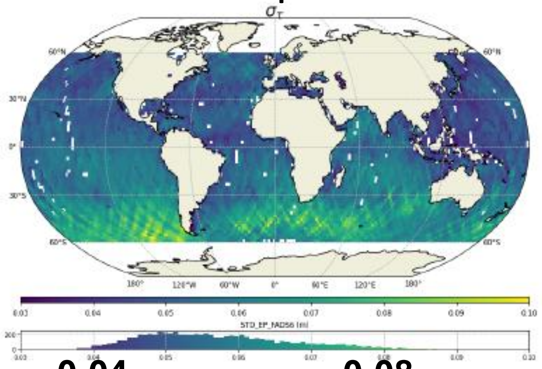
Results for Sentinel-6A Ku band

- **Significant improvement of the weighted estimation: ~60% for SWH, ~12% for SSH up to 50% for σ_0** wrt the **unweighted** num rtk PDAP
- Good fit performances (reduced chi2 compatible with 1) and consistent results with both waveform models Adaptive and Brown
- Estimation of **parameter uncertainties at 20 Hz** as direct **output of the retracker**

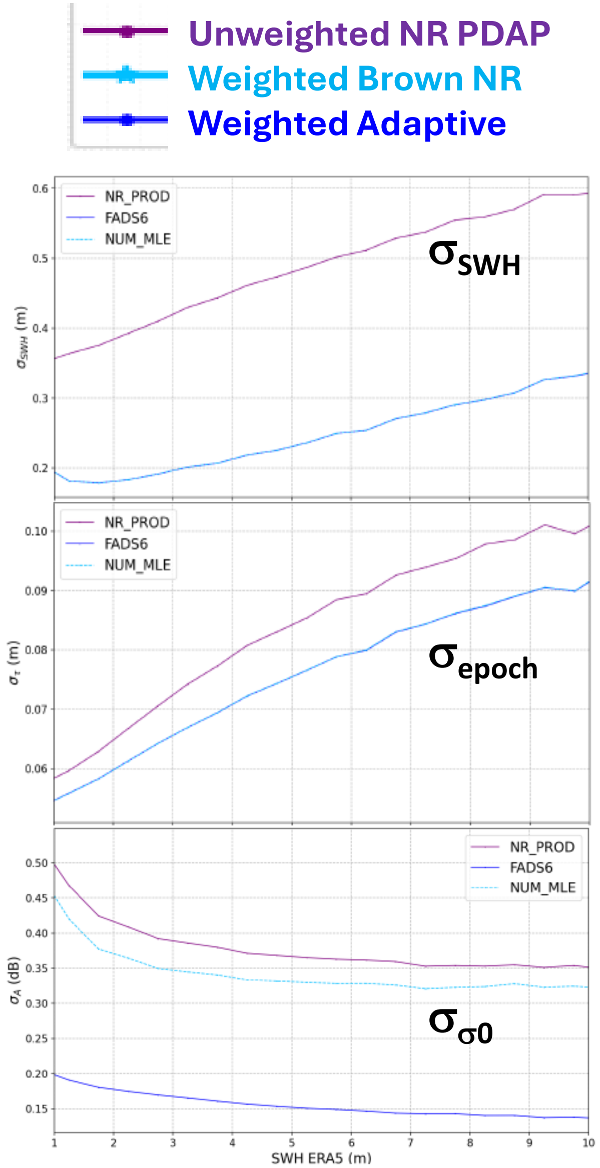
σ_{SWH}

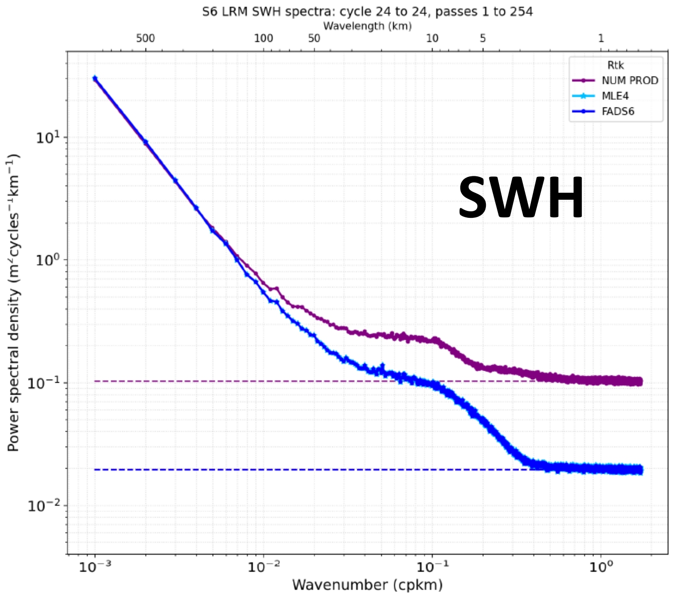


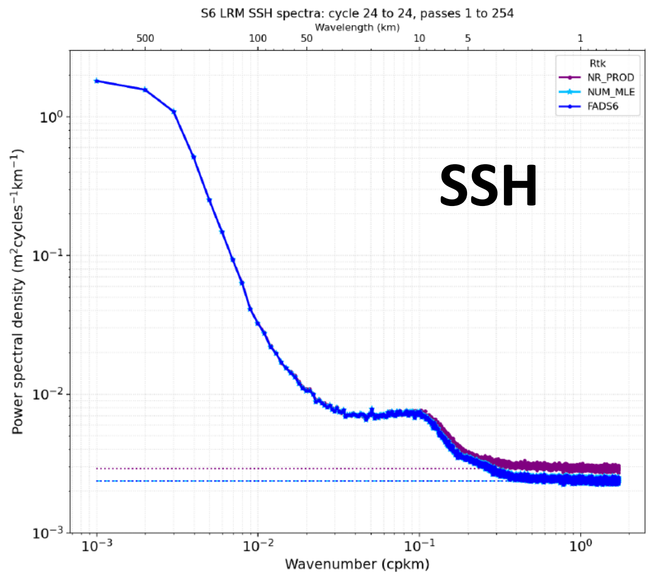
σ_{epoch}



$$\sigma_\theta = \sqrt{diag(P_\theta)} = \sqrt{diag([\mathbf{J}\hat{\mathbf{C}}^{-1}\mathbf{J}]^{-1})}$$







Results for Sentinel-6A Ku band

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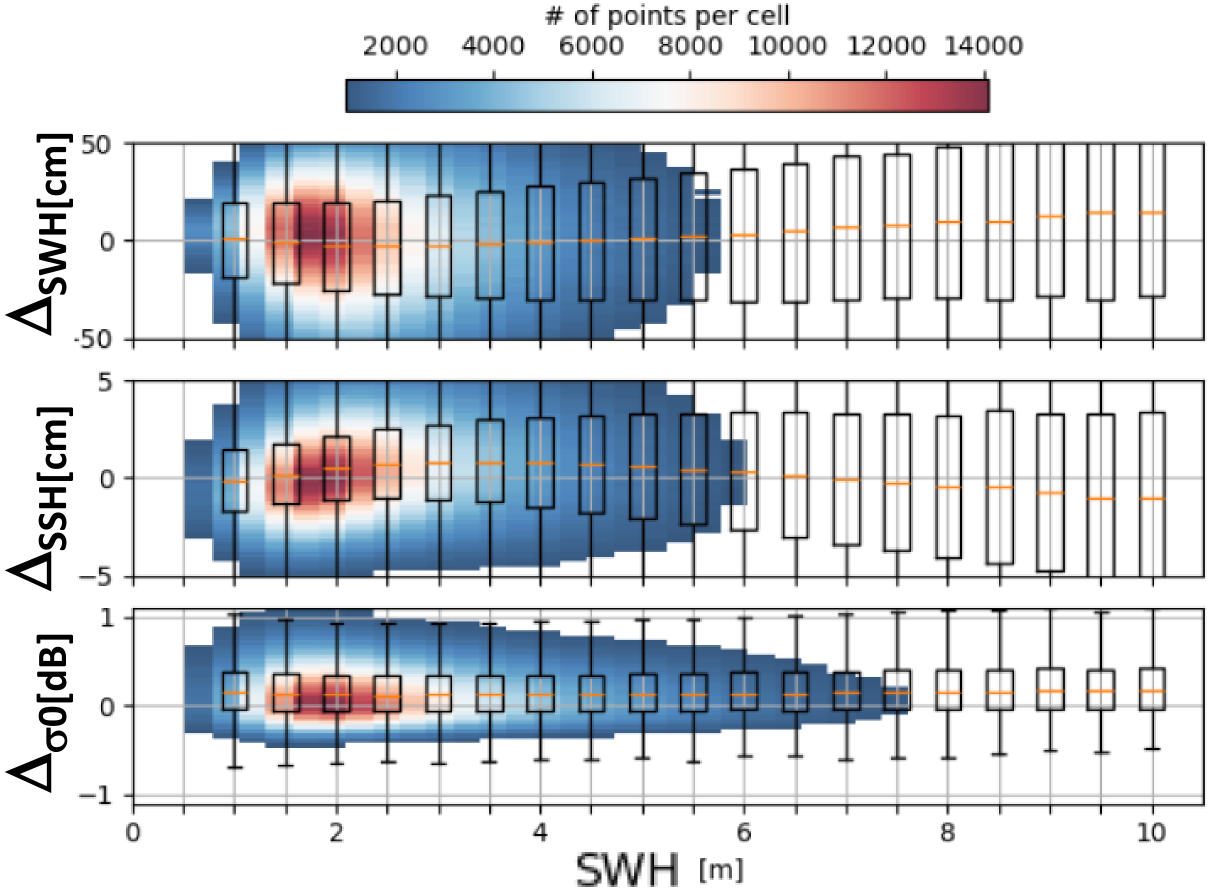
Weighted – Unweighted (PDAP) differences (1 data cycle)



- Evolution of the median of the differences in line with [Egido+2018], partially linked to pulse-to-pulse effect
- Here 2 effects: pulse-to-pulse and estimator change
- @SWH=2 m (peak of the distributions): differences in line with simulations (ΔSSH = 0.0062 m)
- Additional investigations on-going comparing weighted vs unweighted on S6 data at 9kHz and 2kHz

Impact on epoch-SWH correlation

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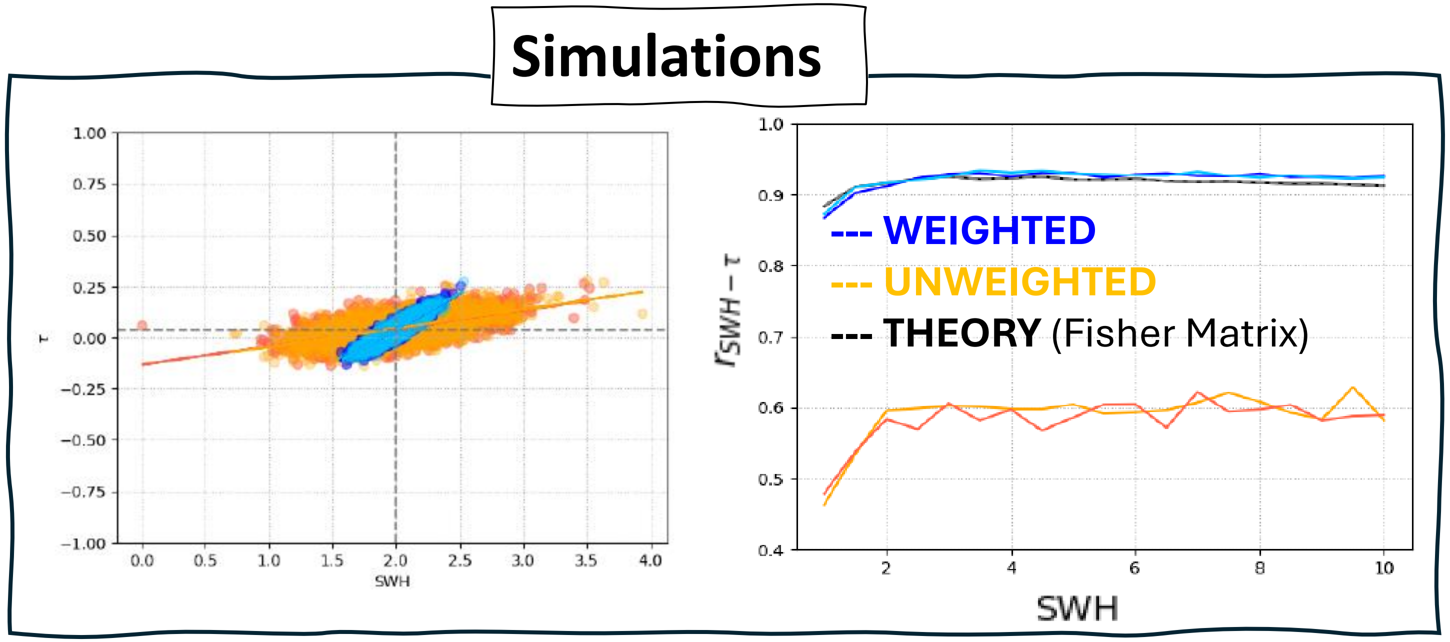

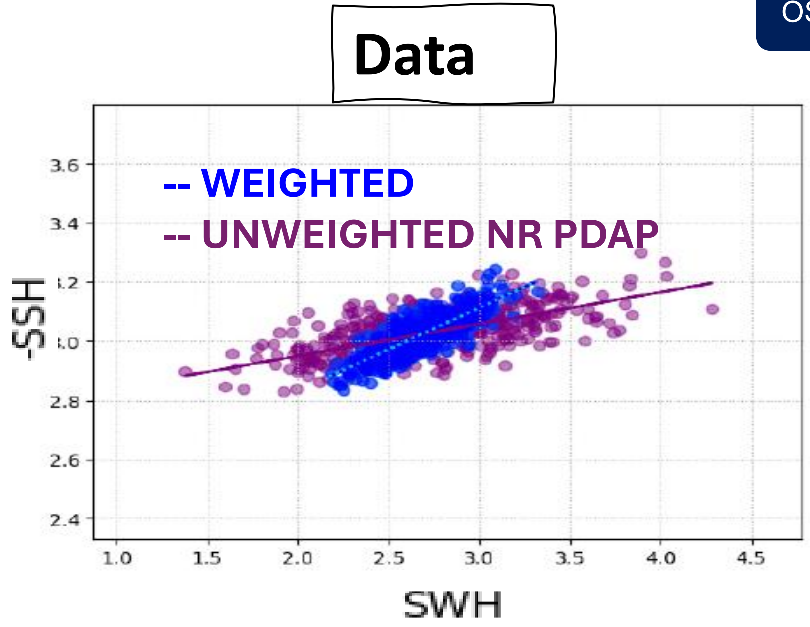

- Because of the **sub-optimality of the unweighted estimation (= larger variance)**, in particular for SWH, the **SWH-epoch correlation is artificially lowered**.
- The correlation is correctly estimated only if the waveform noise is properly accounted in the retracker
- An incorrect estimation of SWH-SSH correlation can have an **impact on the estimation of SSB&HFA correction**
- **Impact on SWH and SSH spectra** should also be further investigated

A new approach: Bayesian retracking

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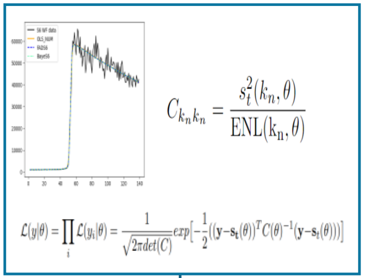
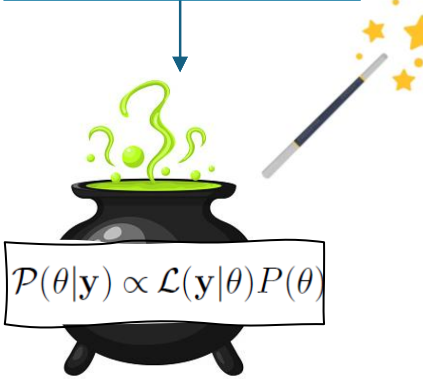
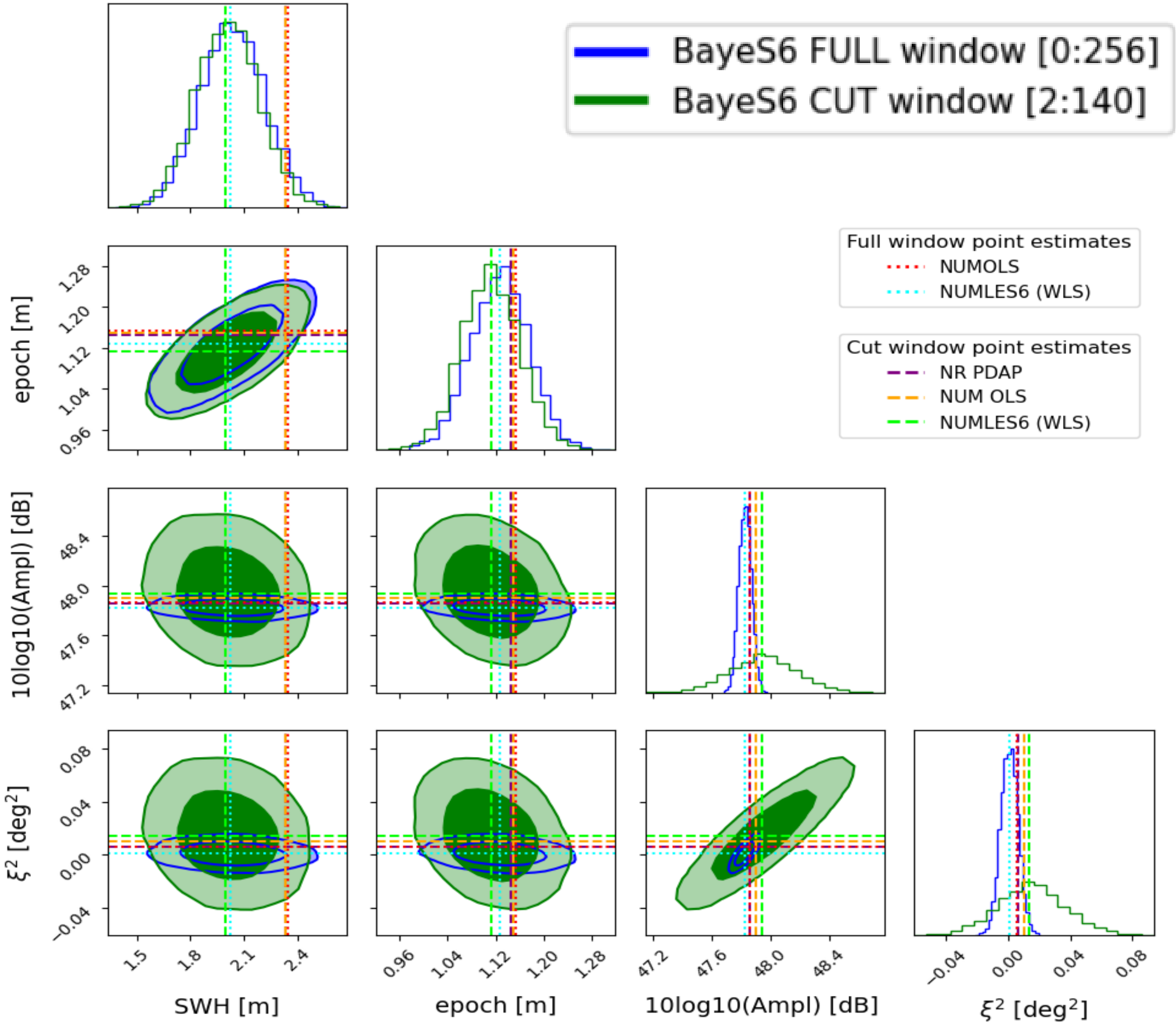
- Provides a **probability distribution of the parameters** rather than a point estimate







More details @ M. Alves S6VT talk !

- Robust estimation of parameters uncertainty and correlations
- WLS results fully consistent
- Compact and physically interpretable summary of parameter estimations.
- Useful to compare different configurations in a glance: full vs cut window example here
- Possible future applications: extended waveform models, full likelihood consistency, HR 2D data

Results on S3 Ku-band PLRM

PRELIMINARY

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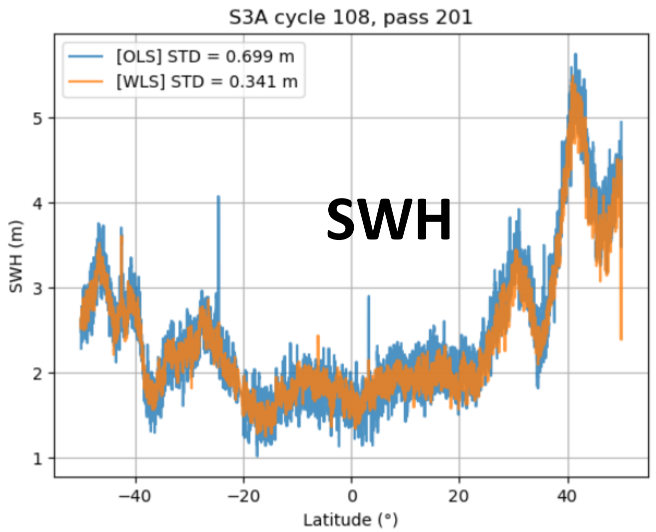
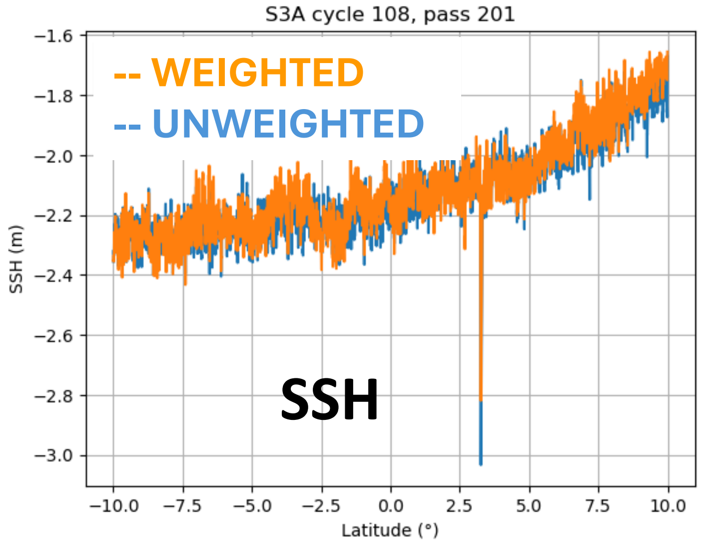
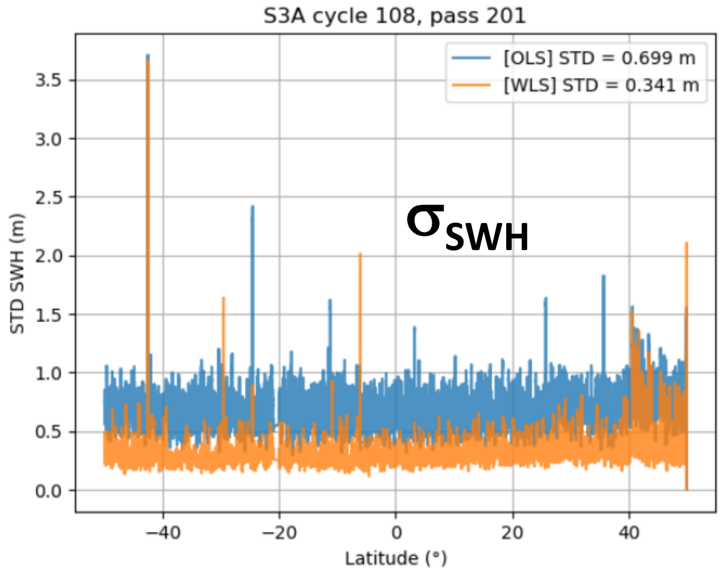
- L2 products based on PLRM data mainly used for consistency (SAR/multi-mission for climate applications) and for S3 wind estimation (WAVE-TAC products): interesting to improve the performance
- S3A 20 Hz PLRM data over ocean processed with the WLS retracker (Brown model, numerical PTR, sinc2 for now) for 1 track (cycle 108, pass 201)
- Significant improvements for all geophysical parameters: reduction of uncertainties of ~50% for SWH and ~10% for SSH in line with expected performances. Improved comparison with HR data?
- Promising results. Validation and more operational implementation ongoing on SPP@CNES.







Conclusions and perspectives

- Radar waveforms are inherently affected by **multiplicative speckle noise** which **must be accounted for in parameter estimation**
- **Optimal retrackers (WLS / Bayesian)** developed for Sentinel-6 Ku-band:
  - Provide unbiased and optimal parameters estimation with **significant precision improvement vs operational solutions** (MLE4,NUM_RTK): ~**60% for SWH**, ~**12% for SSH** and **up to 50% for σ_0**.
  - WLS iterative noise estimation: computation time ~2–3× wrt unweighted numerical solutions
  - Enable a **consistent characterization of parameter correlations** (e.g., ***SWH–epoch***) that are instead underestimated with current operational solutions because of the increased parameter variance.
  - The impact on derived corrections (e.g., **SSB, HFA**) should be further assessed
- Application to **Sentinel-3 PLRM** shows promising results

On going work and perspectives:

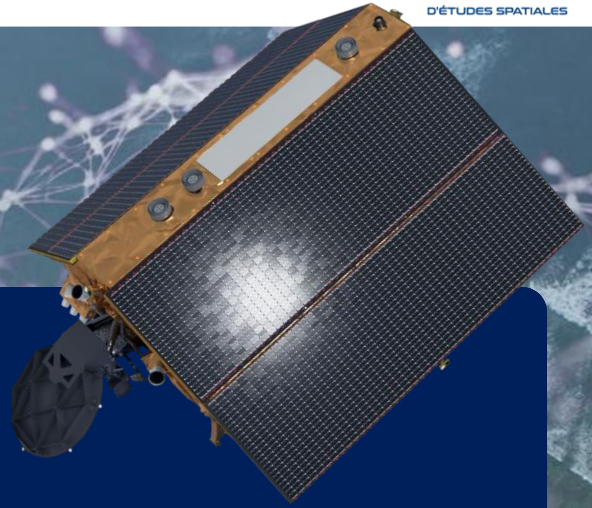
- Implementation of the WLS solution **@SPP (CNES/CLS): S6 Ku band, C-band** + S3 PLRM
- WLS solutions applied to C-band & **S6B data** at 9kHz and 2.2kHz

More details @ S6VT talk !

- Applicability to **SAR data** currently under investigation (**2D SAR S6** R&D study with CNES): key point is the noise characterization

More details @ F. Ehlers talk

- Optimal solutions for **extended waveform model** (skewness, wave groups)



Thank you

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BayeS6 on sims: cut vs full window

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