

Optimal Retracking Solutions for Enhanced Ocean Altimetry



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- Context and motivation
- Waveform noise and estimators
- Optimality and unbiasedness for enhanced parameters estimation over Ocean: Sentinel-6MF Ku band data
- Beyond the standard approach: a Bayesian perspective
- Beyond Sentinel-6: Preliminary results for Sentinel-3 PLRM
- Conclusions

Optimal Estimation of the Geophysical Parameters Over Ocean for the Sentinel-6MF Reference Mission

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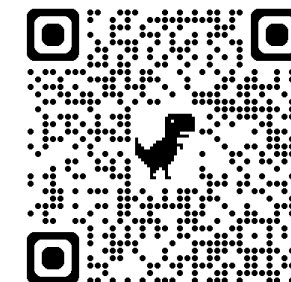
Abstract—Satellite radar altimetry has been used for over 30 years to measure sea surface height (SSH) variations to build records of essential climate variables like the mean sea level (MSL) for a robust assessment of climate change. A key step of the data processing toward this goal is the retracing, that is, the statistical analysis of the radar waveforms to estimate the geophysical parameters. For a robust and optimal estimation, retracing algorithms should account for a reliable waveform model, the time-varying instrumental point target response (PTR), and an accurate description of the waveform noise. However, this is not the case for the operational retracing solutions implemented in the ground segments, which specifically make use of an unweighted estimator, therefore not accounting for the waveform speckle noise. In this article, we present a novel computationally efficient retracing solution for Ku-band low resolution mode (LRM) data that accounts for both the in-flight PTR and a realistic waveform noise through a weighted estimator defined to be statistically equivalent to a maximum likelihood estimator. We consider two waveform models: the Adaptive model and the numerical Brown model, for a consistent comparison with operational retracing solutions. We focus on the current reference mission, Sentinel-6 MF, for which an accurate noise characterization is crucial to account for the pulse-to-pulse correlations resulting from the higher pulse repetition frequency (PRF) compared to conventional configurations. We demonstrate that the novel retracing solution is optimal, providing parameter uncertainties compatible with the Cramér-Rao bounds of minimum variance, and unbiased, with a bias up to 1 cm in the epoch estimation and suboptimality for all parameters is found for the unweighted solutions. We validated the algorithm on realistic simulations and applied it to one cycle of Sentinel-6MF LRM 20-Hz data, demonstrating significant improvements in the precision of estimated parameters: $\sim 60\%$ for significant wave height (SWH), $\sim 12\%$ for the epoch, and $\sim 50\%$ for σ_0 compared to current operational solutions. We report for the first time, geophysical parameter uncertainties consistently computed at 20 Hz as output of the retracker. We also introduce an innovative Bayesian approach for analyzing waveform data to complement current solutions, providing a robust method for the estimation of the parameter uncertainties and correlations. Finally, we perform a comprehensive analysis of parameter correlations on simulations and data, compared to theoretical expectations based on Fisher matrix analysis, demonstrating the importance of optimality for a correct estimation. Specifically, we show that the unweighted, suboptimal retracing solution significantly underestimates the epoch-SWH correlation by a factor of ~ 1.5 in the correlation coefficients, compared to the optimal solution and theoretical expectations. This could significantly impact the estimation of corrections such as sea state bias (SSB) and high-frequency adjustment (HFA), warranting further assessment. Overall, optimal retracing solutions should be considered to derive robust long-term sea-level records for climate research using past, current, and future altimetry missions.

Index Terms—Geophysical parameters, ocean, radar altimetry, reference missions, retracker, Sentinel-6MF.

I. INTRODUCTION
ADVANCING research depends on complex models to make sense of growing datasets, demanding continuous refinement of statistical tools. The key challenge is no longer the availability of the data, but our ability to analyze and interpret it. These considerations fully apply to satellite radar altimetry, from the processing of radar waveforms for surface height estimation to the interpretation of sea-level trends at local and global scales. Indeed, satellite radar altimetry, being the most powerful technique to measure the sea surface height (SSH), plays a crucial role in building robust records of essential climate variables and, more specifically, in accurately measuring sea state and mean sea level (MSL) at global and regional scales, thereby addressing important scientific questions for a rigorous assessment of climate change. The key step toward this goal lies in the accurate estimation of the geophysical parameters, among which are SSB and

the statistical retracing analysis have been made to the reference [1], [4], [5]. These and critical key and precision es, driven by the s. These stringent sions, particularly size and

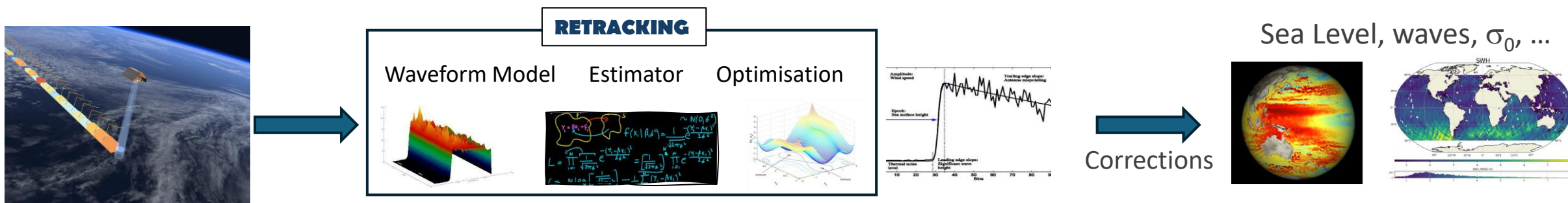
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Context and motivation

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- **Goal of retracking analysis: accurate and efficient estimation of the geophysical parameters.**
- **Key step to build robust records of Essential Climate Variables** from radar Altimetry for a rigorous assessment of the climate change (MSL, SWH, ...)

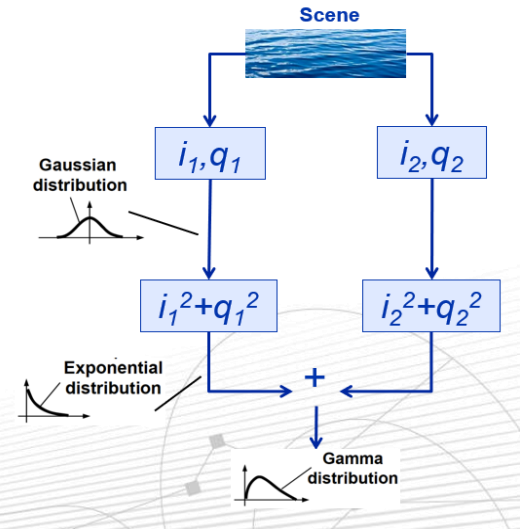


- For long term stability, precision and unbiasedness, **retracking algorithms should account for:**
 - **The real time evolution of instrumental properties**, as the Point Target Response (**PTR**)
 - **a realistic waveform noise characterization**
- Operational retrackers do not (MLE4) or only partially (NUM RTK) account for these features

An accurate noise characterization is necessary to have optimal and unbiased estimation of the geophysical parameters

Waveform noise and estimators

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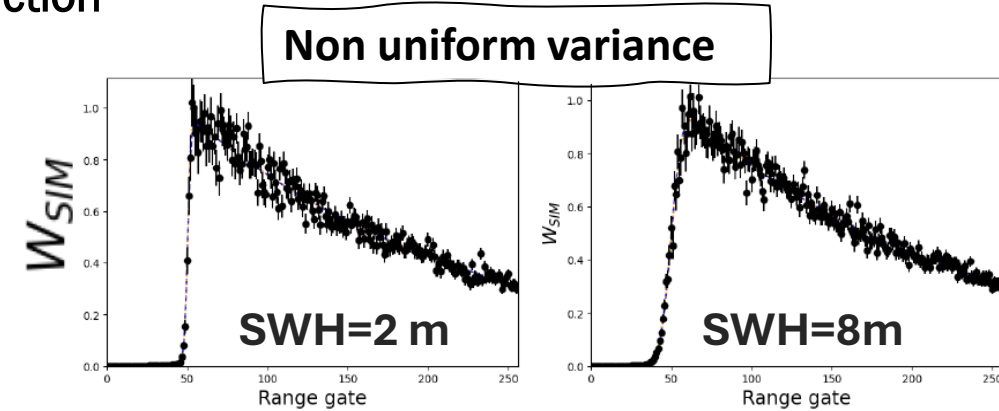


The observed radar waveform is a backscattered radar wave signal corrupted by a **multiplicative speckle noise** whose distribution is described by a **Gamma function**

Theoretical signal
= **waveform model**

Gamma distributed
multiplicative noise

$$S(k, \theta) = \boxed{s_t(k, \theta)} + \boxed{n'(k, \theta)}$$



- **Maximum Likelihood Estimator (MLE): non-Gaussian Likelihood** [Halimi+2015], expansive numerical optimization (ex. **Adaptive** retracker [Tourain+2021, Thibaut+2021] e.g. Jason2&Jason3)

$$-\ln \mathcal{L} = \sum_n L_n \frac{\text{Data } y(k_n)}{\text{Model } s_t(k_n, \theta)} - \sum_n (L_n - 1) \ln(y(k_n)) + \sum_{n|} L_n \ln(s_t(k_n, \theta)) \quad L_n = ENL(k_n, \theta)$$

- **Unweighted Ordinary Least Square estimator** used in **operational retrackers (MLE4, Numerical_RTK)**: assume uniform variance

Cost function : $(y - s_t)^T (y - s_t)$

Waveform noise and estimators

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Theoretical signal
= **waveform model**

$$S(k, \theta) = \boxed{s_t(k, \theta)} + \boxed{n'(k, \theta)}$$

Gamma distributed
multiplicative noise

On board averaging of N
echos. For $N \gg 1$, the
Gamma distribution
converge to a Gaussian
distribution (central limit
theorem)

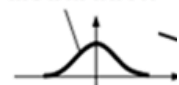
Gaussian
distribution



$$n'(k, \theta) \sim \mathcal{N}\left(0, \frac{s_t^2(k, \theta)}{ENL}\right)$$

- **MLE: likelihood function** can be approximated to a **multi variate Gaussian distribution** [Rodriguez 1998, Halimi et al 2015]

Gaussian
distribution



$$-2\ln\mathcal{L} \underset{L \gg 1}{\sim} (\mathbf{y} - \mathbf{s}_t)^T \mathbf{C}^{-1} (\mathbf{y} - \mathbf{s}_t) + \text{const}$$

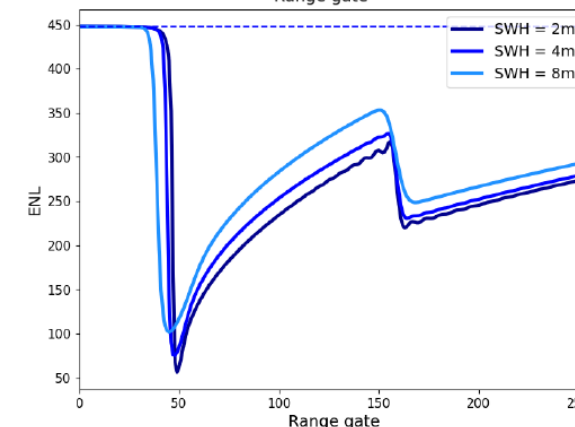
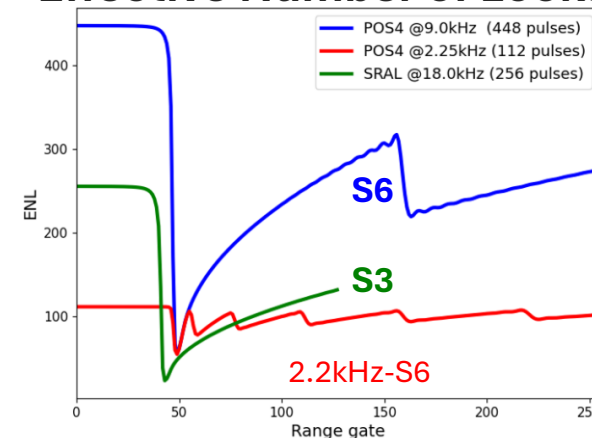
$$C_{k_n k_n} = \frac{s_t^2(k_n, \theta)}{ENL(k_n, \theta)}$$

- MLE equivalent to **Weighted Least Squares (WLS)** estimator:

$$\hat{\theta}_{MLE} = \underset{\theta}{\text{argmin}}(-\ln\mathcal{L}) \sim \underset{\theta}{\text{argmin}}[(\mathbf{y} - \mathbf{s}_t)^T \mathbf{C}^{-1} (\mathbf{y} - \mathbf{s}_t)] = \hat{\theta}_{WLS}$$

- Advantage of WLS approximation: efficient numerical computation with **LS optimisation** (not the case with the full non-Gaussian likelihood)

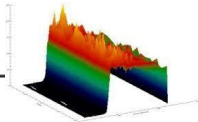
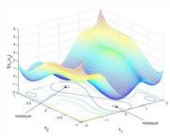
Effective Number of Looks:



Sentinel-6 retracking solutions

Weighted solutions:

- Include model-dependent noise variance directly in the fit as weights
- Main computational cost arises from PTR convolution, not the optimization step
- Iterative noise estimation increases computation time by ~2–3× compared to unweighted numerical solutions

Retracker name	 Model	 Estimator	 Optimization	
FastAdaptiveS6	Adaptive (numerical PTR)	Weighted Least Square (equiv. MLE)	Levenberg–Marquardt	WEIGHTED solutions
RTK_NUMLE_S6	Brown with numerical PTR	Weighted Least Square (equiv. MLE)	Levenberg–Marquardt	
NUM_OLS_AD_S6	Adaptive (numerical PTR)	Unweighted Ordinary Least Square (OLS)	Levenberg–Marquardt	UNWEIGHTED solutions
NUM_OLS_BH_S6	Brown with numerical PTR	Unweighted Ordinary Least Square (OLS)	Levenberg–Marquardt	
Bayes6	Adaptive / Brown (numerical PTR)	Bayesian	MCMC	

Comparison with **operational numerical retracker PDAP NR**

NUM_RTK S6 PDAP	Brown with numerical PTR	Unweighted Ordinary Least Square (OLS)	Levenberg–Marquardt	UNWEIGHTED
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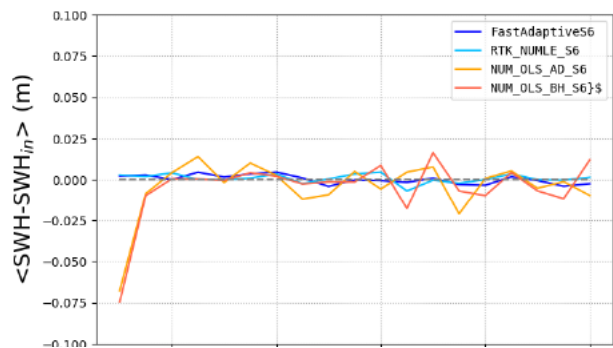
Validation on simulations

Comparison between the **WEIGHTED** and **UNWEIGHTED** (=same model, no weights)

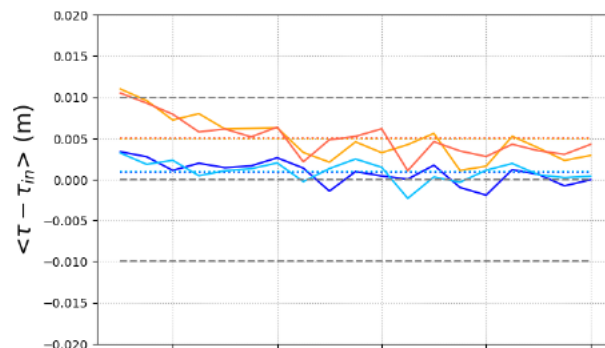
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BIAS

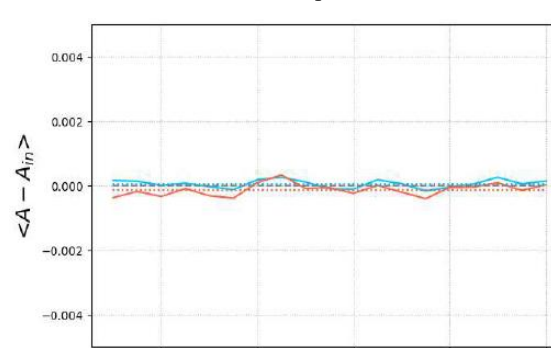
SWH



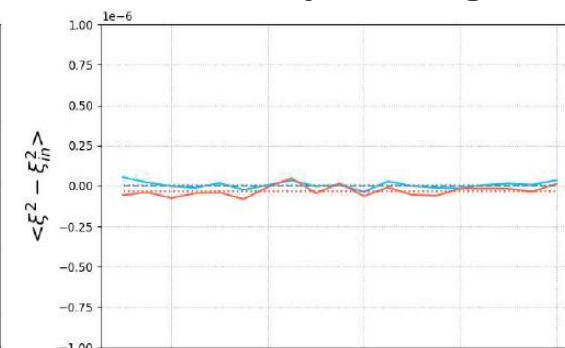
EPOCH



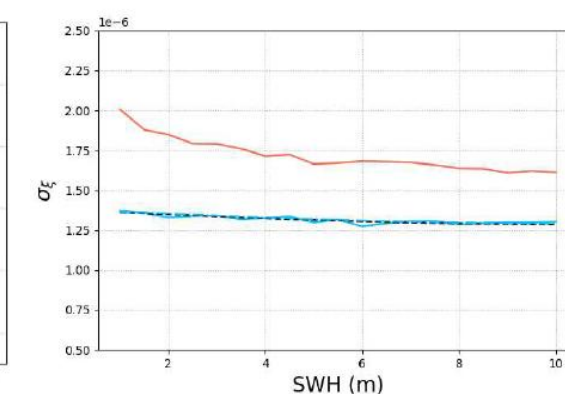
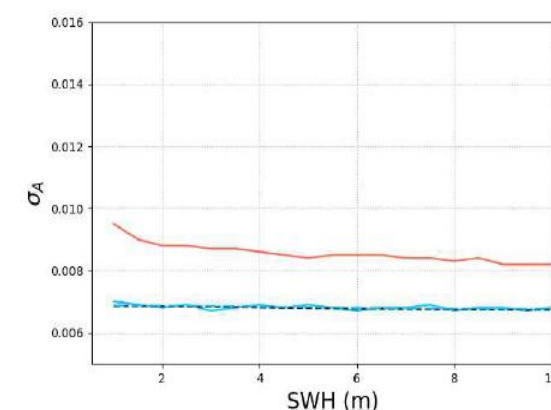
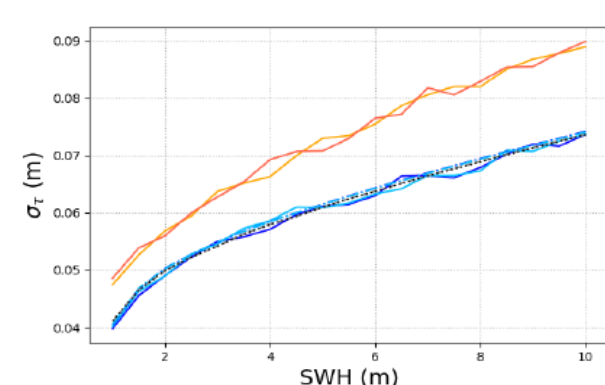
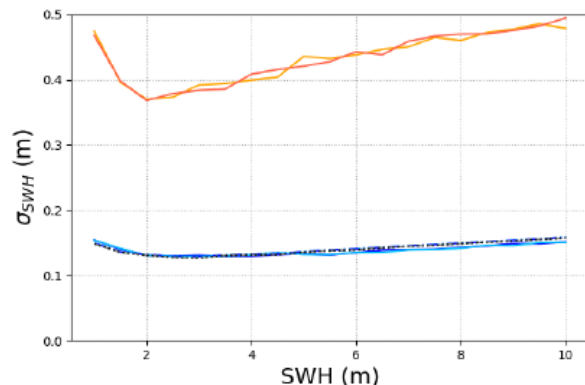
Ampl



Mispointing



STD



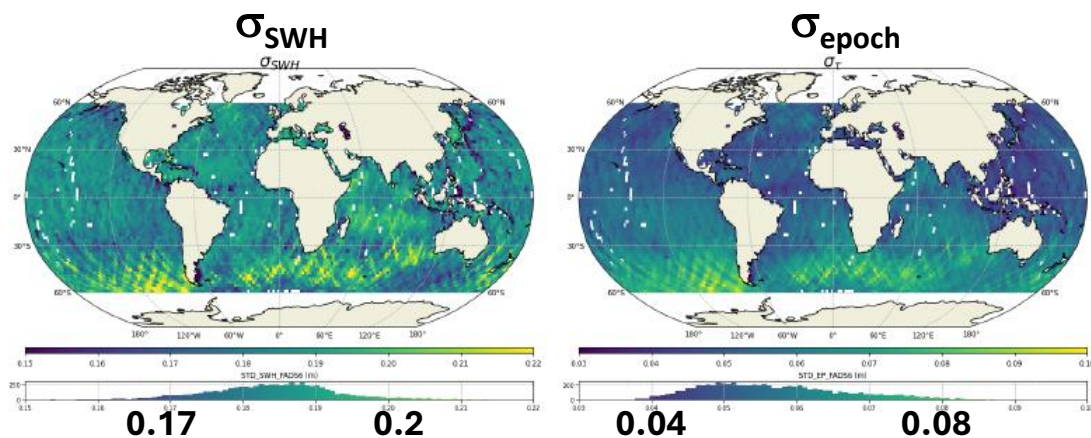
----- Theory: Cramer-Rao (CR) bounds of minimum variance

WEIGHTED solutions: unbiased and optimal=parameters std compatible with Cramer-Rao bounds. Significant noise reduction for all the parameters wrt unweighted OLS.

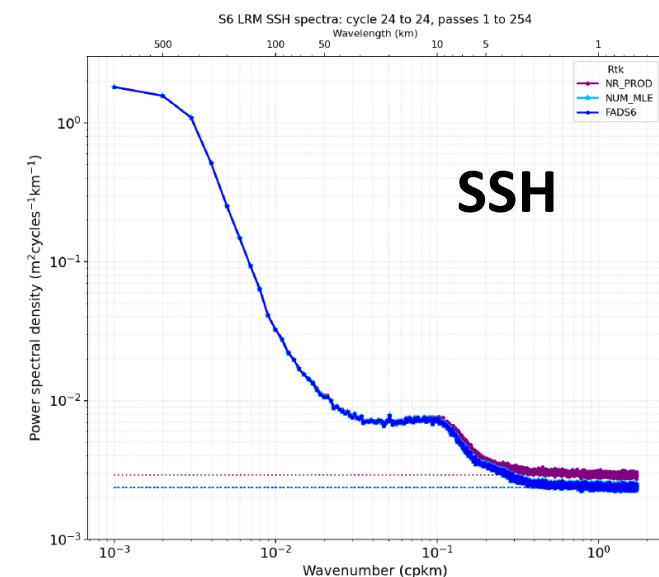
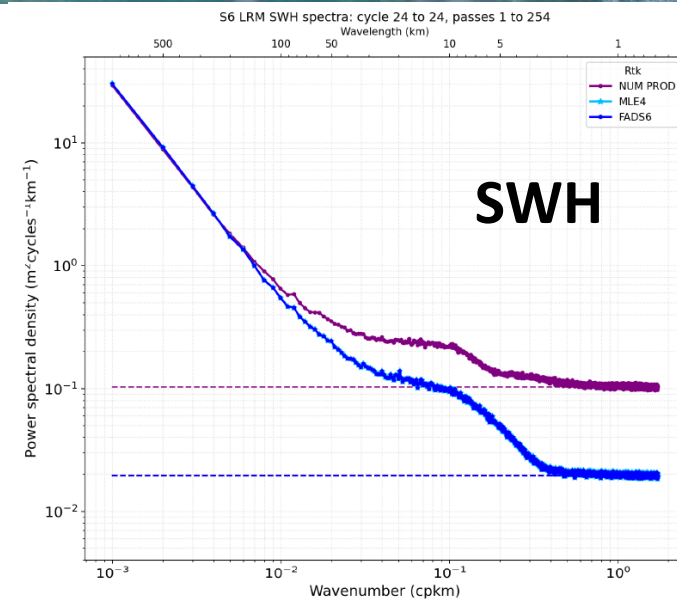
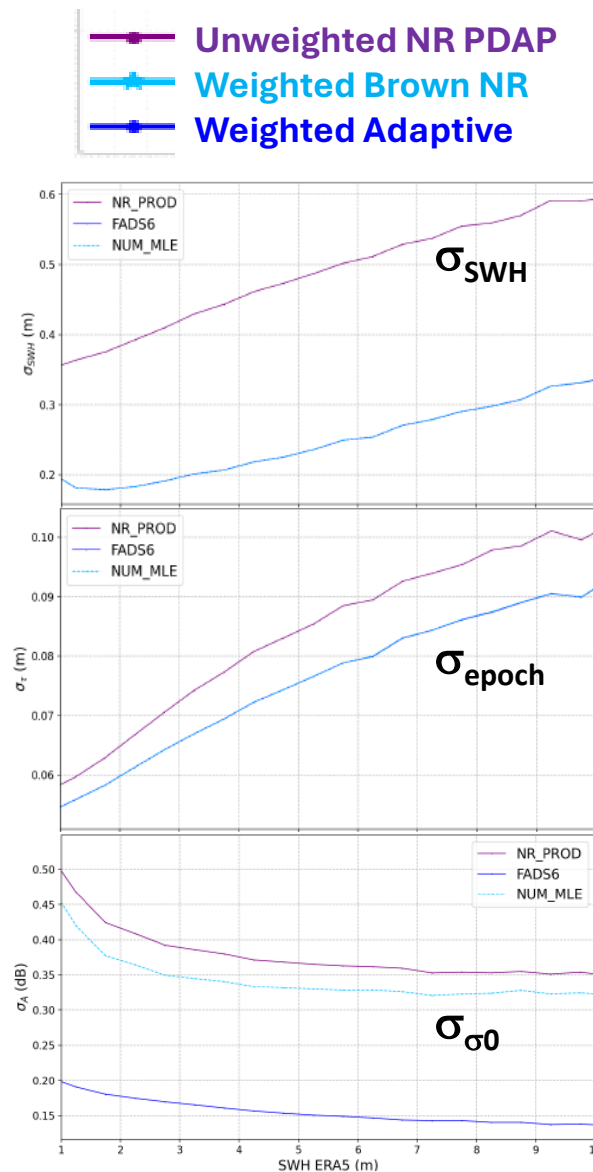
UNWEIGHTED: suboptimal=parameters variance significantly higher than CR bounds, epoch bias up to ~1cm and SWH bias of ~6 cm for small SWH=1m

Results for Sentinel-6A Ku band

- Significant improvement of the **weighted** estimation: ~60% for SWH, ~12% for SSH up to 50% for σ_0 wrt the **unweighted** num rtk PDAP
- Good fit performances (reduced chi2 compatible with 1) and consistent results with both waveform models **Adaptive** and **Brown**
- Estimation of **parameter uncertainties at 20 Hz** as direct **output of the retracker**



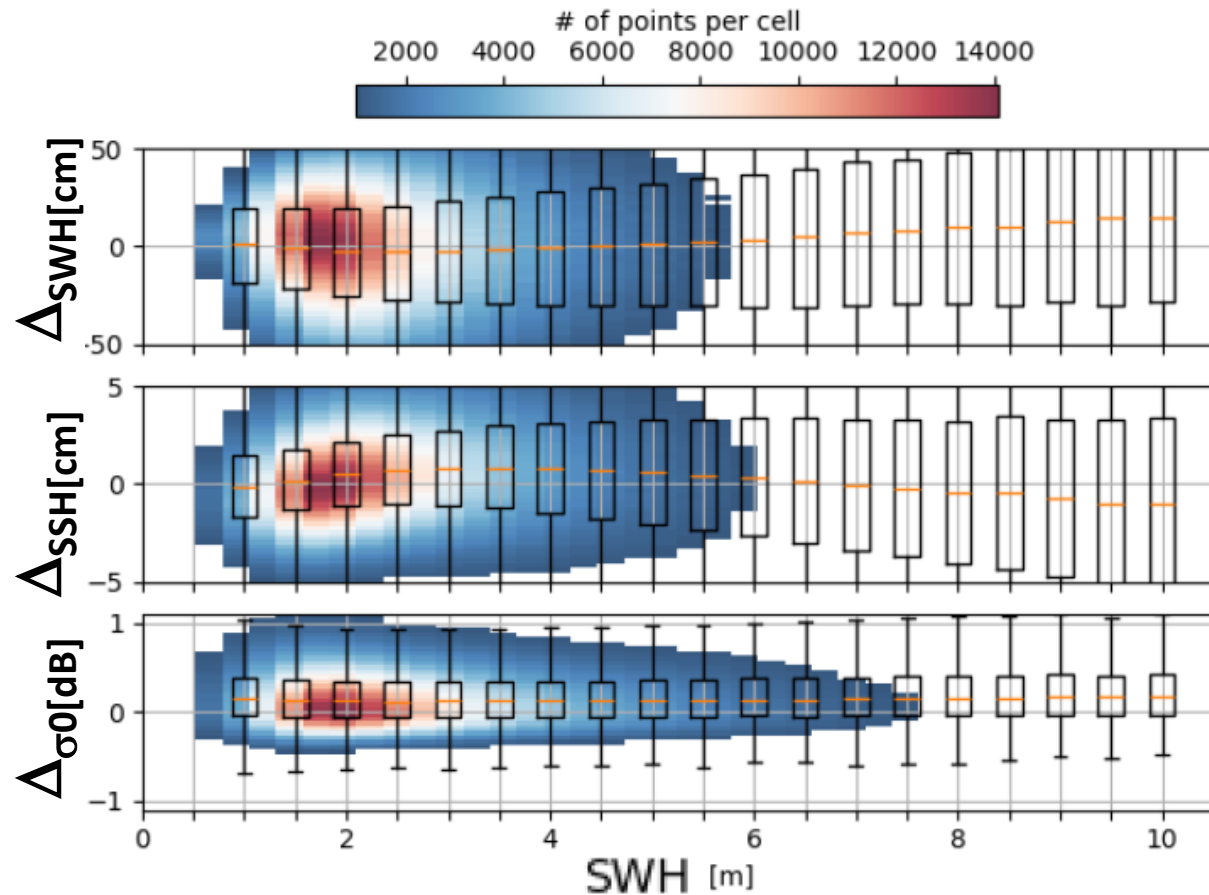
$$\sigma_{\theta} = \sqrt{\text{diag}(P_{\theta})} = \sqrt{\text{diag}([J\hat{C}^{-1}J]^{-1})}$$



Results for Sentinel-6A Ku band

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Weighted – Unweighted (PDAP) differences (1 data cycle)

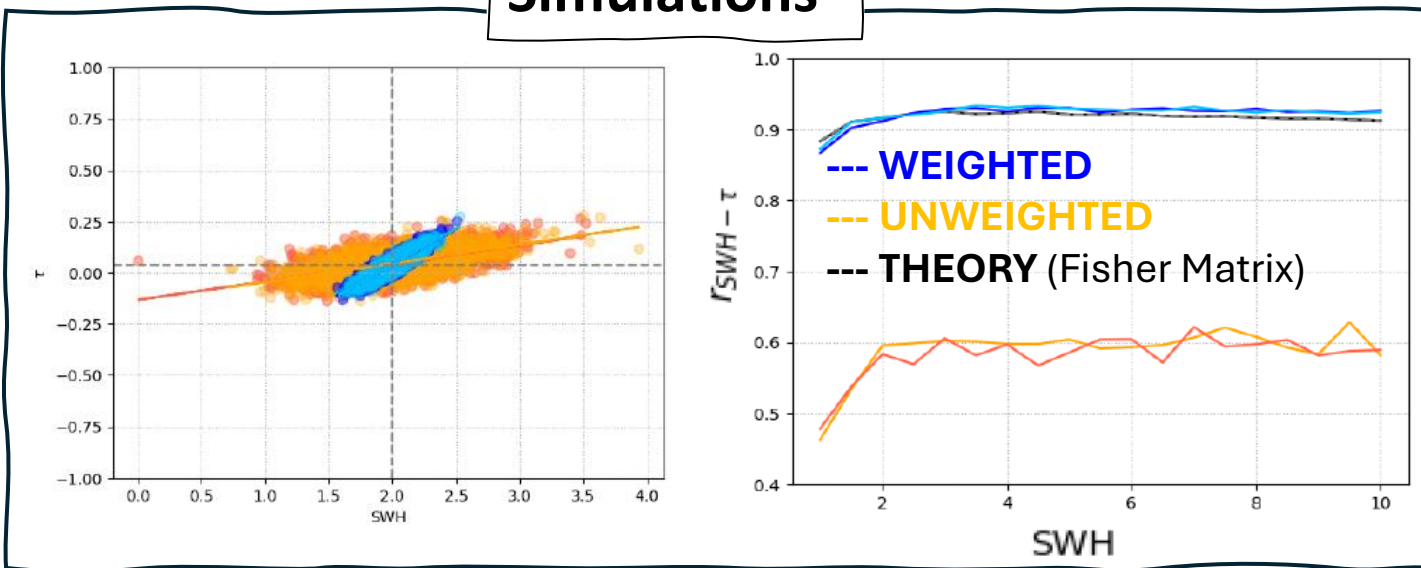


- Evolution of the median of the differences in line with [Egido+2018], partially linked to pulse-to-pulse effect
- Here 2 effects: pulse-to-pulse and estimator change
- @SWH=2 m (peak of the distributions): differences in line with simulations ($\Delta_{SSH} = 0.0062$ m)
- Additional investigations on-going comparing weighted vs unweighted on S6 data at 9kHz and 2kHz

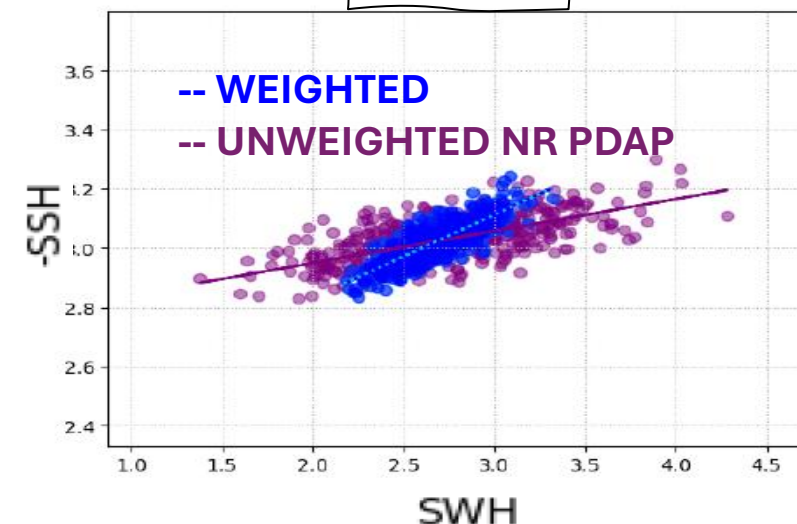
Impact on epoch-SWH correlation

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Simulations



Data



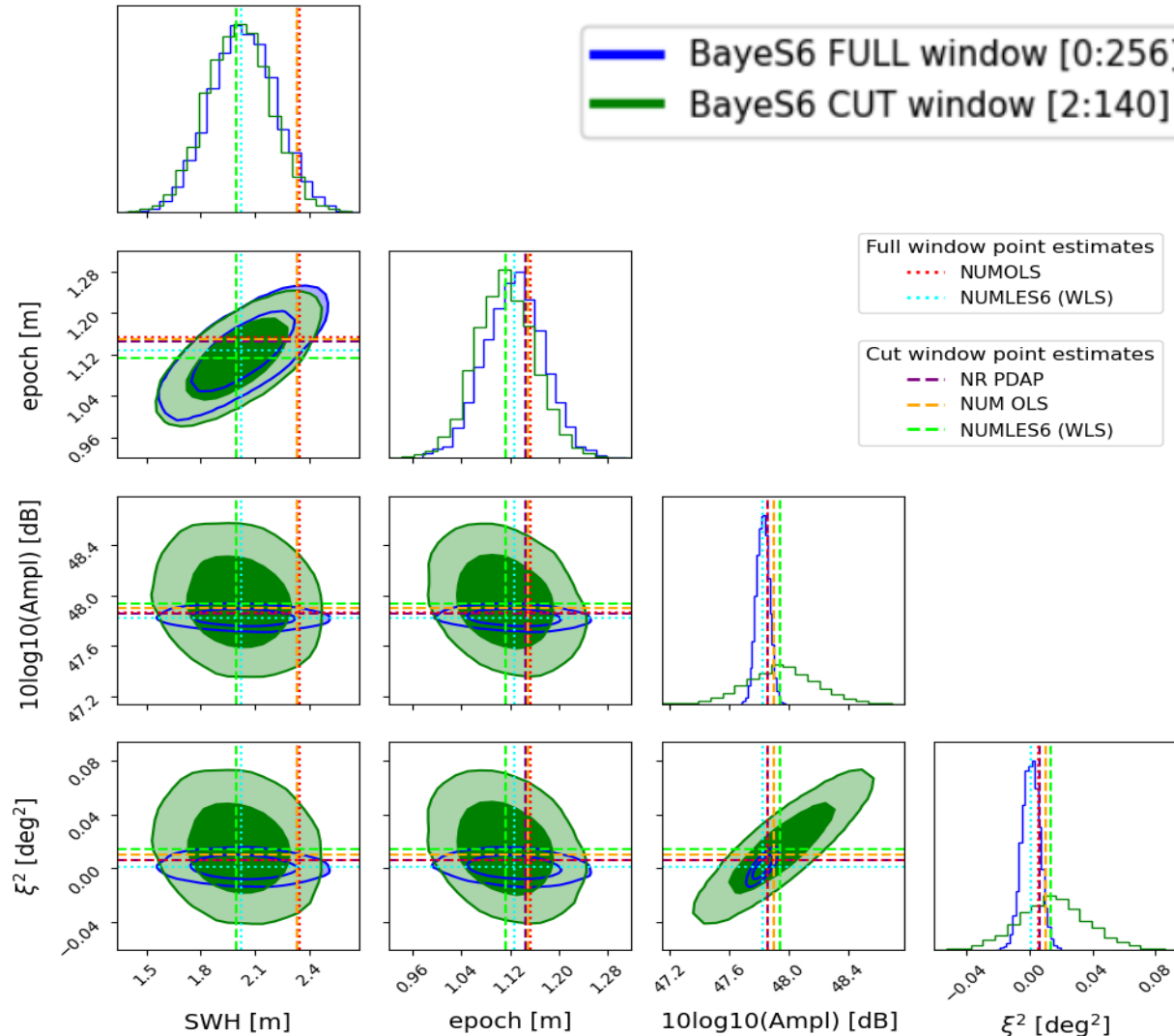
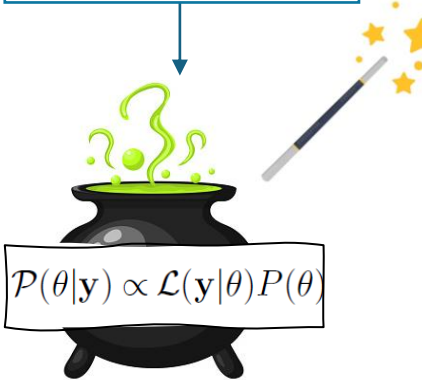
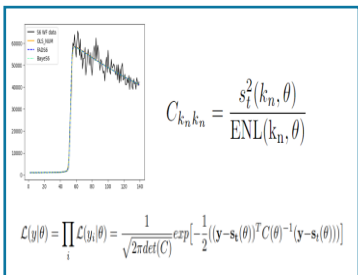
- Because of the **sub-optimality of the unweighted estimation (= larger variance)**, in particular for SWH, the **SWH-epoch correlation is artificially lowered**.
- The correlation is correctly estimated only if the waveform noise is properly accounted in the retracker
- An incorrect estimation of SWH-SSH correlation can have an **impact on the estimation of SSB&HFA correction**
- **Impact on SWH and SSH spectra** should also be further investigated

A new approach: Bayesian retracking

- Provides a **probability distribution of the parameters** rather than a point estimate

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More details @ M. Alves S6VT talk !



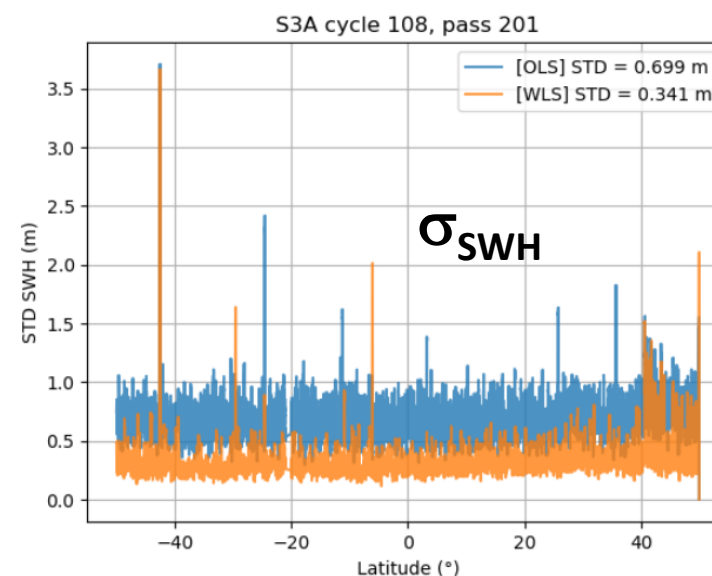
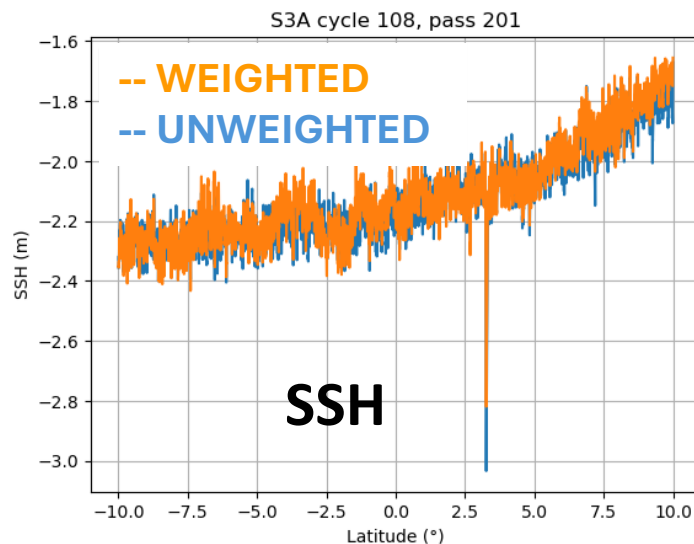
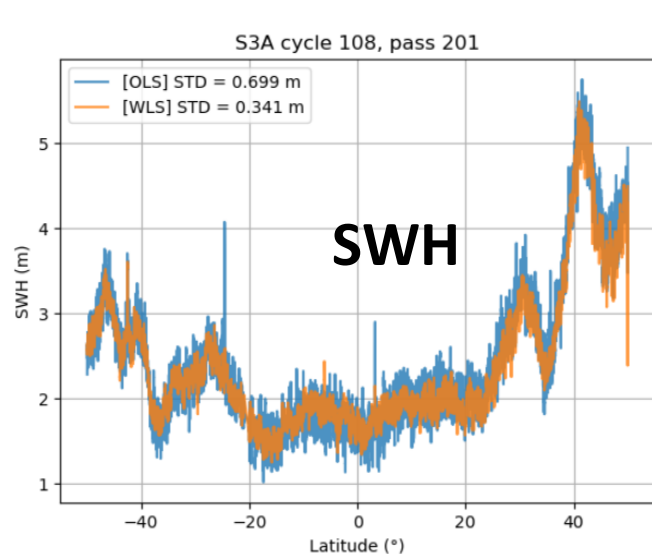
- Robust estimation of parameters uncertainty and correlations
- WLS results fully consistent
- Compact and physically interpretable summary of parameter estimations.
- Useful to compare different configurations in a glance: full vs cut window example here
- Possible future applications: extended waveform models, full likelihood consistency, HR 2D data

Results on S3 Ku-band PLRM

PRELIMINARY

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- L2 products based on PLRM data mainly used for consistency (SAR/multi-mission for climate applications) and for S3 wind estimation (WAVE-TAC products): interesting to improve the performance
- S3A 20 Hz PLRM data over ocean processed with the WLS retracker (Brown model, numerical PTR, sinc2 for now) for 1 track (cycle 108, pass 201)
- Significant improvements for all geophysical parameters: reduction of uncertainties of ~50% for SWH and ~10% for SSH in line with expected performances. Improved comparison with HR data?
- Promising results. Validation and more operational implementation ongoing on SPP@CNES.

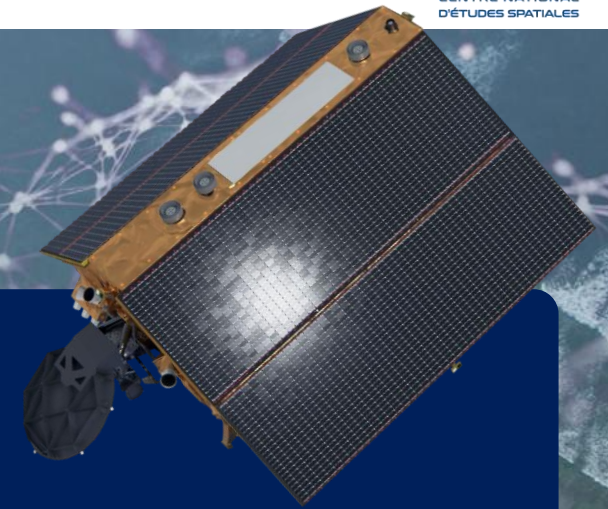


Conclusions and perspectives

- Radar waveforms are inherently affected by **multiplicative speckle noise** which **must be accounted for in parameter estimation**
- **Optimal retrackers (WLS / Bayesian)** developed for Sentinel-6 Ku-band:
 - Provide unbiased and optimal parameters estimation with **significant precision improvement vs operational solutions** (MLE4, NUM_RTK): **~60% for SWH**, **~12% for SSH** and **up to 50% for σ_0** .
 - WLS iterative noise estimation: computation time $\sim 2-3\times$ wrt unweighted numerical solutions
 - Enable a **consistent characterization of parameter correlations** (e.g., **SWH-epoch**) that are instead underestimated with current operational solutions because of the increased parameter variance.
 - The impact on derived corrections (e.g., **SSB**, **HFA**) should be further assessed
- Application to **Sentinel-3 PLRM** shows promising results

On going work and perspectives:

- Implementation of the WLS solution **@SPP (CNES/CLS): S6 Ku band, C-band** + S3 PLRM
- WLS solutions applied to C-band & **S6B data** at 9kHz and 2.2kHz *More details @ S6VT talk !*
- Applicability to **SAR data** currently under investigation (**2D SAR S6** R&D study with CNES): key point is the noise characterization *More details @ F. Ehlers talk*
- Optimal solutions for **extended waveform model** (skewness, wave groups)



Thank you



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BayeS6 on sims: cut vs full window

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Comparison of Full and Cut Window: S6 SIM

