

Presentation

Altimeter and profile observations working together

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IN PARTNERSHIP WITH



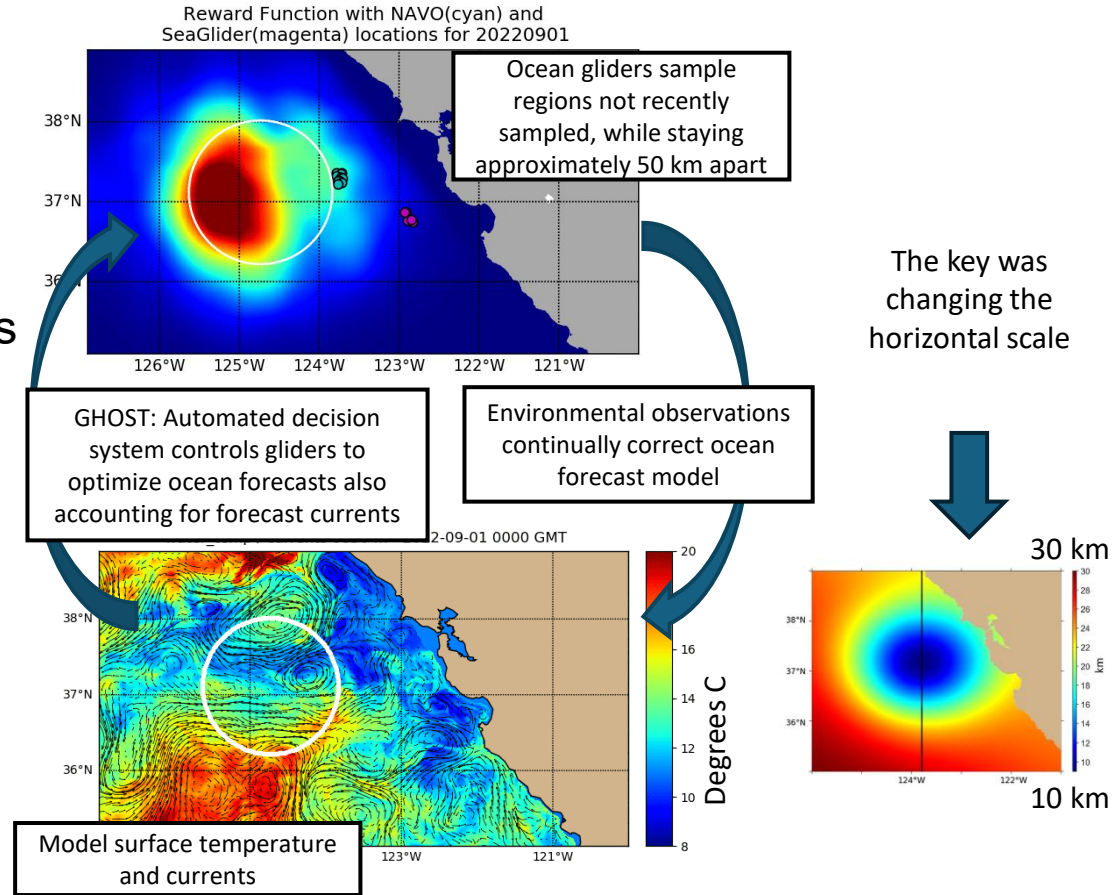
JPL



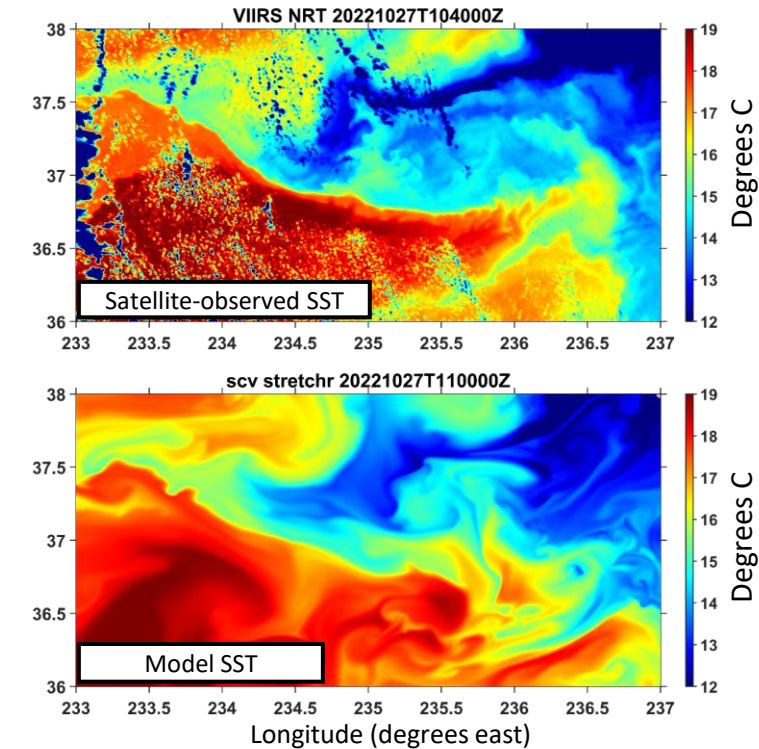
Motivation

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Horizontal data distribution
“Patchy dense” data occurs
every day
SWOT sampling, local
observations



Results indicate observed fronts within 15 km



Similar considerations apply in the vertical
Using satellite and in situ observations together

Jacobs, Gregg, Joseph D'Addezio, Brent Bartels, Chris DeHaan, Charlie Barron, Matthew Carrier, Andrey Shcherbina, and Mathieu Dever. "Adapting constrained scales to observation resolution in ocean forecasts." *Ocean Modelling* 186 (2023): 102252.

Vertical inference

- Given a SSH from altimetry, many approaches convert this to a temperature and salinity profile (GEM, MODAS, ISOP, NeSPReSO, SSOP, ...)
- Inferred profiles are vertically very smooth
- Used with a cycling ocean forecast system, fine details can be removed

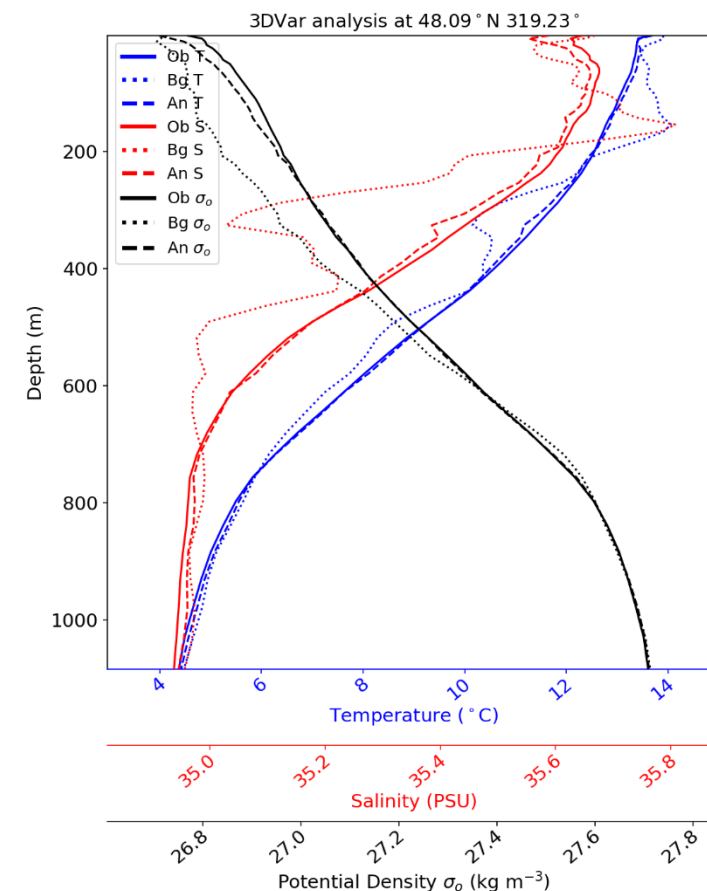
Questions needing answers

1. ARGO resolves small vertical scales, what scales are resolved by satellite SSH?
2. Can multiscale methods reduce assimilation errors in either SSH or ARGO T&S?
3. Can we prevent inferred profiles from overly smoothing model forecasts?

Answers

1. SSH detects a subspace of the lowest vertical modes
2. Slightly, yes
3. Yes, and the results from 1 and 2 shed light on the path

Detrimental effects using vertical inference profiles in assimilation



Ocean vertical structure challenge

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Observed

— Temperature
— Salinity
— Potential density σ_0

Climatology

... Temperature
... Salinity
... Potential density σ_0

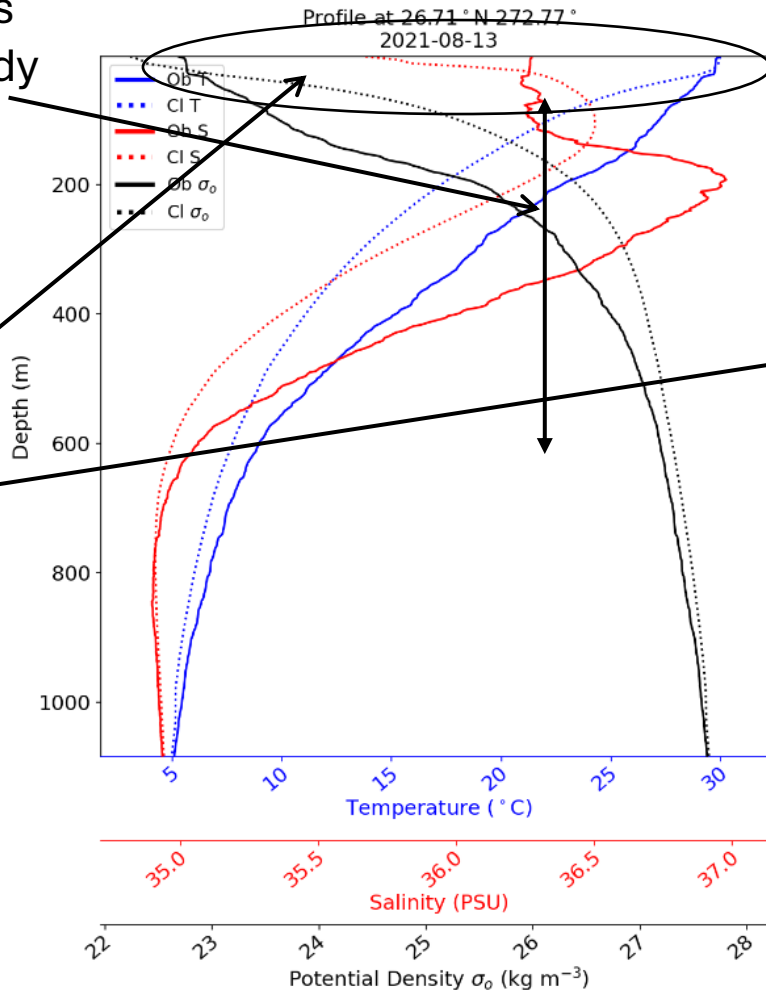
Large vertical scales

- Loop Current Eddy (>100 meters)

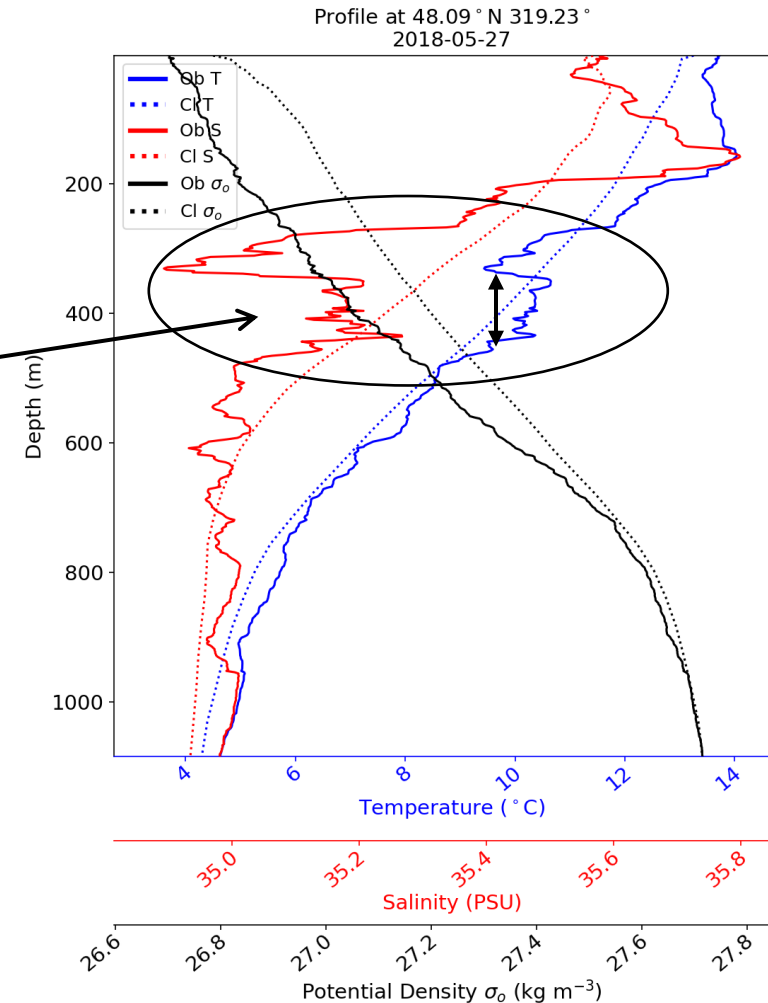
Small vertical scales

- Mixed layer
- Compensating layers

Gulf of Mexico



North Atlantic Current



Assimilation approaches

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Standard variational

$$\delta \mathbf{x} = \mathbf{B} \mathbf{H}^T (\mathbf{H} \mathbf{B} \mathbf{H}^T + \mathbf{R})^{-1} \mathbf{d}$$

Multiscale

$$\delta \mathbf{x}_L = \mathbf{B}_L \mathbf{H}^T [\mathbf{H} \mathbf{B}_L \mathbf{H}^T + (\mathbf{R} + \mathbf{H} \mathbf{B}_S \mathbf{H}^T)]^{-1} \mathbf{d}$$

$$\delta \mathbf{x}_S = \mathbf{B}_S \mathbf{H}^T [\mathbf{H} \mathbf{B}_S \mathbf{H}^T + (\mathbf{R} + \mathbf{H} \mathbf{B}_L \mathbf{H}^T)]^{-1} \mathbf{d}$$

$$\delta \mathbf{x} = \delta \mathbf{x}_L + \delta \mathbf{x}_S$$

$$\mathbf{d}_1 = \mathbf{y}_1^o - \mathbf{H}_1 \mathbf{x}^{bg}$$

$$\delta \mathbf{x}_1 = \mathbf{B} \mathbf{H}_1^T (\mathbf{H}_1 \mathbf{B} \mathbf{H}_1^T + \mathbf{R}_1)^{-1} \mathbf{d}_1$$

$$\mathbf{B}_a = (\mathbf{I} - \mathbf{B} \mathbf{H}_1^T (\mathbf{H}_1 \mathbf{B} \mathbf{H}_1^T + \mathbf{R}_1)^{-1} \mathbf{H}_1) \mathbf{B}$$

Sequential (Bayesian)

$$\mathbf{x}_2^{bg} = \mathbf{x}_1^{bg} + \delta \mathbf{x}_1$$

$$\mathbf{d}_2 = \mathbf{y}_2^o - \mathbf{H} \mathbf{x}_2^{bg}$$

$$\delta \mathbf{x}_2 = \mathbf{B}_a \mathbf{H}_2^T (\mathbf{H}_2 \mathbf{B}_a \mathbf{H}_2^T + \mathbf{R}_2)^{-1} \mathbf{d}_2$$

$$\delta \mathbf{x} = \delta \mathbf{x}_1 + \delta \mathbf{x}_2$$

All three produce the same result given the same

- Observation sampling $\mathbf{H}$ and innovations $\mathbf{d}$
- Background errors $\mathbf{B}$
- Observation errors $\mathbf{R}$

UNLESS

Observations are separated by scales, which reduces scale aliasing in multiscale assimilation, and observations are not complete

If all three are the same, why bother? They give options once we understand the data.

Observation errors

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R

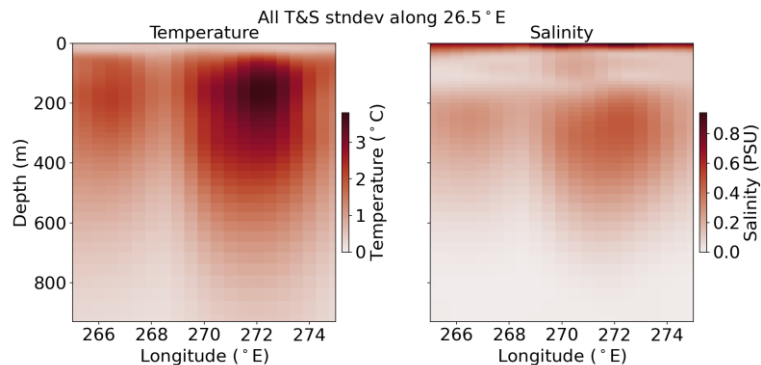
Instrument error

- Temperature: 0.001°C
- Salinity: 0.01 PSU
- Steric height: 0.03 cm

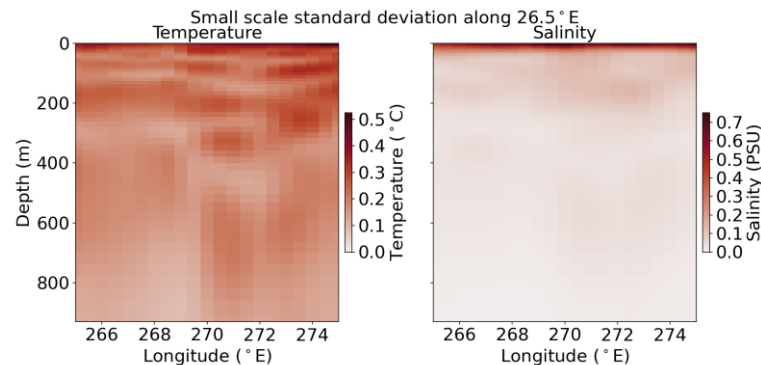
Representativeness error

- Variance not resolved by model
- Dependent on vertical grid
- These experiments: 53 layers, 5 m top layer, expanding to ~50 m at 1000 m depth
- Estimated by a Gaussian 2-point filter on the WOD23 data at the analysis grid sampling

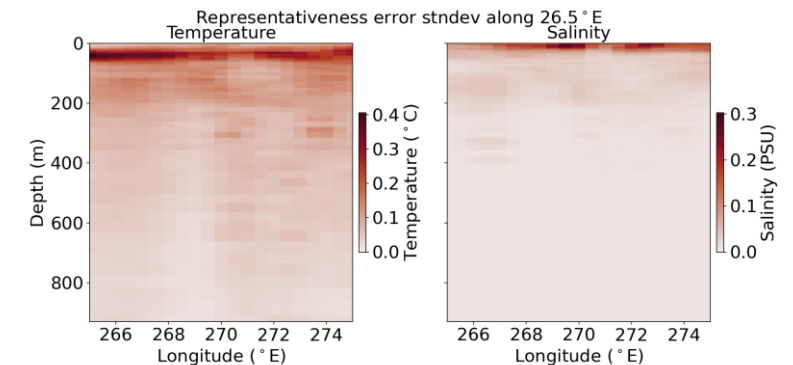
TS standard deviation
from WOD 23



Small scale (next slide)



Representativeness
error



On average, this is the limit
analysis errors should reach

Background errors

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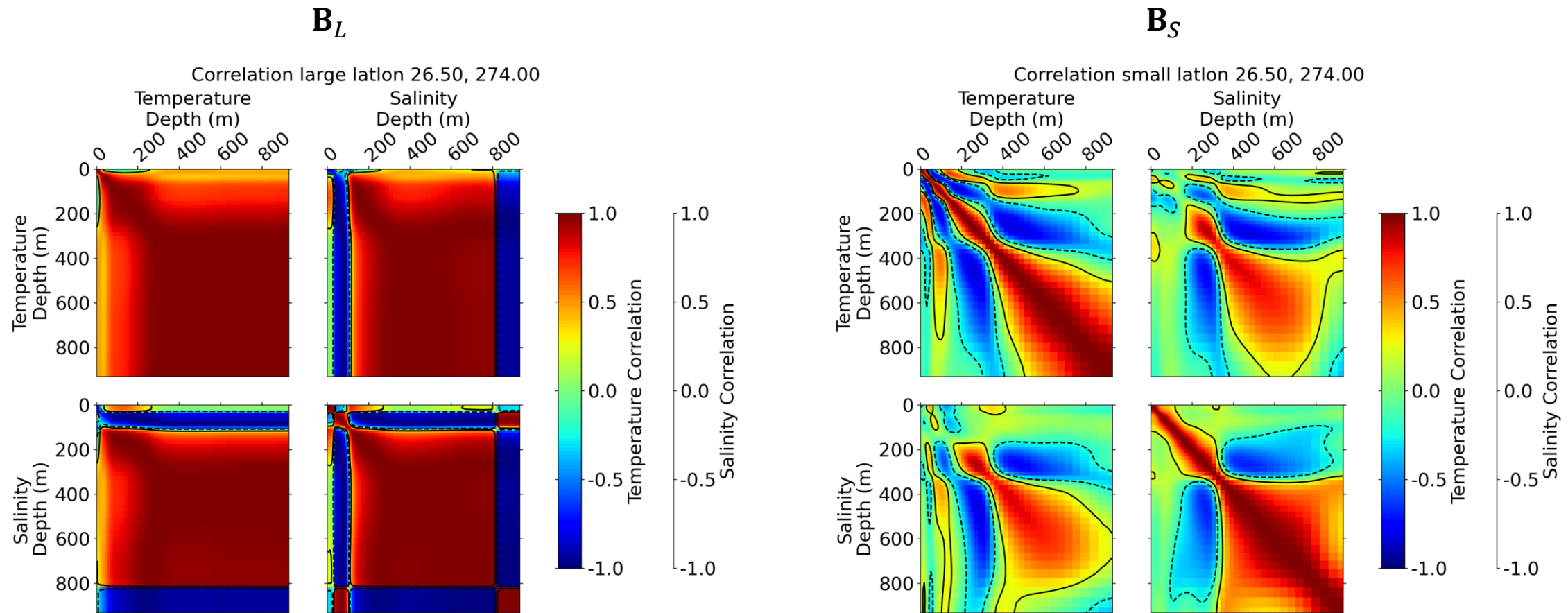
$\mathbf{B} = \mathbf{B}_L + \mathbf{B}_S$ on a 0.5° lat,lon grid

Background is climatology from WOD23

Background error is variance from climatology

Large scale is first eigenmodes providing at least 95% of variance (4 modes in both areas)

Small scale is the remainder



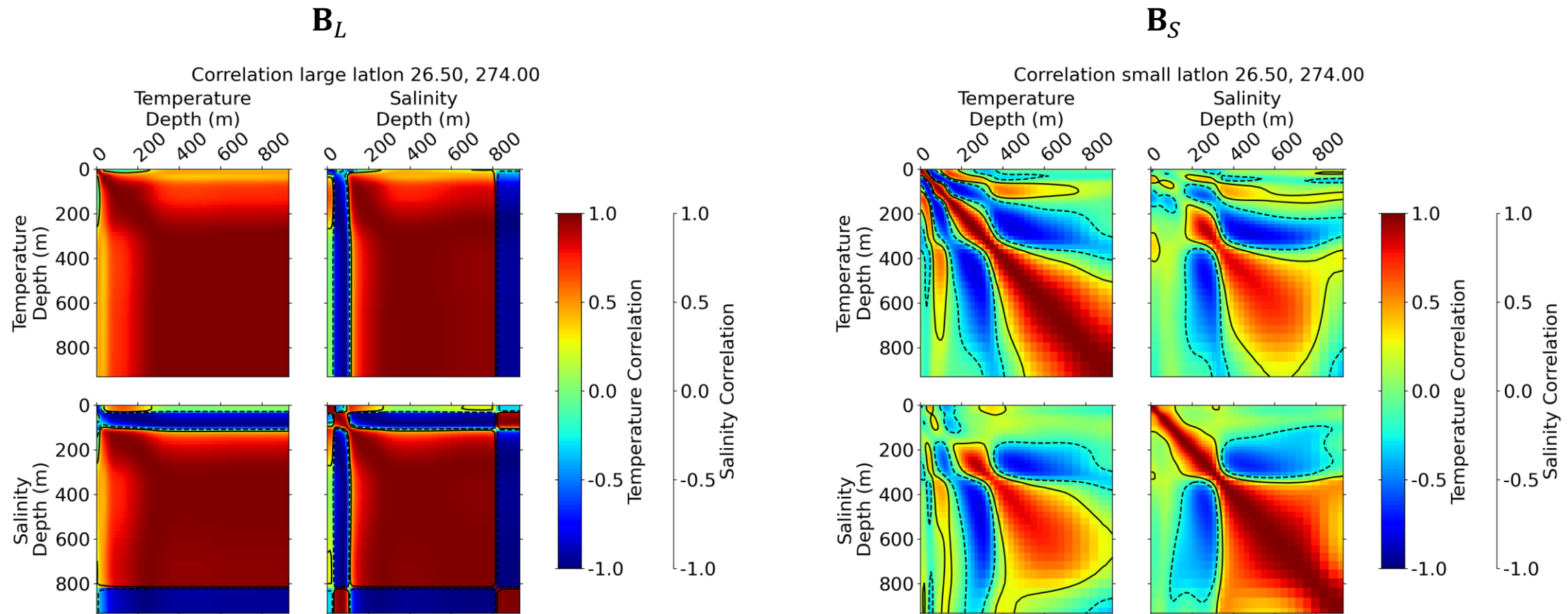
Background errors

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Cross covariances indicate the system variations based on the historical stratification

Maintaining TS cross covariances is critical

If cross covariances are set to 0, temperature and salinity solutions have opposite signs when using SHA



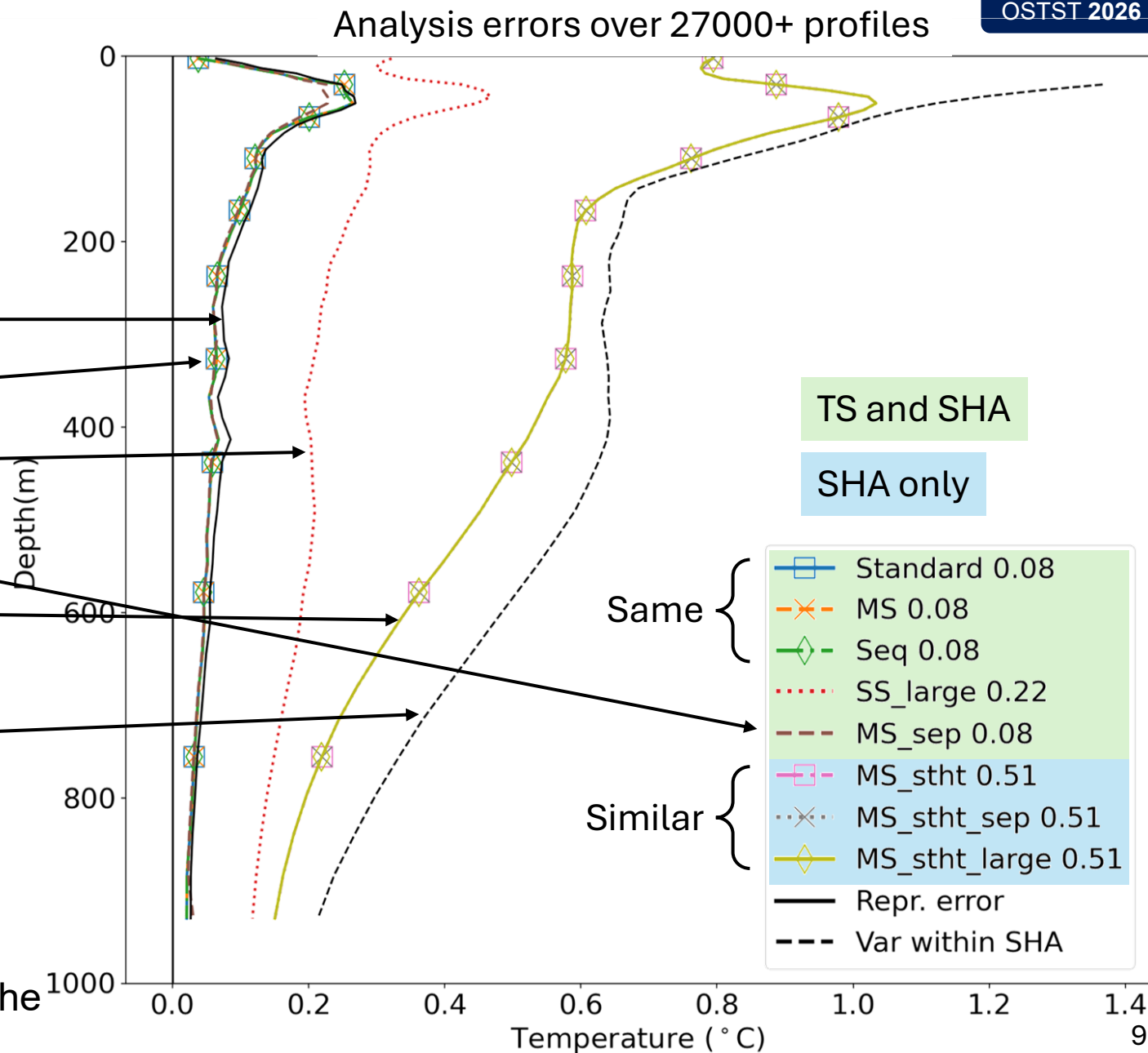
Summary results

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- 930 m reference for steric height (allows more profiles)
- 1D analysis for each WOD23 profile to 930 m
- > 27000 profiles
- Average representativeness error
- Different approaches, same results
- Using only large scales B_L
- Separating innovations by scales
- Using only steric height
- Variance of temperature in 0.03 m steric height bins (steric height observability limit estimate)

Steric height observations are limited to a subspace of only the large scales

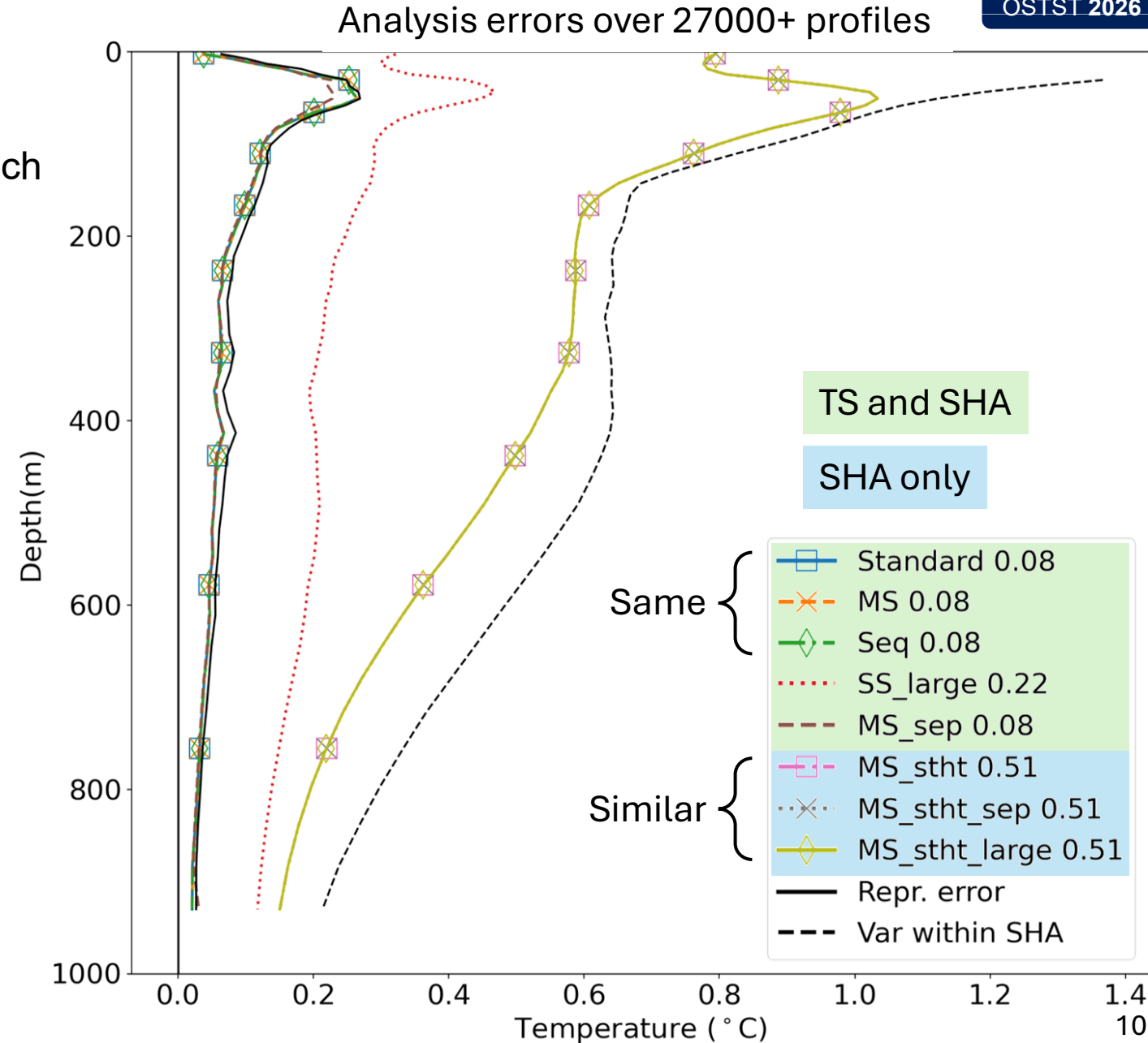
Innovation scale separation decreases errors in the MLD range



Considerations

- Steric height capability is a subspace of low modes
- We get the same solution regardless of approach

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Example approach

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Sequential analysis

1. Assimilate steric height

- Use a multiscale approach separating large and small vertical scales
- Project inferred TS innovation on to large vertical scales
- There is no small scale information, so the small scale step is skipped

2. Assimilate TS profiles

- Can use a multiscale approach, though the improvement may not be worth the computational cost

1 can be made more efficient

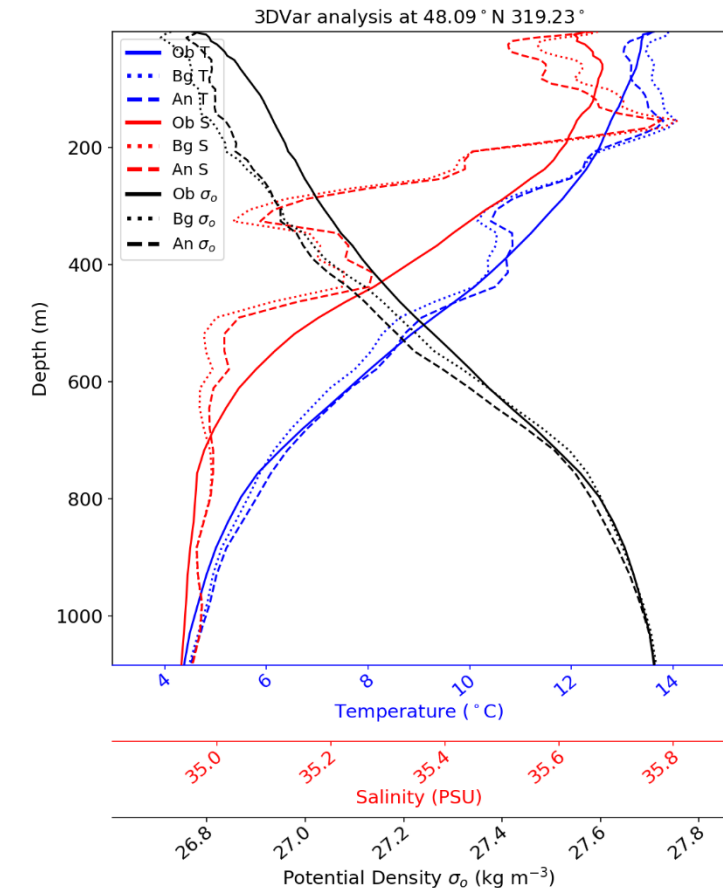
Given a separable form for the background and observation functions, it can be reduced to a 2D inversion rather than 3D and then essentially a vertical projection

That's great!

SSH observations far outnumber profiles

Avoid creating inferred profiles and using in a 3D analysis saves cost

Effects of scale consideration in steric height assimilation



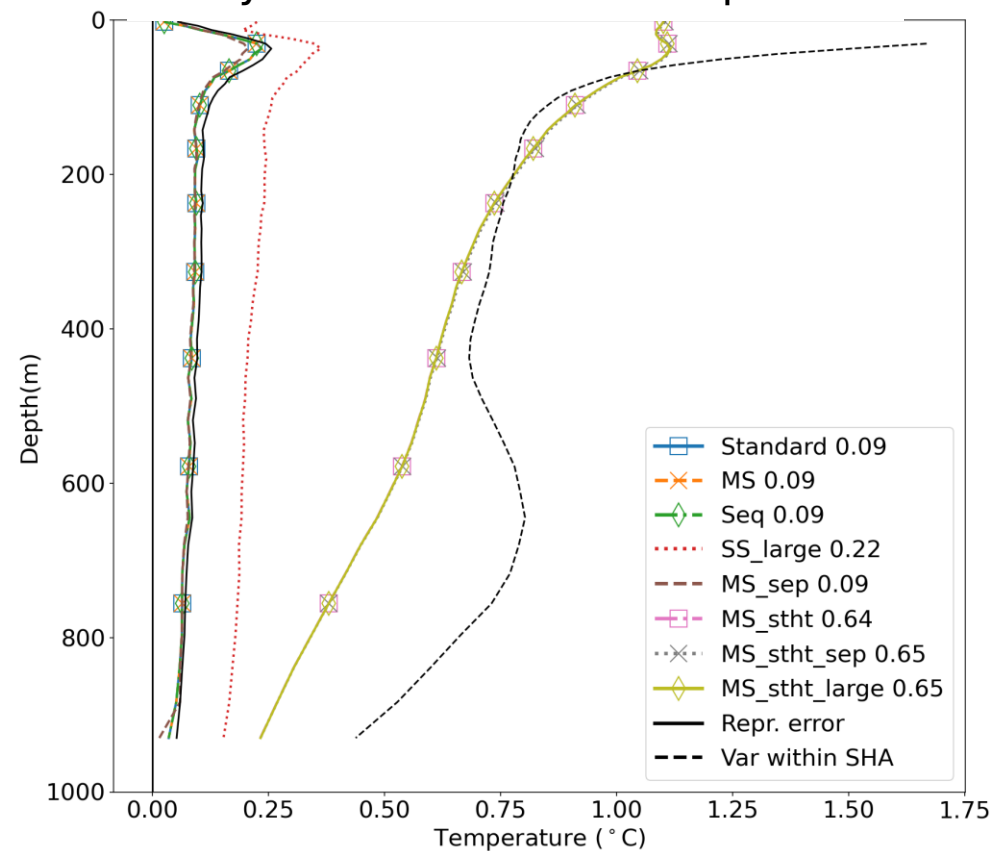
Considerations

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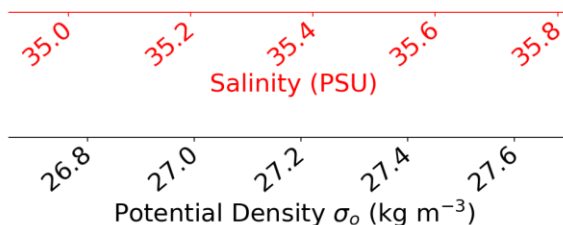
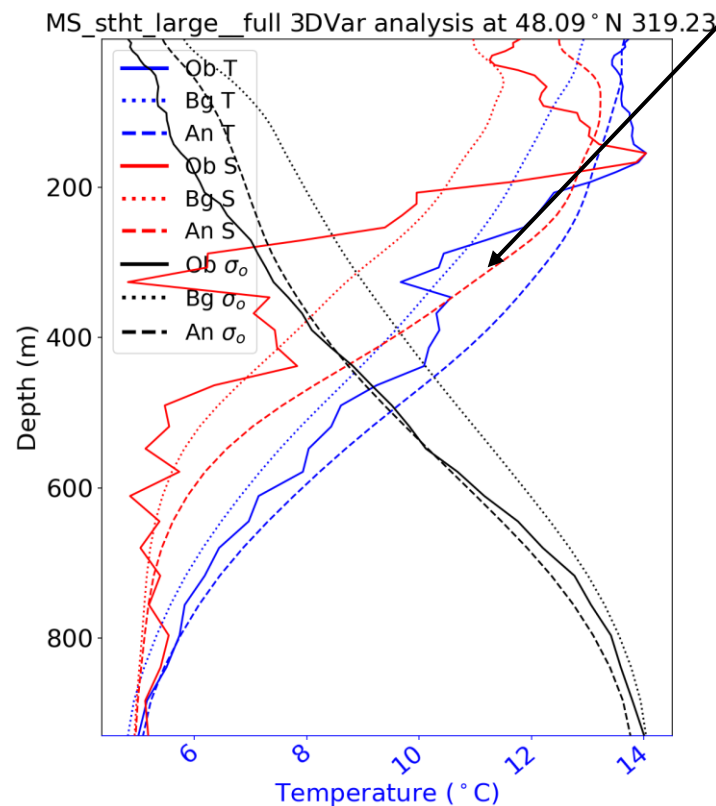
- North Atlantic is more challenging
- Vertical resolution in the analysis (**dictated by the model**) is a significant limitation
- Motivation for pushing vertical resolution

North Atlantic profile analysis example

Analysis errors over 17000+ profiles



Given only SHA



Given TS + SHA

