Templex-detected patterns and their connection to elephant seal behaviour

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Introduction

The topological approach called *templex* (Charó et al., 2022) introduces the use of a cell complex endowed with a directed graph to characterize the structure of a flow in phase space. Templex properties are shared by fluid particles exhibiting a certain qualitative behavior, and therefore can be used to detect Lagrangian patterns. When applied to single particles, the approach requires long time windows to reconstruct the flow in phase space. In order to compensate for the low-term stability of ocean particle behavior, we develop a strategy that uses a bundle of several particles in the neighborhood of a point. The topological classes found with this strategy applied to surface geostrophic velocities maps derived from satellite altimetry enable identifying eddies and other patterns, which have been studied in connection with elephant seal behaviour.

Topological Grids

The first scope in this work is to compute the topological grid for the South Atlantic region in a time window ranging from 20 October to 6 December, i.e. a map of the daily topological classes observed when the virtual particles visit the point of the grid as shown in Figure (1). To achieve this goal, a method for integrating Lagrangian particles from satellite altimetry velocity fields was developed.

Once the particles are advected in the vicinity of each grid point (distance $R = 0.1°$), the next step is to embed the time series using time delays. This produces a multi-dimensional point cloud whose topological properties have to be determined. The workspace defined by the four-dimensional delay embedding of each pure topological class encountered in a forward time window of 10 days from 23 November 2018 is shown in Figure (2).

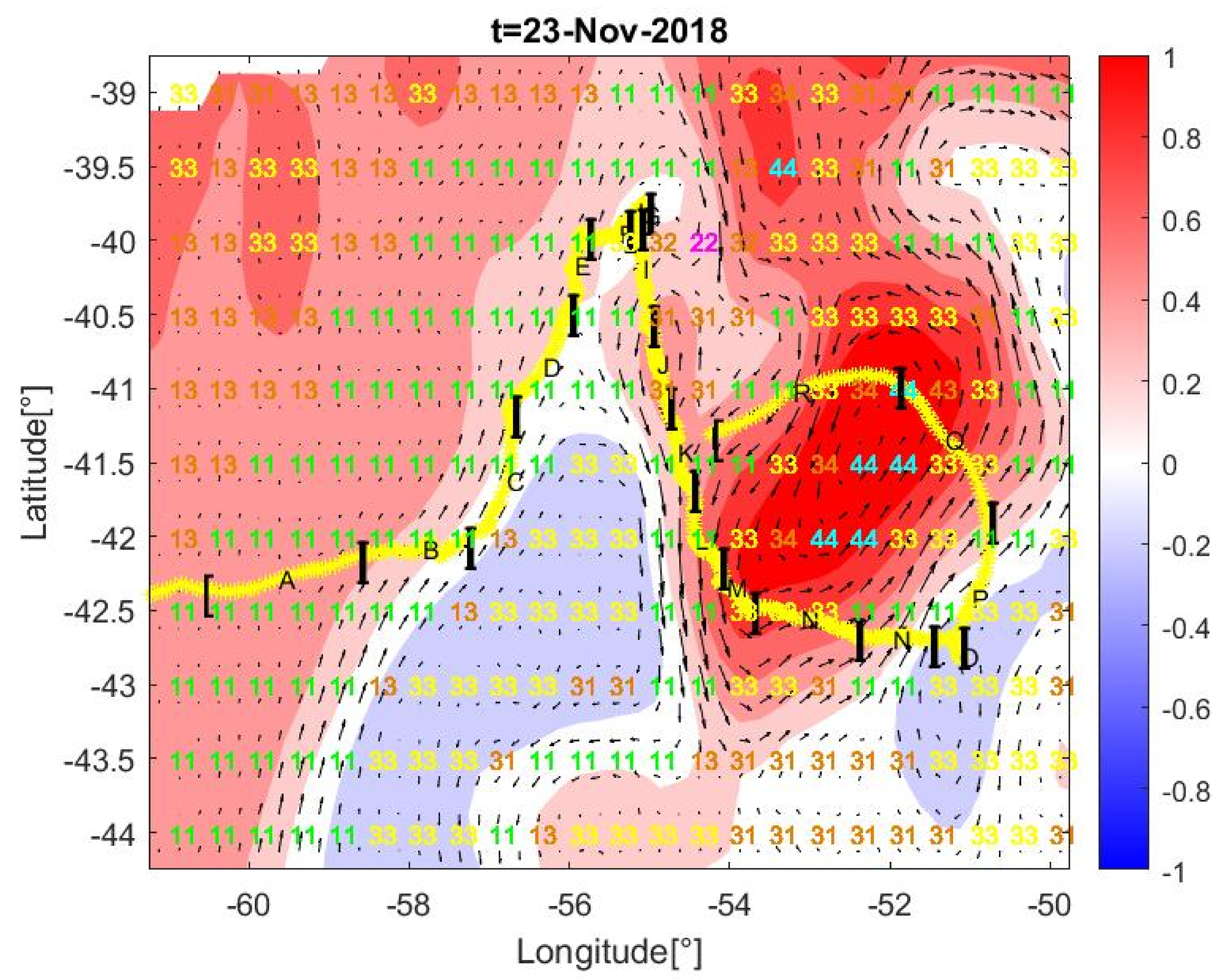

Figure 1: ADT (Absolute Dynamic Topography) height values in [metres], superimposed with the altimetric velocity fields. The elephant seal's trajectory is in yellow and the indexes correspond to the different topological class found at each grid point in a forward time window of 10 days. Each number corresponds to a topological class, but as topology can change within the window, two indices are used. Pure topologies are represented by two identical numbers. For example 11 represents the cylinder, 22 a mutiple 1-hole topology, 33 the Möbius strip, and 44 the "Torus". Hybrid topologies are represented by two different numbers tagging the topological types at the beginning and at the end of the window.

Templex-detected patterns

The point cloud is used to build a cell complex K. Homology groups $H_i(K) (i = 0, 1, 2)$ can be computed to determine the Betti's numbers β_k, the orientability chains o, the weak boundaries w and therefore the topological class to which the time series belongs. The complex can be endowed with a digraph to form a templex and provide further details on the flow structure (not shown).

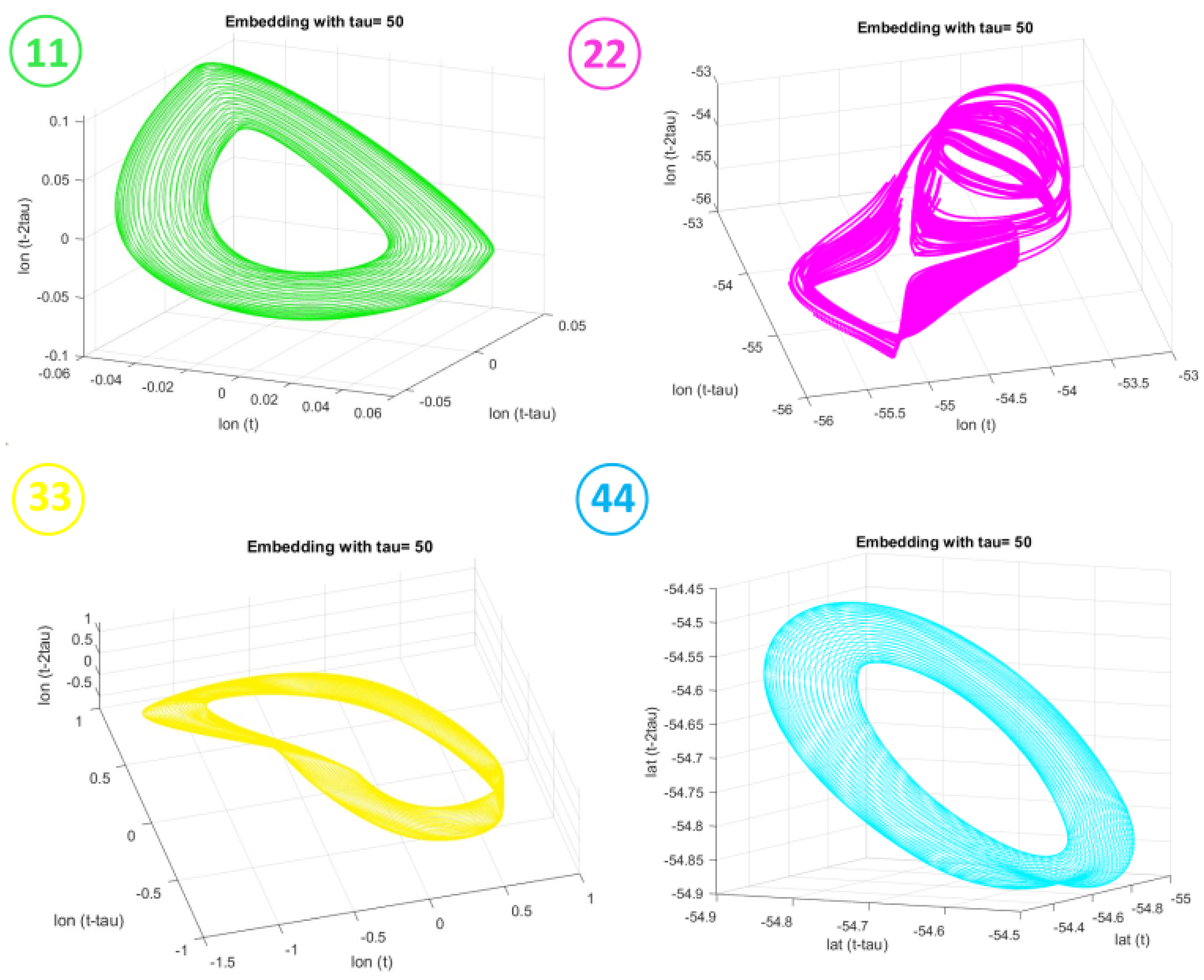

Figure 2: 3D projections of 4D reconstructions corresponding to topological classes $11, 22, 33, 44$ using a forward time window of 10 days forward from 23 Nov. 2018.

The connection with elephant seal behaviour

In order to investigate the connection between mesoscale ocean dynamics with a marine predator behaviour, GPS and accelerometry data in the same region and time window were considered. We compute the topological classes along the seal's trajectory and we find that they are mostly those typically observed in jets (11) and on the edge of eddies (33). In sections labeled (A,B,C,D,E,F,G,H,I,K,L,M,N,O,P,Q) prey capture attempts increase, the seal's behaviour is largely affected by oceanic currents and can be characterized as quasi-planktonic (Allice Della Penna et al., 2015). This is in line with a foraging strategy driven by mesoscale structures.

Cell complex	β_0	β_1	β_2	o	w	Top. class	Section of traj. seal	Colour
$\mathbb{K}_1$	1	1	0	x	x	11	A,B,C,D,E,I,K,L,Ñ	Green
$\mathbb{K}_2$	1	2	1	x	x	22	-	Magenta
$\mathbb{K}_3$	1	3	0	✓	x	33	F,G,H,M,N,O,P,Q	Yellow
$\mathbb{K}_4$	1	3	0	✓	x	44	R	Light Blue

References

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