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Gridding Sea Level Anomalies using Co-Kriging with collocated Sea Surface Temperatures

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Motivation:

Sea Surface height (SSH) from along-track altimetry is characterised by restrictions in their spatial and temporal resolution. Gridded SSH maps are thus crucial for studying global and regional sea level variations. Grids of Sea level anomalies (SLA) are obtained by, e.g., optimal interpolation schemes that perform strong smoothing on the data, such that resolution of the obtained maps is greatly reduced compared to the along-track data (Ballarotta et al., 2019).

Other observables can be measured at a higher resolution and their high-frequency variability is related to SLA variation towards smaller scales, which the common maps often lack. Using the correlation of SLA with several other variables with higher observation density can be used to obtain more detailed maps, e.g., towards better resolution of the data in mesoscale sales as well as in regions where SLA are potentially lacking in resolution and quality such as coastal and continental shelf areas. In this study, we propose the method of *intrinsic collocated Co-Kriging (ICCK)* using Sea Surface temperature (SST) as a Co-variable for mapping of SLA in these regions.

The Data and Methods:

Data:

As input data we use along-track (L3) data provided by CMEMS (Copernicus Marine Service), as well as the corresponding daily grids (L4) for comparison. The SST are obtained from ERA-5 reanalysis and tide gauge data from GESLA-3 are used for comparison.

Co-Kriging:

Co-Kriging is an extension of Ordinary Kriging (OK) - a widely used, geostatistical method - that predicts the target varible (Z) at unobserved location (u_0) by incorporating spatial autocorrelations per distance (h).

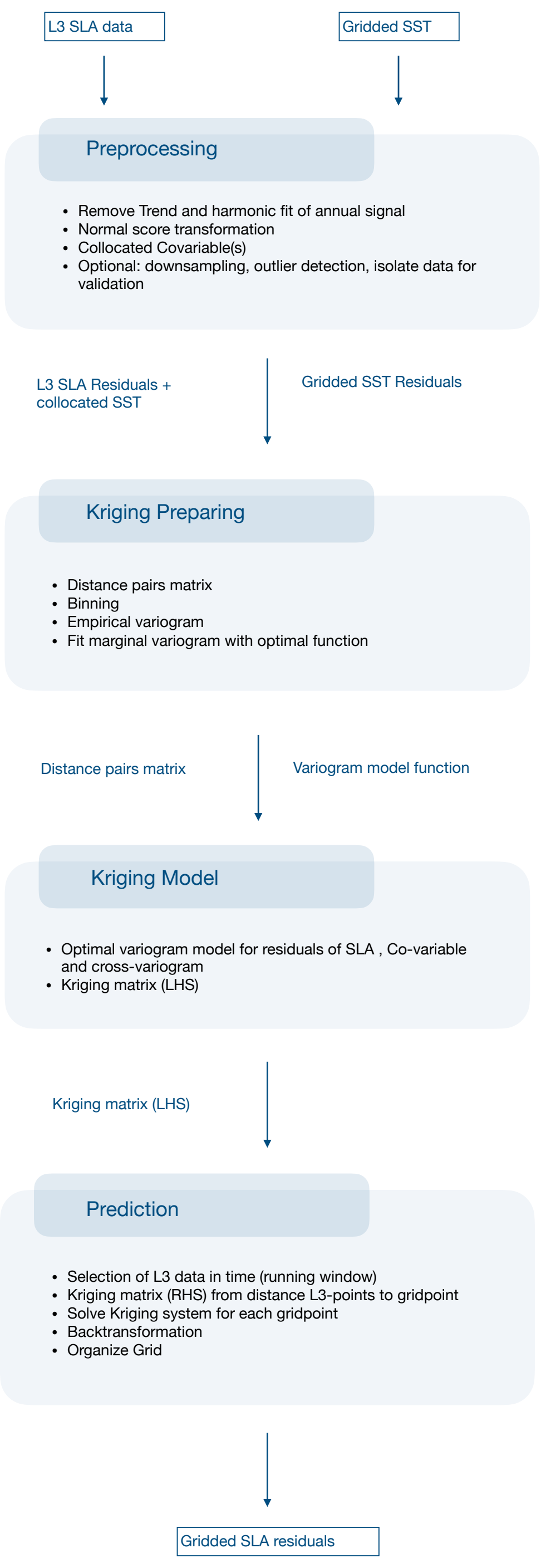
For Co-Kriging, we additionally model the spatial (cross-) covariance as a semivariogram (γ) ((1); Figure 2(b)) for an auxiliary variable (Y) as the following:

$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} (Z(x_i) - Z(x_i + h))^2 \qquad (1)$$

Co-Kriging minimizes the prediction variance by incorporating the observed data Z and their spatial relationships, providing a spatially weighted (λ) average for predictions at $Z^*(u_0)$. The optimization of predictions is based on the variogram structure.
In this approach we use an intrinsic collocated Co-Kriging (ICCK) approach (Babak & Deutsch, 2009), where the Co-variable is considered at each point of the target variable (collocated) as well as at the point of prediction:

$$Z^*(u_0) = \sum_{\alpha=1}^{n} \lambda_{Z,\alpha} Z(u_\alpha)) + \sum_{\alpha=1}^{n} \lambda_{Y,\alpha} Y(u_\alpha)) + \lambda_{Z,0} Y(u_0)) \qquad (2)$$

The predicted point is thus a weighted mean from all obervations of the target variable Z at all N points, as well as the collocated Co-variable Y, and Y at the point of prediction. The set of weights for each predicted point is obtained by solving a 2N+1 Kriging-system containing the Kriging matrix. The dataset of N along-track data points is selected as a running window of 3 days around the target grid point, with an enlarged area of along-track data for avoiding boundary effects.


L3 SLA data
Gridded SST

Preprocessing
- Remove Trend and harmonic fit of annual signal
- Normal score transformation
- Collocated Covariable(s)
- Optional: downsampling, outlier detection, isolate data for validation

L3 SLA Residuals + collocated SST
Gridded SST Residuals

Kriging Preparing
- Distance pairs matrix
- Binning
- Empirical variogram
- Fit marginal variogram with optimal function

Distance pairs matrix
Variogram model function

Kriging Model
- Optimal variogram model for residuals of SLA , Co-variable and cross-variogram
- Kriging matrix (LHS)

Kriging matrix (LHS)

Prediction
- Selection of L3 data in time (running window)
- Kriging matrix (RHS) from distance L3-points to gridpoint
- Solve Kriging system for each gridpoint
- Backtransformation
- Organize Grid

Gridded SLA residuals

Figure 1: Workflow schematic

Preprocessing:

For Co-Kriging, data is required to follow second order stationarity. Thus, Co-Kriging is done using only residuals of the original data reduced by annual cycle and trends. Further, we make a normal score transformation (Figure 2 (a), (b)) for SLA and SST residuals. Key to this approach is a proper variogram model, which is obtained from fitting a suiting empirical (cross-) variogram. In this example the variograms are fitted with the Matérn-function is used for fitting the model. Downsampling of the L3-data was applied in order to decrease the complexity of the Kriging system.

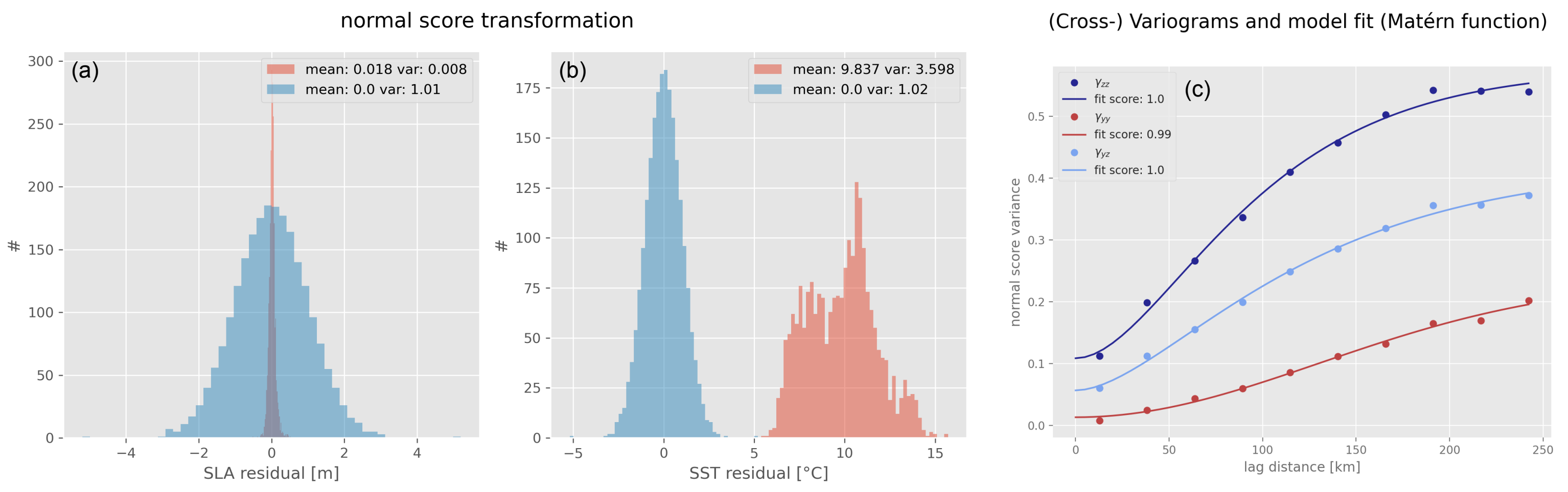


Figure 2: Preprocessing and variogram model of Co-Kriging: Histogram of normal score quantile transformation before (orange) and after (blue) for (a) SLA residuals and (b) collocated SST residuals, (c) empirical variogram and variogram fit (Matern function) for target variable (γ_{ZZ}), Co-variable (γ_{YY}) and their cross-variogram (γ_{YZ}) per distance lag in km.

Resulting Grid:

Maps:

We compare the gridded SLA from ICCK and OK with corresponging CMEMS L4 gridded SLA at the same day on the Patagonian Continenal Shelf (Figure 3).

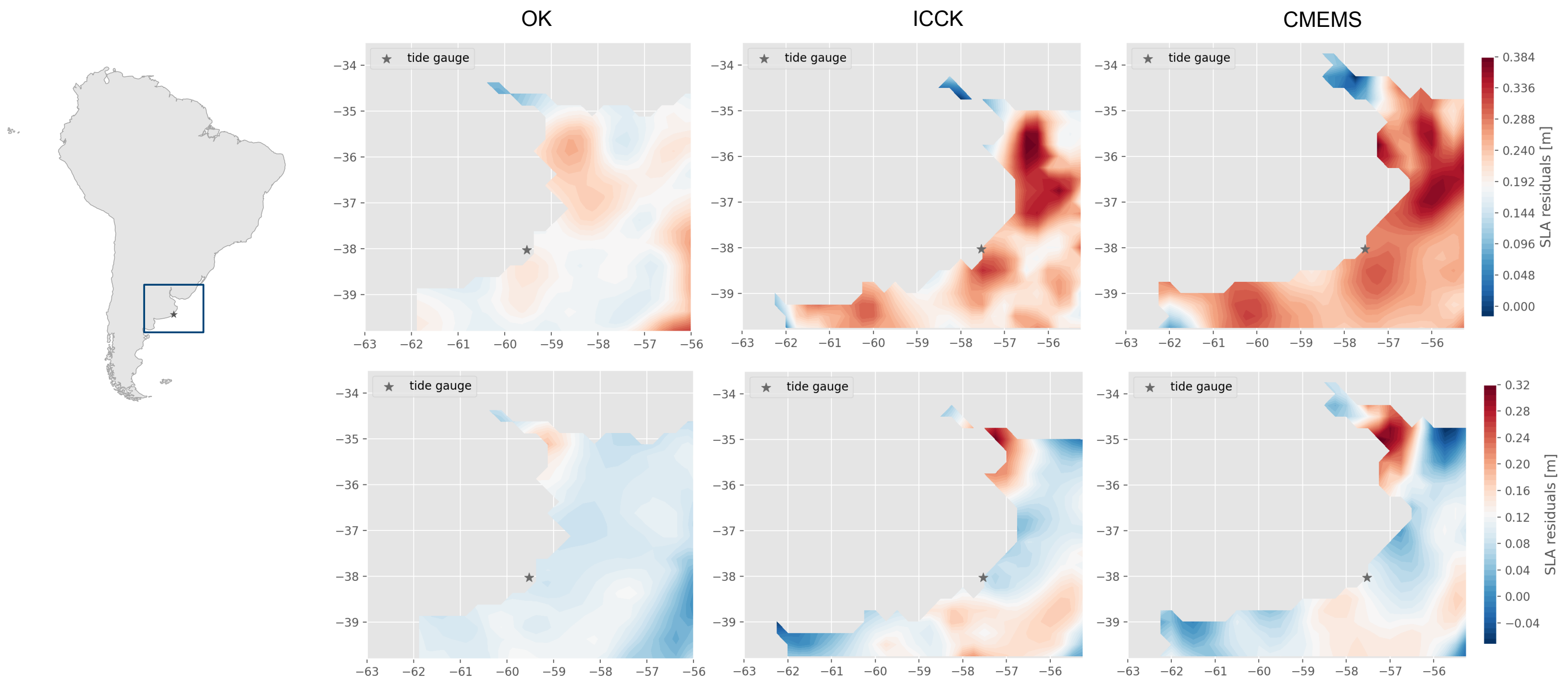


Figure 3: SLA on the Patagonian Continental Shelf. Left to right: Grid from Ordinary Kriging, Grid from intrinsic collocated Co-Kriging using SST and associated CMEMS L4 SLA map of the same day.

Time Series:

Additionally we compare the predicted grids (OK, ICCK) with tide-filtered tide gauge data at Mar del Plata (see Figure 3) and CMEMS L4 time series. The power spectral density (PSD) of the data is provided.

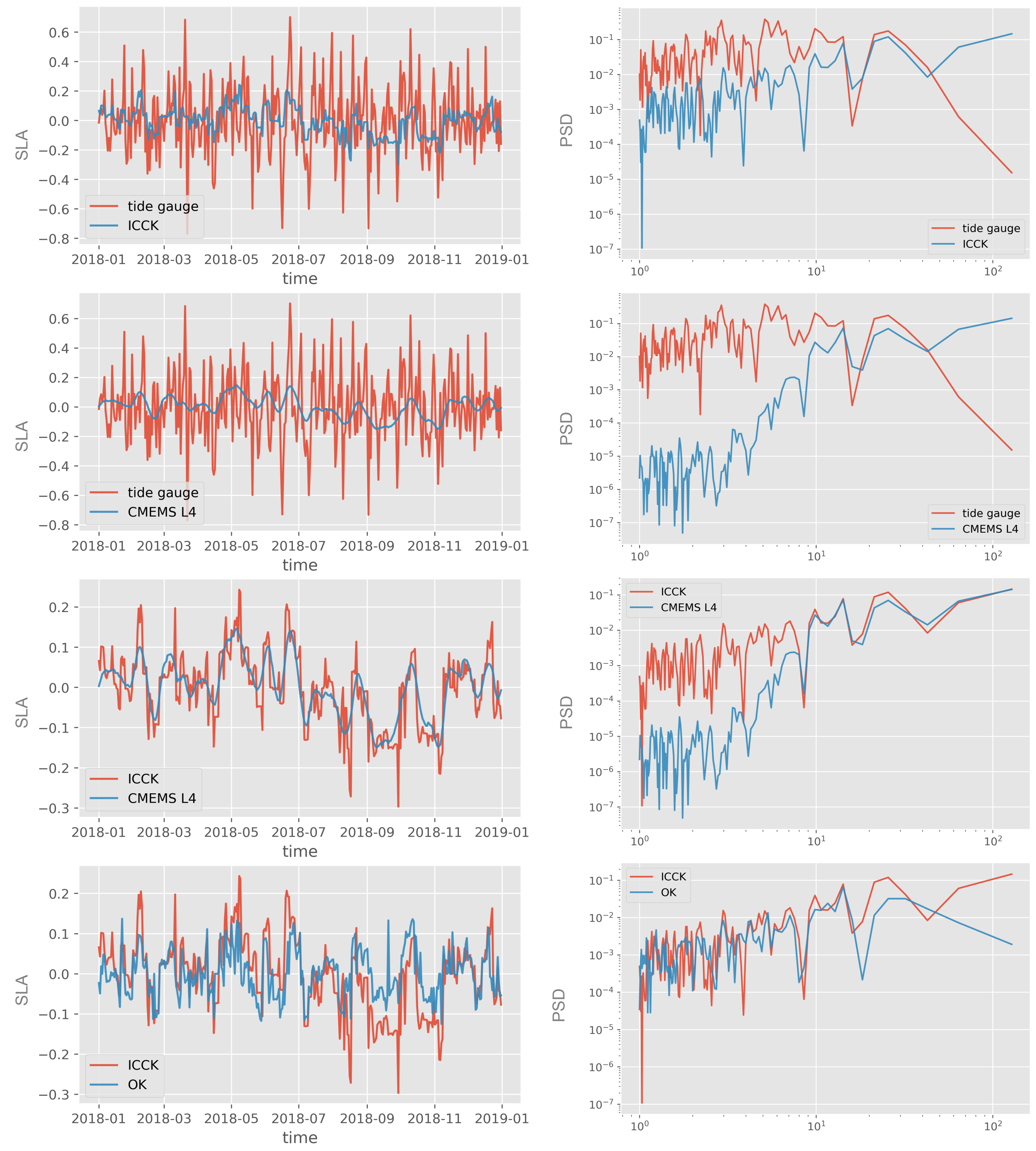


Figure 4: Left: Comparison of time series at tide gauge station Mar del Plata (see Figure 3). Right: Associated power spectral density $[m^2/d]$. Tide filtered tide gauge data from Gesla-3, Intrinsic collocated Co-Kriging (ICCK), Ordinary Kriging (OK) and CMEMS L4 gridded SLA.

Outlook:

As an conclusion, Co-Kriging is a feasible tool for mapping of SLA considering related variables. The ICCK approach was shown here to be feasable of producing gridded SLA. Moreover, they showed to hold more energy on small temporal scales at the coast than CMEMS L4 maps.
A more detailed model will be obtained using a spatial-temporal (cross-) variogram model with a three dimensional variogram model.
Further, the approach potentially allows for the combination of more than one Co-variable and combine variogram models on different temporal and spatial scales, which could be benefical for regional grids in coastal and continental shelf regions.

References:

Ballarotta, M., Ubelmann, C., Pujol, M. I., Taburet, G., Fournier, F., Legeais, J. F., ... & Picot, N. (2019). On the resolutions of ocean altimetry maps. *Ocean Science*, *15*(4), 1091-1109.

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This reserach is part of the DGF-funded project CIRCOS.

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