Models of the sea-surface height expression of the internal wave continuum



R. M. Samelson¹ and J. T. Farrar²

¹Oregon State University, Corvallis OR USA ²Woods Hole Oceanographic Institution, Woods Hole MA USA

Ocean Surface Topography Science Team Meeting - 7-11 November 2023 - Puerto Rico

Summary

Should we expect to see the background internal wave field in SWOT data? If we do, to what theoretical model should we compare the observed internal wave signal?

We discuss several models for the sea-surface height (SSH) signature of the interior-ocean internal-wave continuum. Most are based on the Garrett-Munk internal-wave model. The different models are all plausibly consistent with accepted descriptions of the interval-wave climatology in the interior ocean, but they result in different predictions for SSH spectral energy levels. The differences arise in part from differences in the treatment of near-surface stratification, and a major source of uncertainty comes from lack of knowledge about the energy in the low-vertical-mode internal-wave field. Most of these models suggest that the SSH signature of the internal-wave continuum will be visible in SWOT SSH measurements.

This poster sketches some ideas that are discussed more fully in a manuscript submitted for publication (Samelson and Farrar).

(1) The Garrett-Munk model of the internal wave field applied to SSH

The Garrett and Munk internal wave model provides a reasonably accurate description of internal wave properties in the interior ocean. It can be used to make predictions of properties like the kinetic energy spectrum or pressure spectrum that are accurate within an order of magnitude.

There are three main things we wonder about for applying it to SSH:

(1) The GM spectrum uses an exponential buoyancy frequency profile that is a reasonable description of deep ocean stratification but is grossly inaccurate in the upper ocean (see SPURS-1 profile figure, for example). (2) The SSH signal is very sensitive to the assumed form of the vertical mode spectrum; the low-mode part of this spectrum is not well constrained and while this doesn't matter for most applications of GM, it does matter for SSH. (3) The GM model uses the WKB approximation to translate between depths, such that internal wave kinetic energy at depth 2 is N₂/N₁ times the energy at depth 1. This is usually adequate but may not work well in the upper ocean where N(z) changes rapidly. There are additional physics (e.g., D'Asaro, 1978 or Levine, 1987) that can enhance the upper-ocean wave amplitude beyond what WKB would predict.

We considered a few model variants that account for (1)



If we ignore (1), (2), and (3) above, the simplest form of the GM model predicts a wavenumber magnitude spectrum (not 1D wavenumber



Buoyancy frequency profile from the SPURS-1 mooring site in the subtropical North Atlantic (blue line). The canonical Garrett-Munk exponential buoyancy frequency profile is shown for comparison (orange line).

spectrum) of the form:

 $\hat{F}_{\eta}(K)|_{0} = \frac{2E_{0}b^{2}N_{0}^{2}}{\pi g^{2}} \sum_{i=1}^{J} \frac{c_{j}^{5}K^{2}f}{(c_{i}^{2}K^{2} + f^{2})^{2}} H_{j}.$

This term is taken to be constant

c, is the gravity wave speed for mode j. H_i is the vertical mode spectrum

Conclusion

This suggests internal wave SSH signals may be highest in the western parts of the midlatitude gyres, but there are many potentially important effects that could modify this picture. There could be major effects from: (1) Directionality of the IW field (2) Intermittency (3) Variations in low-mode energy

(2) Implied geographical variability of the SSH signal of the background internal wave field

The equation above will have geographic variability because of geographic variations in f and c_i. We used the Chelton et al (1998) values for c₁ and the



Estimated standard deviation of SSH (cm) caused by internal waves with wavelengths of 15-100 km, based on the "direct extrapolation" form of the GM model above. For comparison, the SWOT error requirement for these wavelengths has a standard deviation of 0.47

approximation $c_i = c_1/j^2$.

0°E 100°E 60°W 160°W

1.4 1.6 RMS SSH fluctuation, 15-100 km wavelengths (cm)

The bottom line is that we will learn a lot from the similarities and differences from GM.

(The magenta circles mark the location of the SPURS-1 mooring in the North Atlantic that was used for the buoyancy frequency profile above and the Stratus mooring used by Callies and Wu (2019).)

References

cm.

Callies and Wu (2019): Some expectations for submesoscale sea surface height variance spectra. Journal of Physical Oceanography, 49 (9), 2271–2289.

D'Asaro (1978): Mixed layer velocities induced by internal waves. Journal of Geophysical Research, 83 (C5), 2437–2438.

Munk (1981): Internal waves and small-scale processes. *Evolution of Physical Oceanography*, B. A. Warren, and C. Wunsch, Eds., MIT Press, Cambridge, MA, 623 pp.

Samelson and Farrar (submitted to JPO): Models of the sea-surface height expression of the internal-wave continuum.