Improvement of global mean sea level trend and acceleration uncertainties from satellite altimetry

Anna Mangilli[1], Pierre Prandi[1], Benoit Meyssignac [2] , Victor Quet[1], Frank Octau[1], Sylvie Labroue[1], Michael Ablain[3], Anne Barnoud[3], Gerald Dibarboure[2]

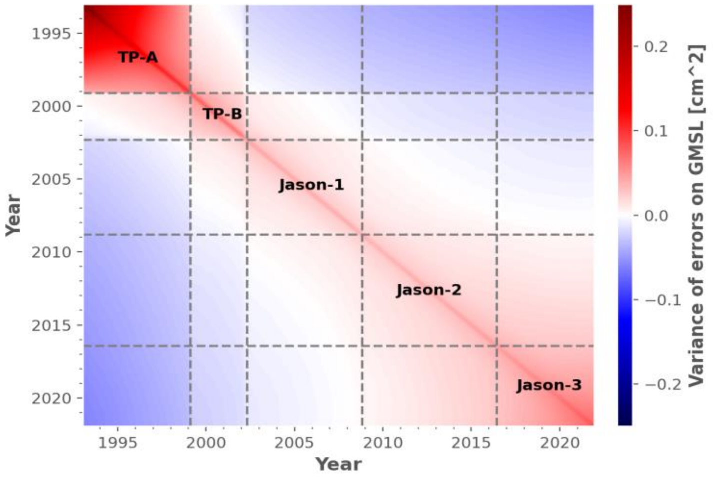
[1]CLS (France),[2] LEGOS (France), [3] Magellium (France), [4] CNES (France)

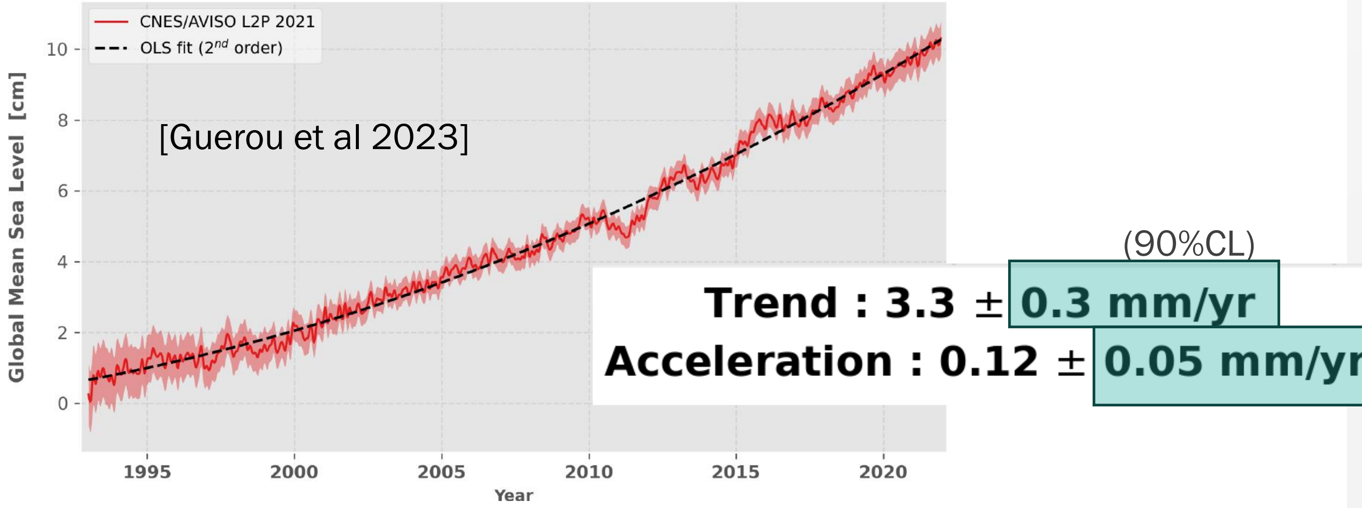
OUTLINE

- Context and motivation of revisiting the GMSL's trend and acceleration estimation
- General Least Square and Bayesian estimation
- Conclusion and perspectives

Context and motivation

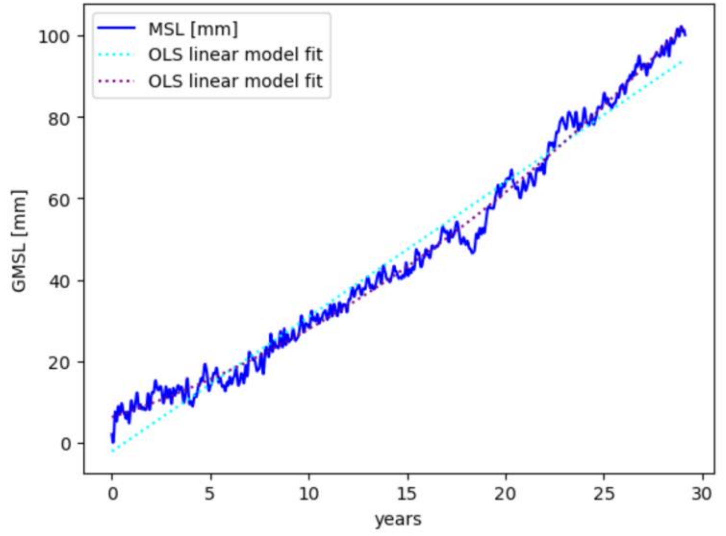
- The accurate measurement of Global Mean Sea Level (GMSL) record and the estimation of GMSL rate and acceleration are key goals of high precision satellite altimetry
- Great efforts have been put in the last decade to better understand and improve the uncertainties associated to the GMSL measurements from radar altimetry leading to the design of an error variance-covariance matrix describing the temporal correlations of the GMSL uncertainty budget [Ablain et al 2009, Ablain et al 2019, Guerou et al 2023]





- **Further improvements are still required to address three main scientific questions:** 1) closure of the sea level budget, 2) detection, and attribution of the signal in sea level that is forced by greenhouse gases emissions (GHG) and 3) estimate of the current Earth Energy Imbalance [Meyssignac et al 2023].
- Meeting such requirements will require improving the accuracy and precision of satellite altimetry data but also **further improving the error description and the statistical analysis.**

Context and motivation



Current GMSL analyses are done by using an **Ordinary Least Square (OLS)**

$$\hat{\beta} = argmin[(\mathbf{y} - \mathbf{m})^T (\mathbf{y} - \mathbf{m})]$$

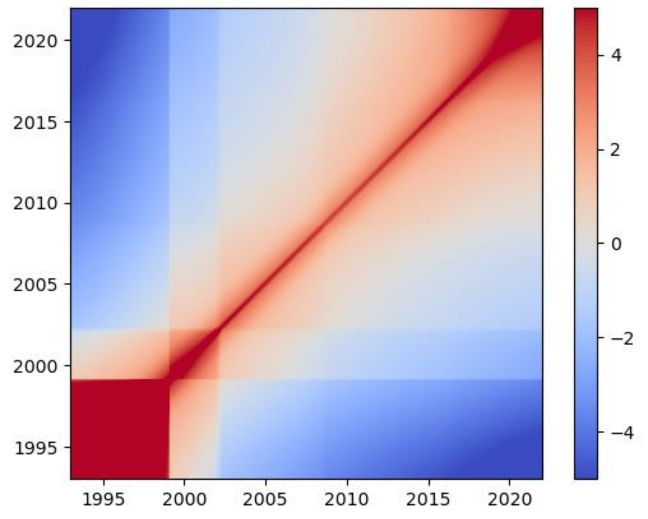
Residuals

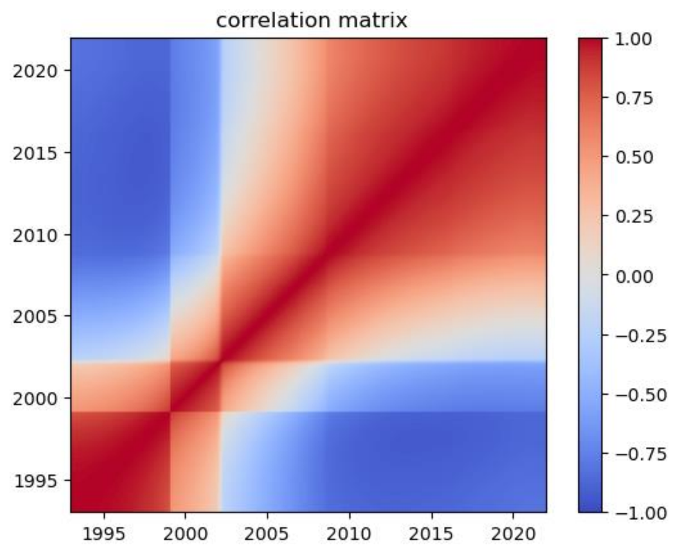
Data

Model (linear/quadratic)

$$R_{ij} = \frac{C_{ij}}{\sqrt{C_{ii}C_{jj}}}$$

BUT





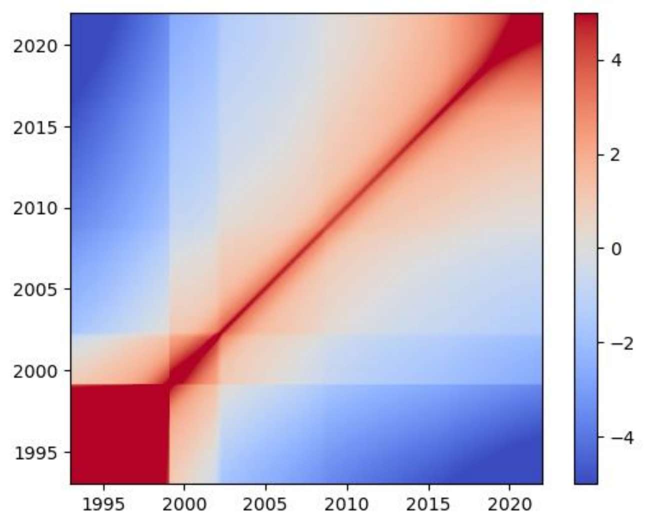
GMSL error variance-covariance matrix:
Heteroskedastic and highly correlated noise

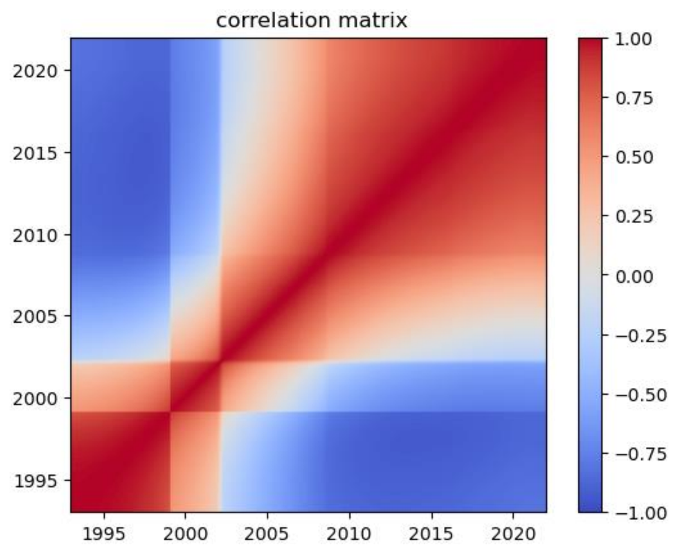
OLS is no longer adapted: sub-optimal estimation

General Least Square Estimator (GLS)

$$\hat{\beta} = argmin[(\mathbf{y} - \mathbf{m})^T \Sigma^{-1} (\mathbf{y} - \mathbf{m})]$$

Data Model (linear/quadratic) Inverse of the Covariance matrix

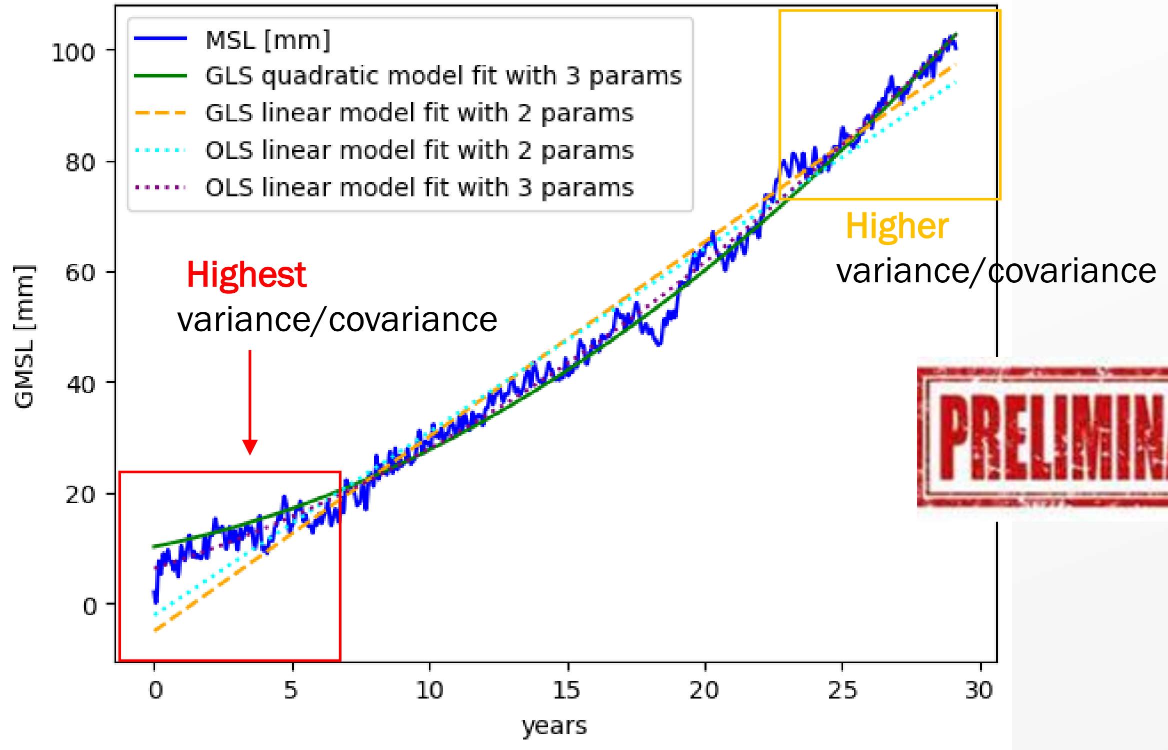


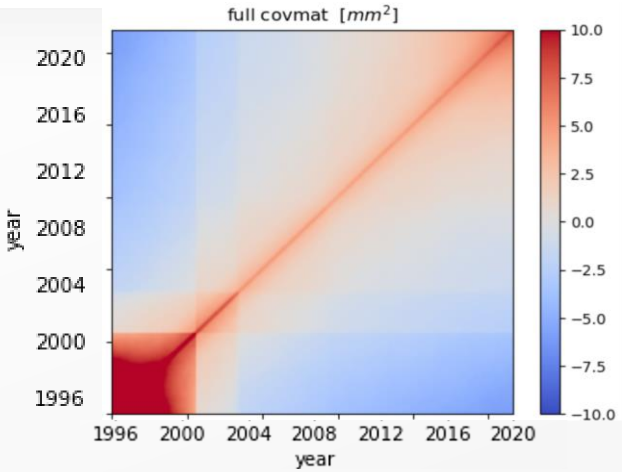


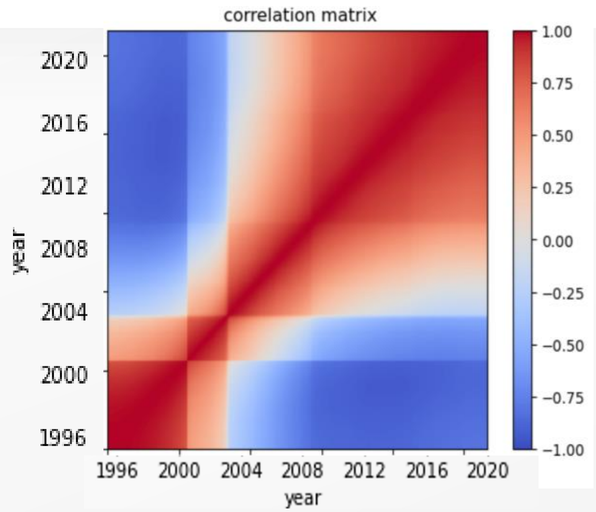
GMSL error variance-covariance matrix:
Heteroskedastic and highly correlated noise

GLS: optimal estimation of trend and acceleration

GLS fit with full covariance (vs OLS) fit





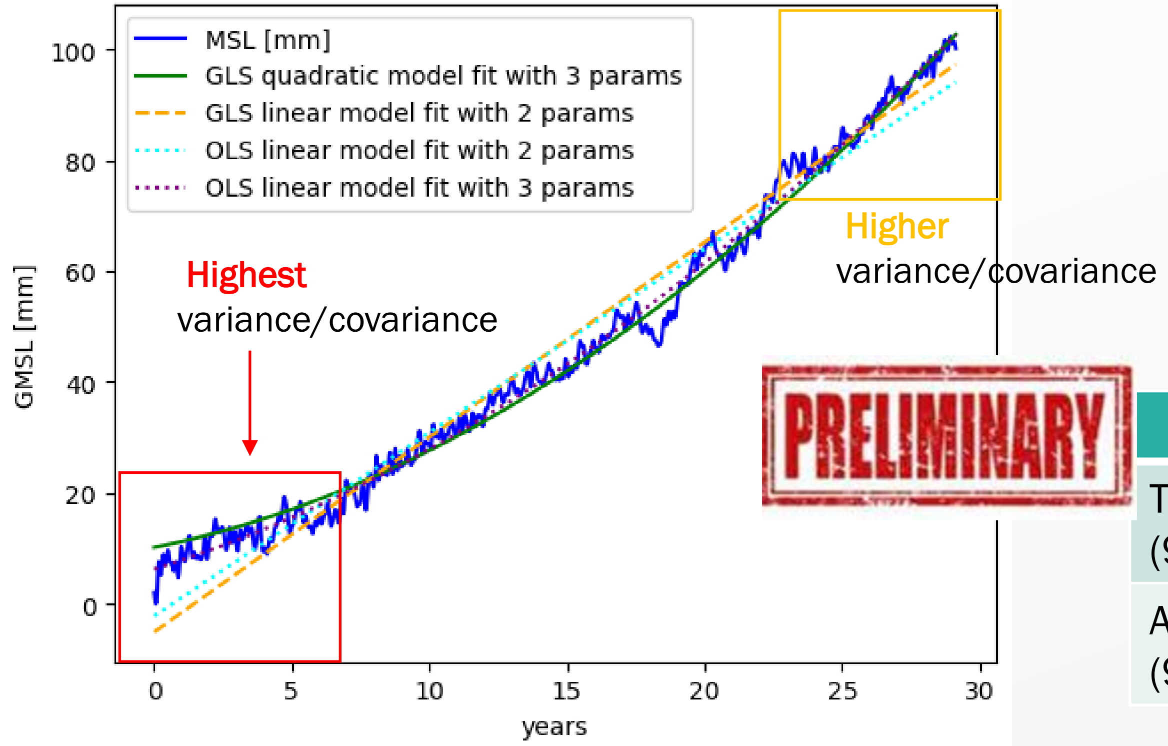


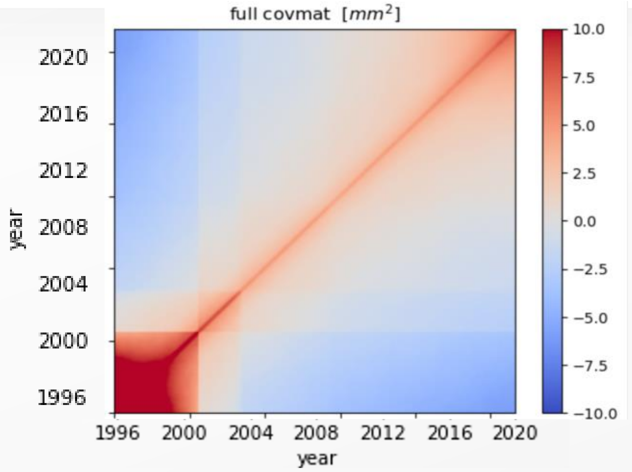
	GLS estimation
Trend [mm yr-1]	3.51 +/-0.16
Acceleration [mm yr-2]	0.14 +/-0.026

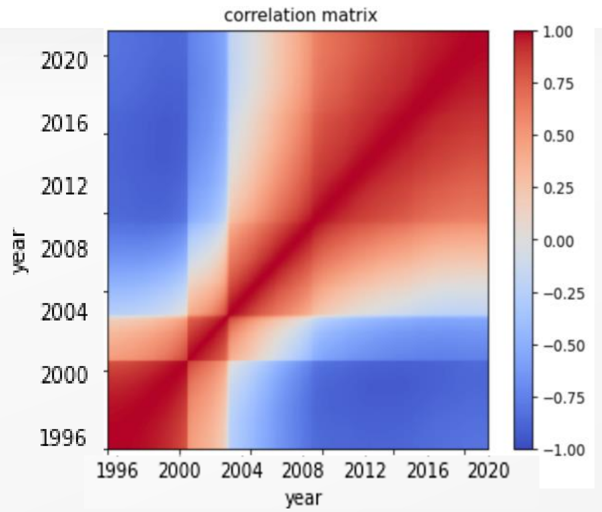
- Because of the large covariance (and variance) of the data the GLS fit gives different results wrt the OLS

GLS: optimal estimation of trend and acceleration

GLS fit with full covariance (vs OLS) fit







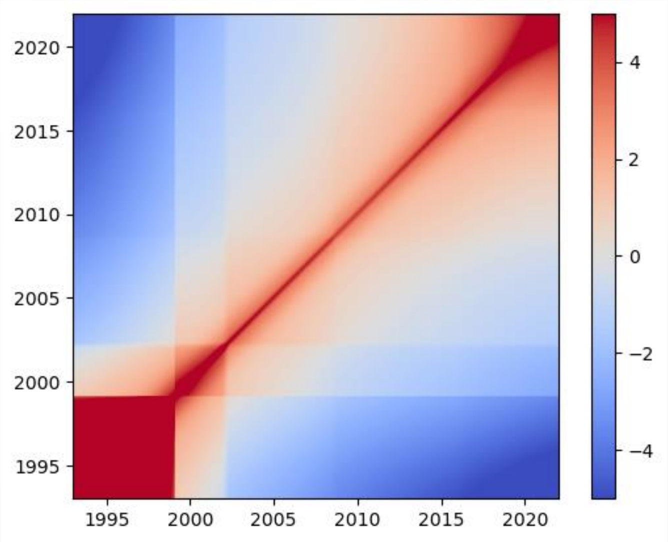
	Current [Guerou et al 2022]	GLS estimation
Trend (90%CL) [mm yr-1]	3.3 +/- 0.30	3.51 +/-0.26
Acceleration (90%CL) [mm yr-2]	0.12 +/-0.05	0.14 +/-0.04

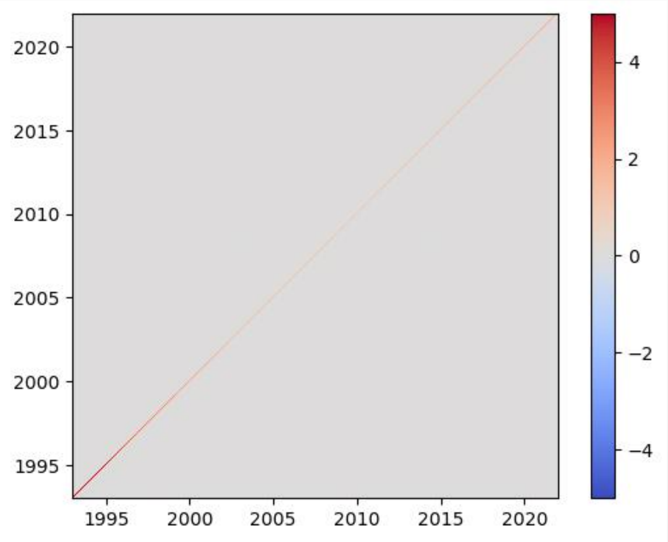
- Because of the large covariance (and variance) of the data the GLS fit gives different results wrt the OLS
- GLS provides with an optimal estimation of the GMSL trend and acceleration given the data variance/covariance: The parameters uncertainties are significantly reduced wrt to previous estimates : ~15% better for trend and ~20% better for acceleration

Impact of the noise correlations on parameters estimation

Look at the difference between the GLS results obtained with the full covariance matrix wrt the ones obtained with the diagonal covariance matrix (=only the variance)

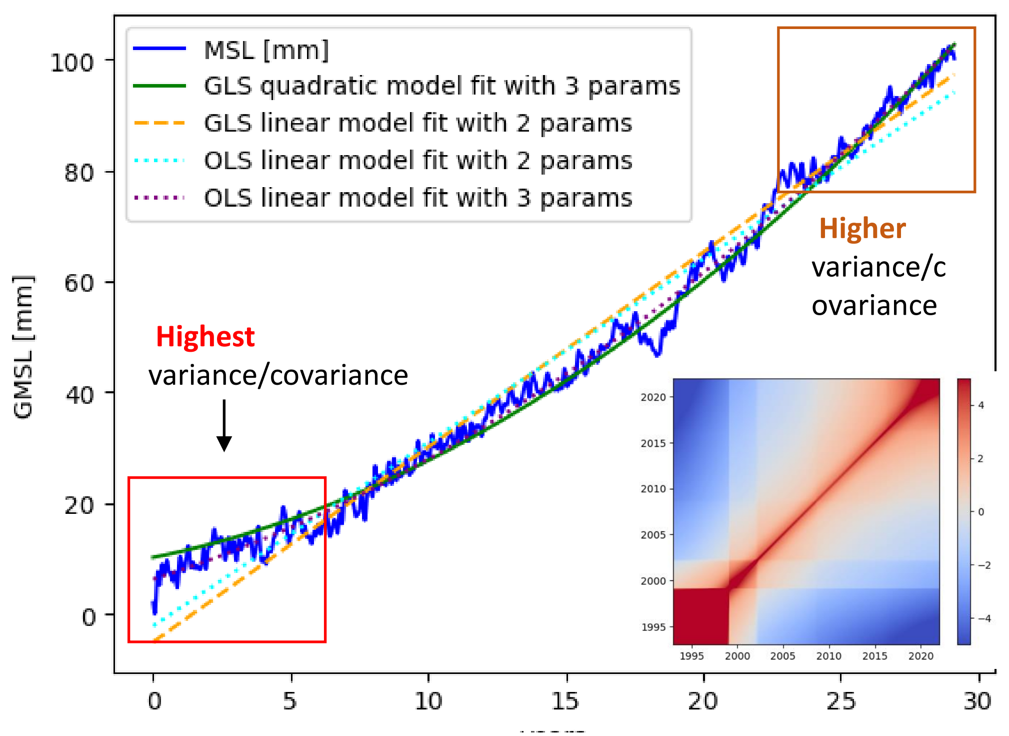
$$\hat{\beta} = argmin[(\mathbf{y}-\mathbf{m})^T \Sigma^{-1} (\mathbf{y}-\mathbf{m})]$$





Impact of noise correlations on parameters estimation

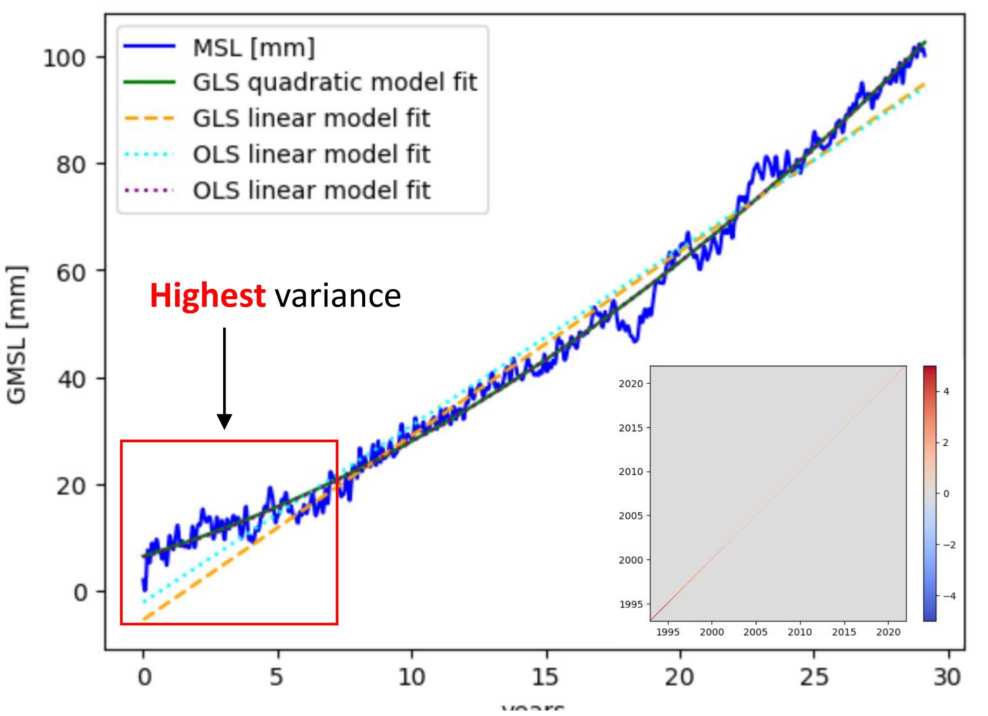
GLS fit with *full covariance* matrix



$$trend \pm \sigma_{trend} = 3.51 \pm 0.16 \text{ mm y}^{-1}$$
$$acc \pm \sigma_{acc} = 0.149 \pm 0.026 \text{ mm y}^{-2}$$

PRELIMINARY

GLS fit with *diagonal covariance* matrix



$$trend \pm \sigma_{trend} = 3.44 \pm 0.01 \text{ mm y}^{-1}$$
$$acc \pm \sigma_{acc} = 0.12 \pm 0.0028 \text{ mm y}^{-2}$$

- GLS results with diagonal covariance matrix are overall compatible with OLS results (yet small differences are seen in the highest variance regions)
- Noise correlations have a significative impact on parameter estimation (both parameters' values and errors)

GMSL trend and acceleration estimation: Bayesian

$$\boxed{P(\theta|y)} = \frac{\mathcal{L}(y|\theta)\,P(\theta)}{P(y)} \simeq \boxed{\mathcal{L}(y|\theta)}\boxed{P(\theta)}$$

Posterior

Likelihood

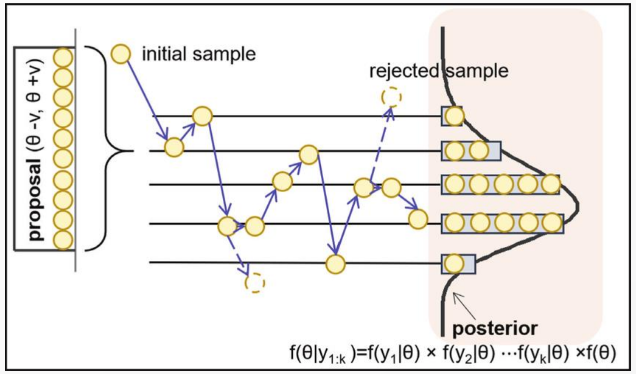
Prior = « probability of the model »

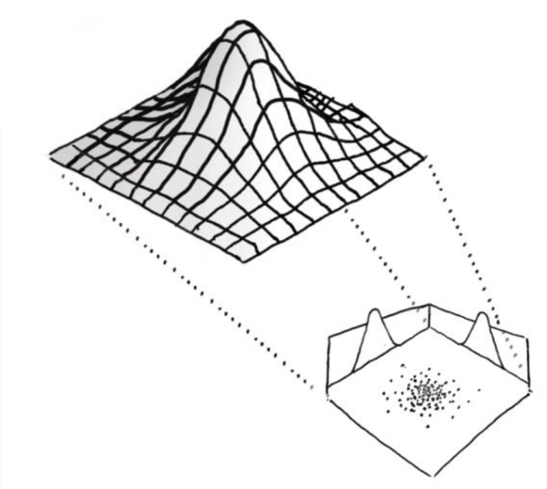
Multivariate Gaussian likelihood

$$\mathcal{L}(y|\theta) = \prod_i \mathcal{L}(y_i|\theta) = \frac{1}{\sqrt{2\pi det(\Sigma)}} exp\left[-\frac{1}{2}((\mathbf{y}-\mathbf{m})^T \boxed{\Sigma^{-1}}(\mathbf{y}-\mathbf{m}))\right]$$

Inverse of the Covariance matrix

Sampling of the posterior distribution with Markov Chain Monte Carlo (MCMC) algorithms

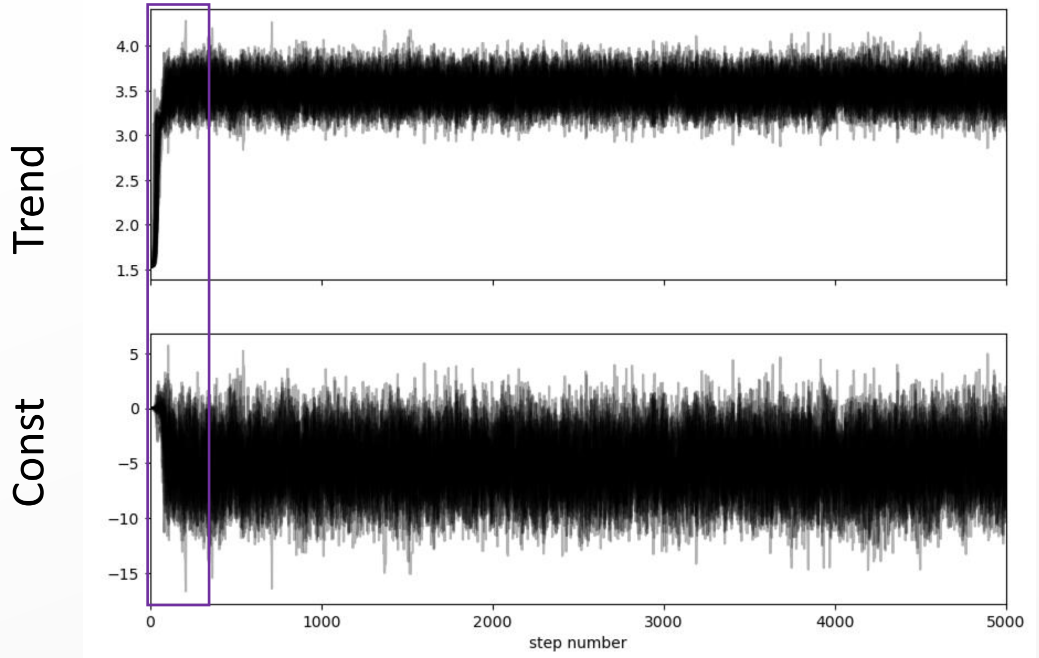




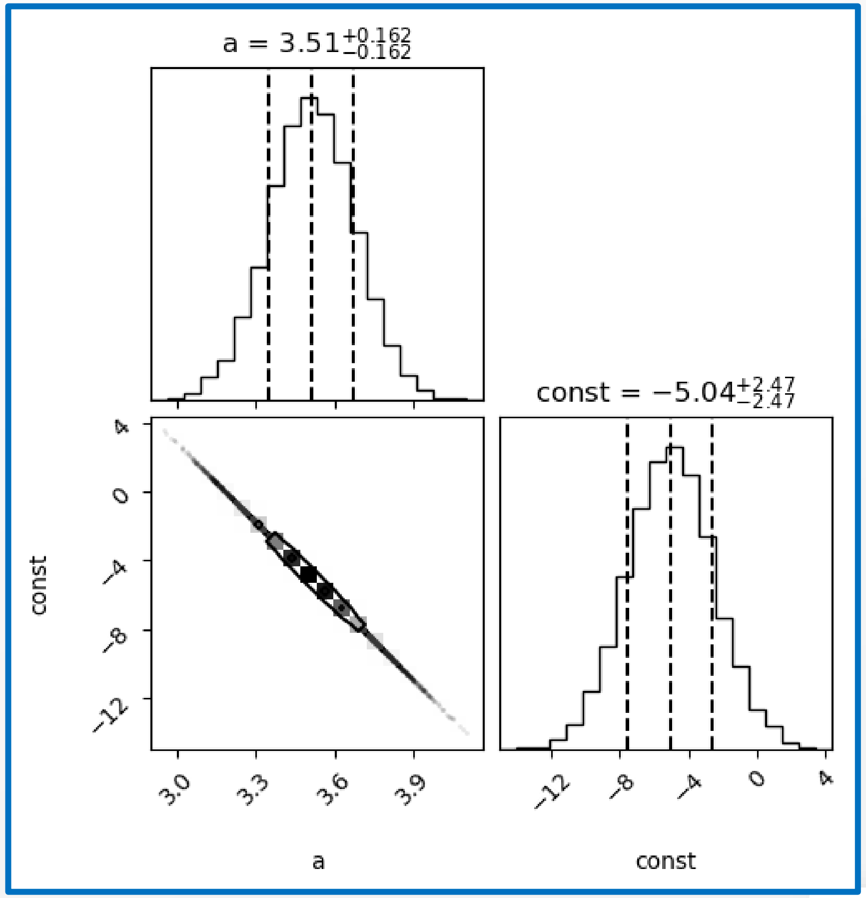
Allow to robustly estimate and propagates the parameters uncertainities

GMSL: Bayesian analysis for trend estimation

Should give the same results as GLS with full covariance matrix, and it does!



= burn-in (this part is removed from the chain for parameter estimation)



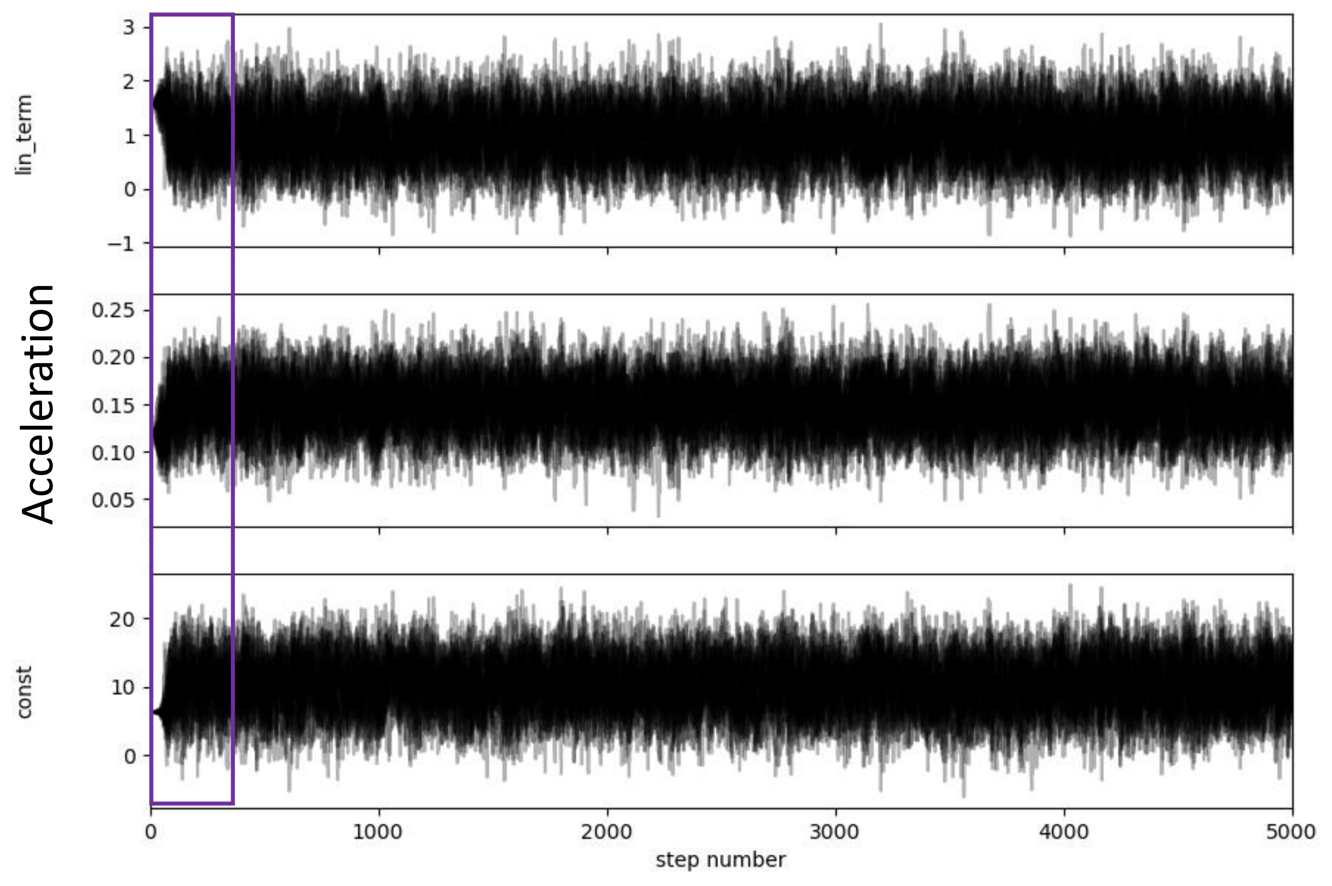
"corner" plot:

- two dimensional projections of the posterior probability distributions of the parameters
- Displays the covariances between parameters

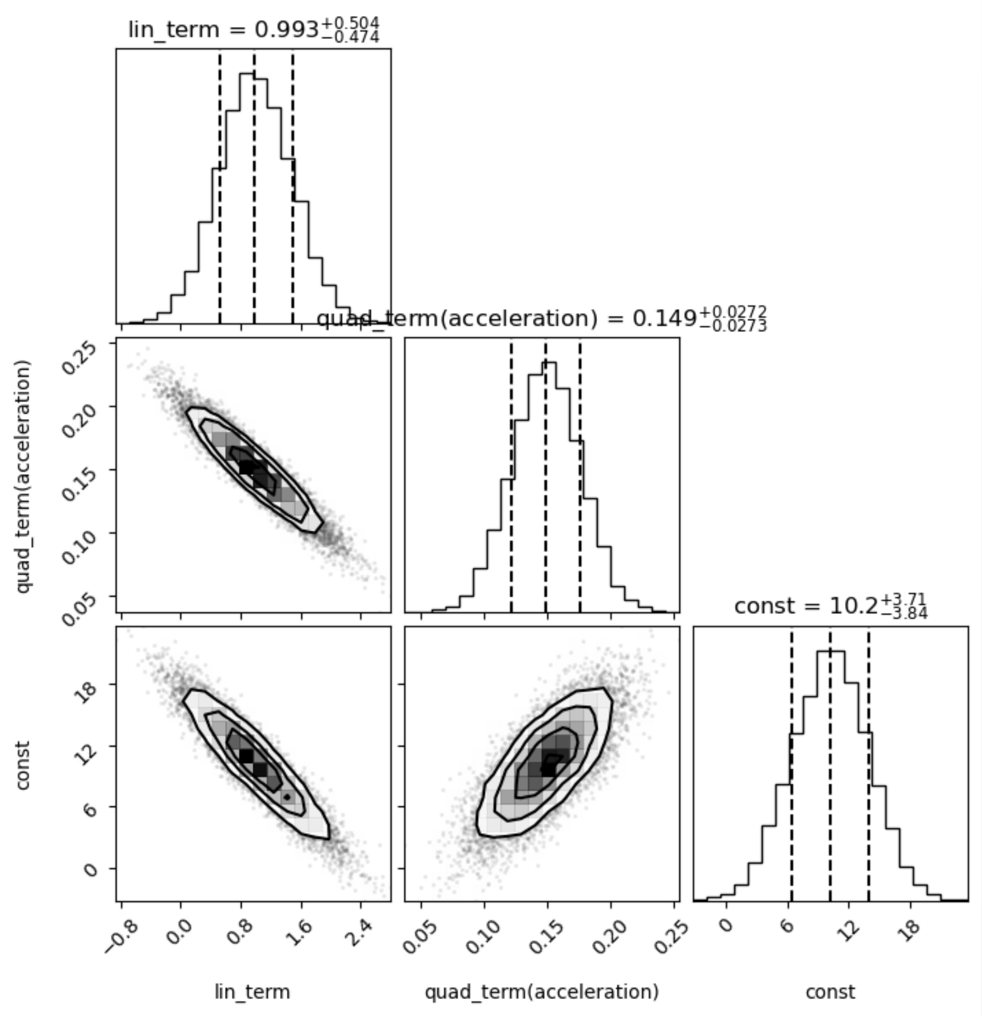
CLS
COLLECTE LOCALISATION SATELLITES

GMSL: Bayesian analysis for the acceleration estimation

Should give the same results as GLS with full covariance matrix, and it does!

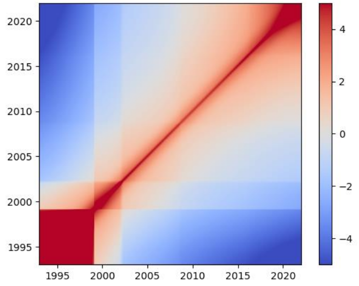


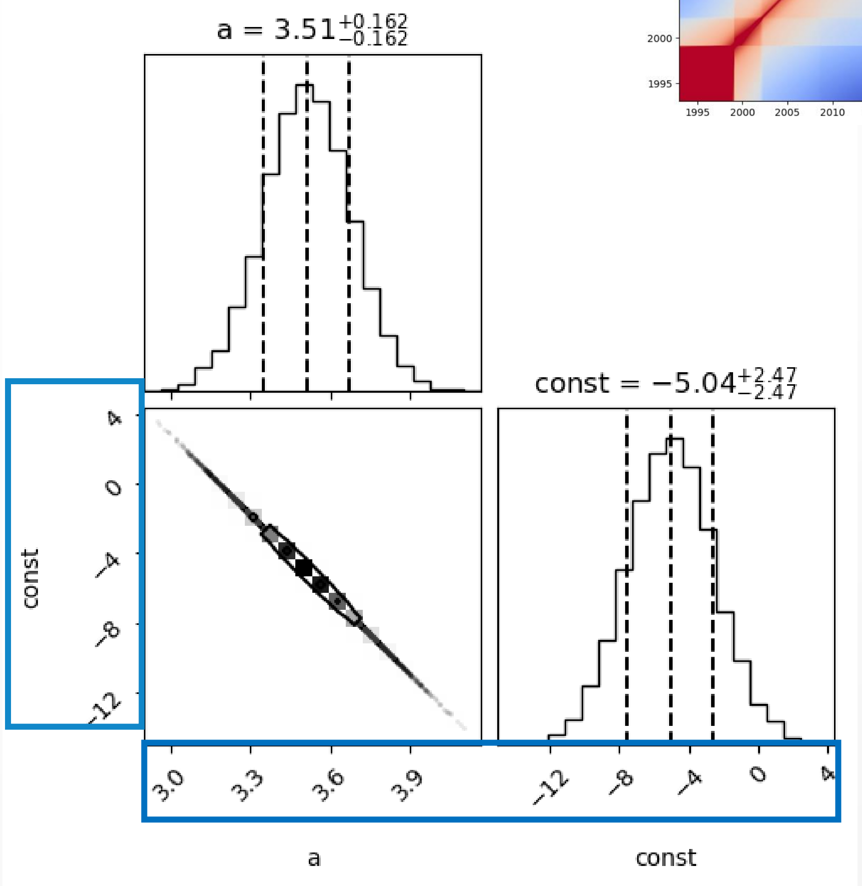
= burn-in (this part is removed from the chain for parameter estimation)



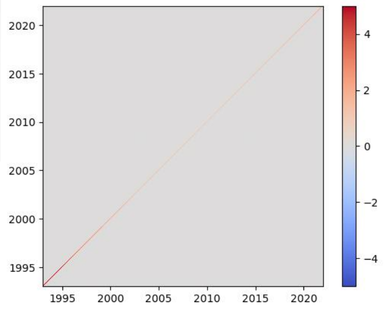
Impact of noise correlations (Bayesian analyisis): trend

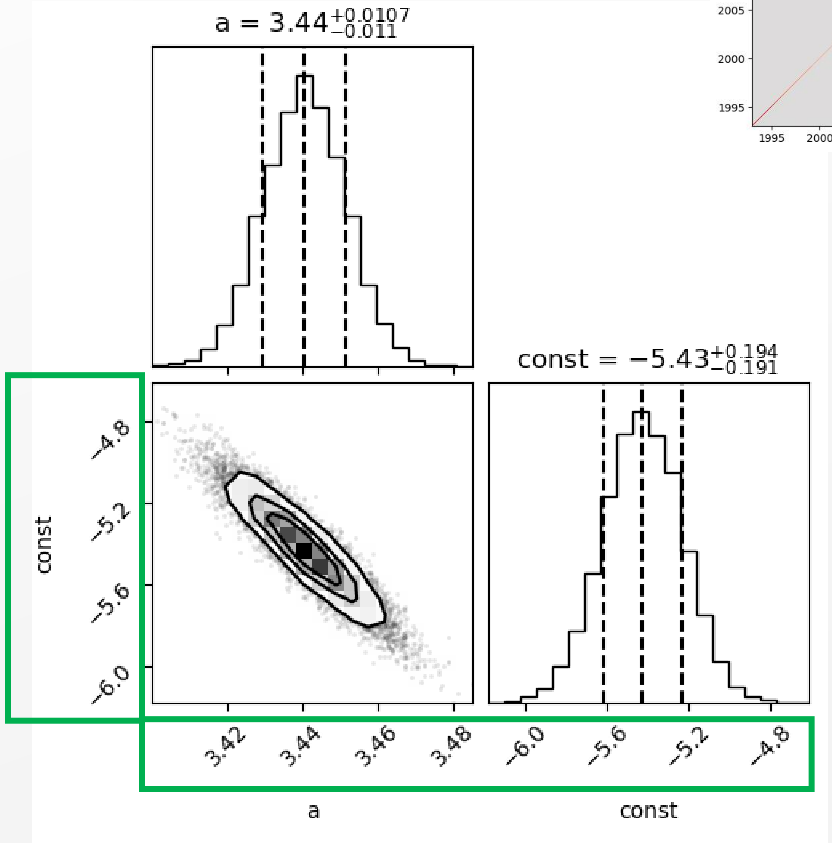
full covariance matrix





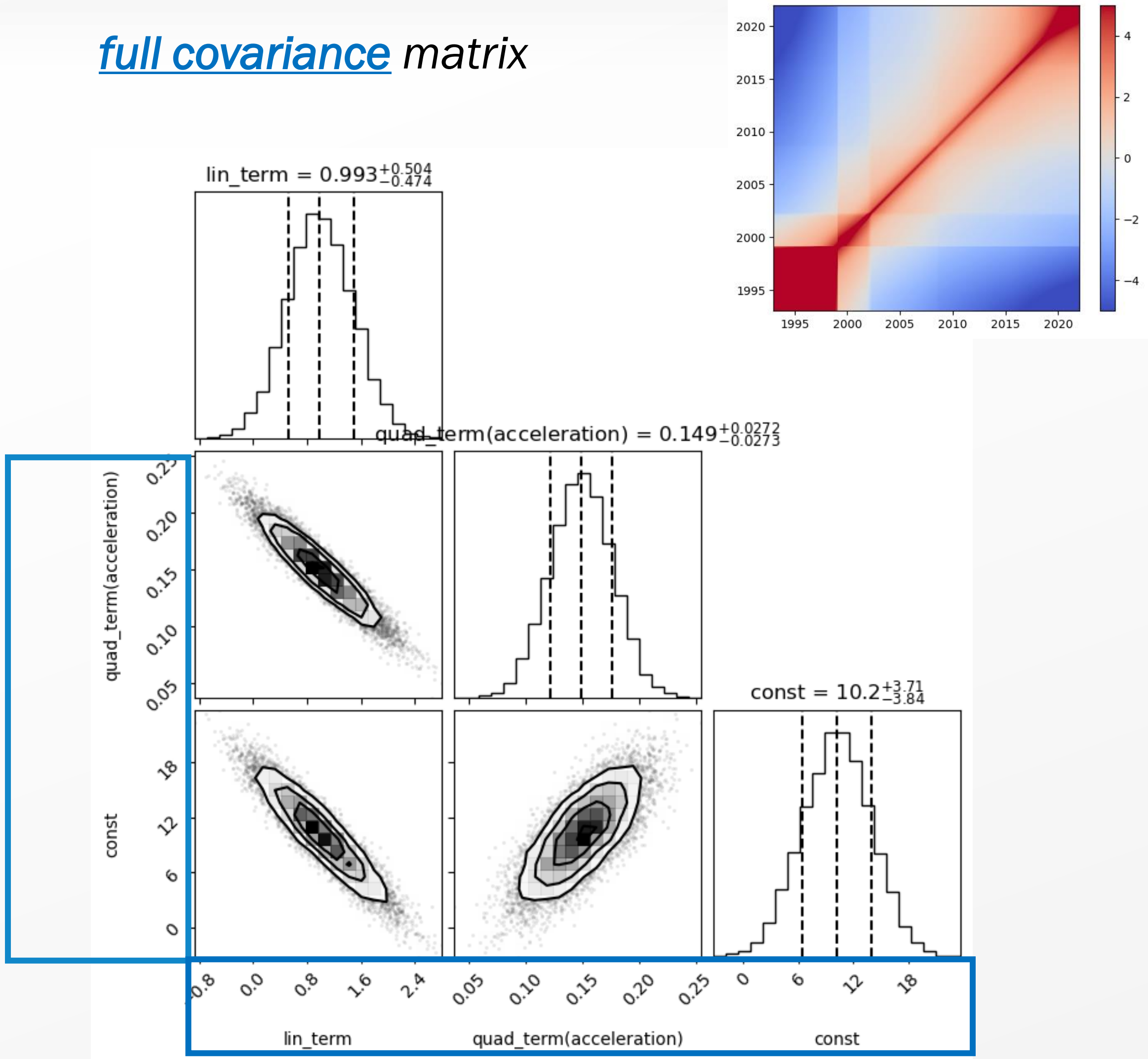
diagonal covariance matrix



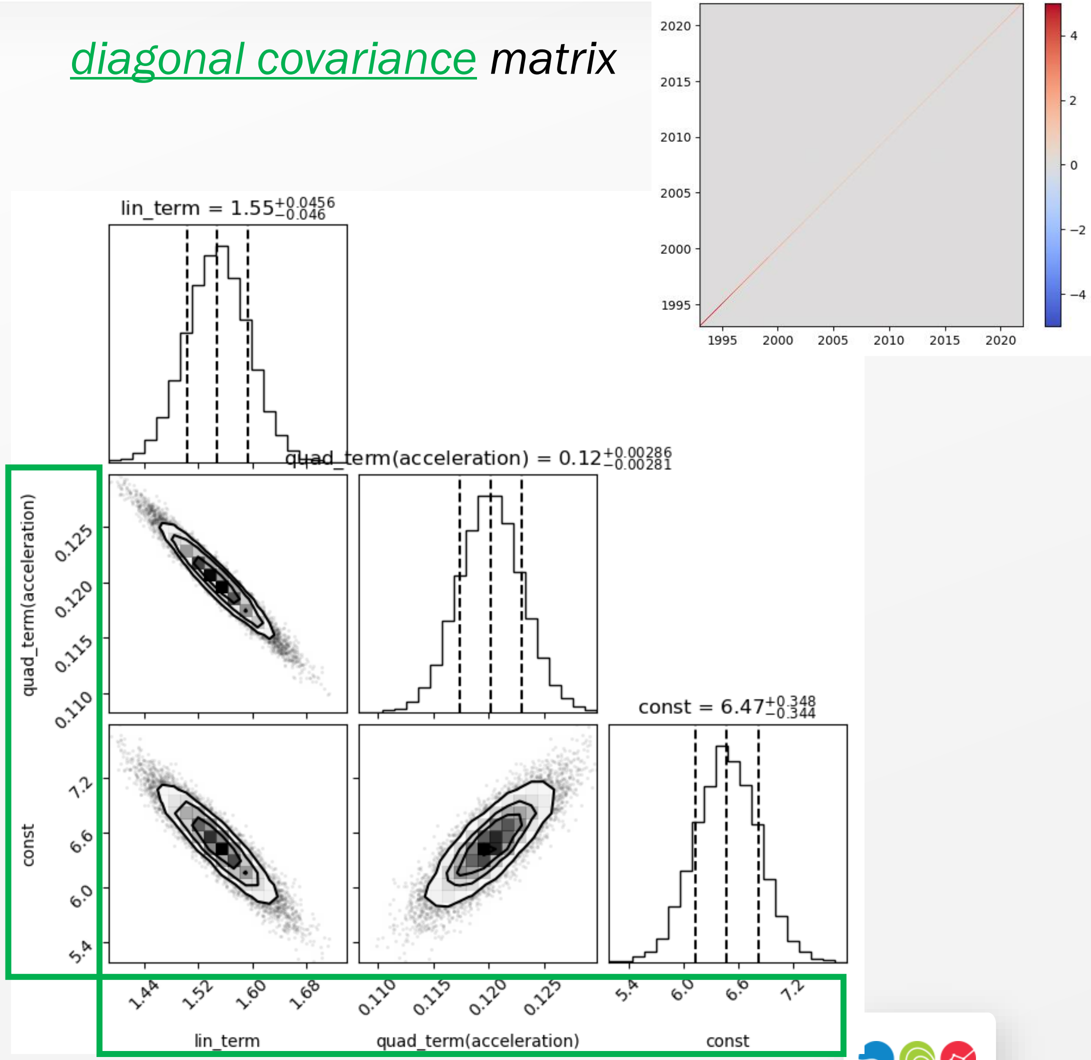


Impact of noise correlations (Bayesian analyisis): acceleration

full covariance matrix



diagonal covariance matrix

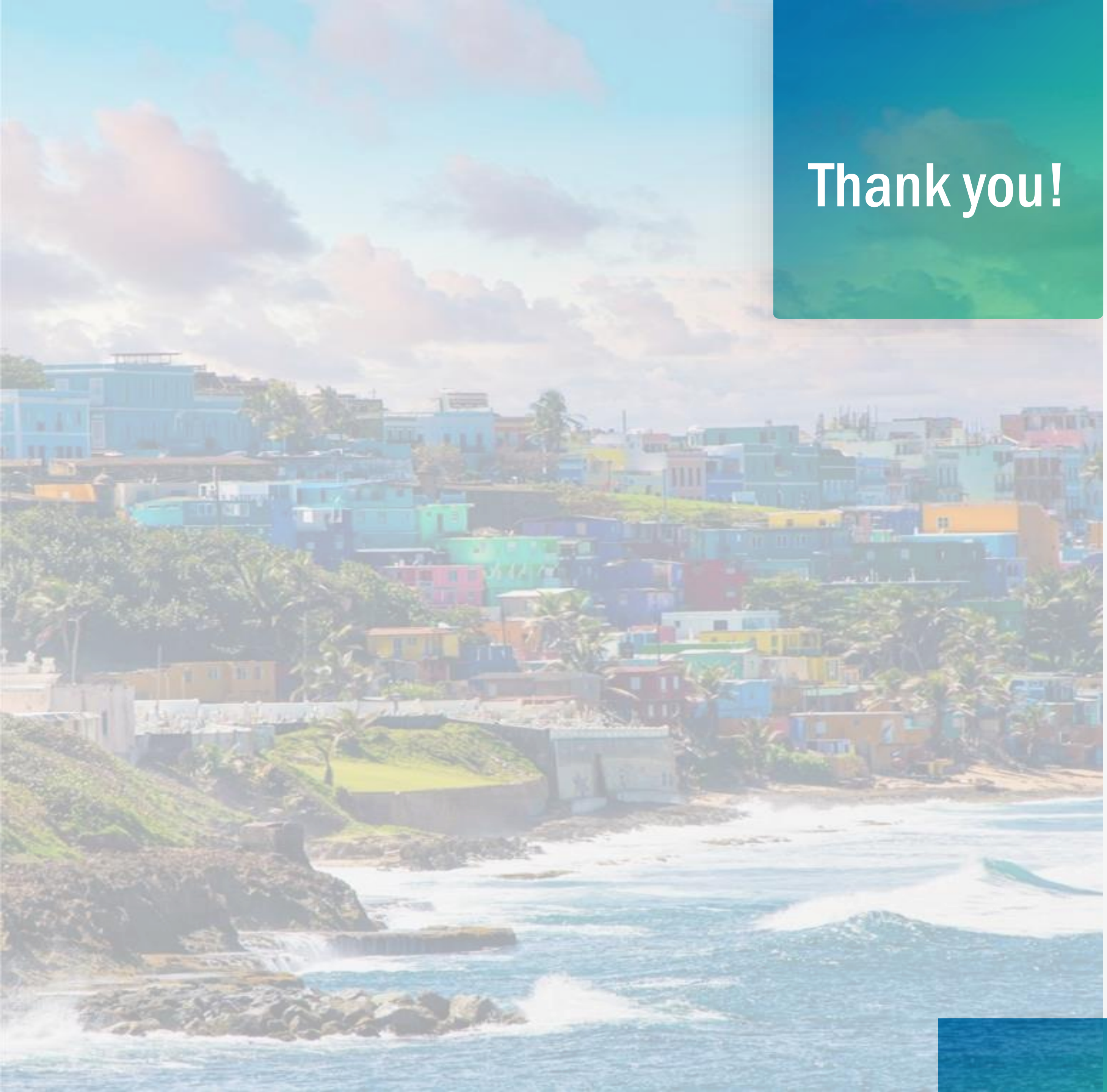


Conclusions and Perspectives

- We revisited the GMSL analysis and demonstrated that **an optimal estimation of the GMSL data (General Least Square and Bayesian approach) allows to have a ~15% improvement of the trend uncertainty and ~20% improvement of the acceleration uncertainty** with respect to current estimates
- **Noise correlations have a significant impact on parameter estimation** (both parameters' mean and error): accurate modelling and assessment of the GMSL error variance/covariance is crucial

Perspectives

- Further tests and assessment of the GLS approach: once consolidated adopt as baseline for future GMSL analysis for an optimal estimation of GMSL trend and acceleration
- The GLS method should be applied to improve the ocean heat content trend and acceleration derived from space geodetic measurements
- Further explore the benefits of the Bayesian approach for future analysis
- The results presented here are for global GMSL but the analysis can also be applied at local scales



Thank you!

amangilli@groupcls.com

CLS
COLLECTE LOCALISATION SATELLITES