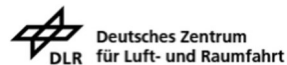




# CROSS-SPECTRAL ANALYSIS OF SAR ALTIMETRY WAVEFORM TAILS

08 November 2023

Partners

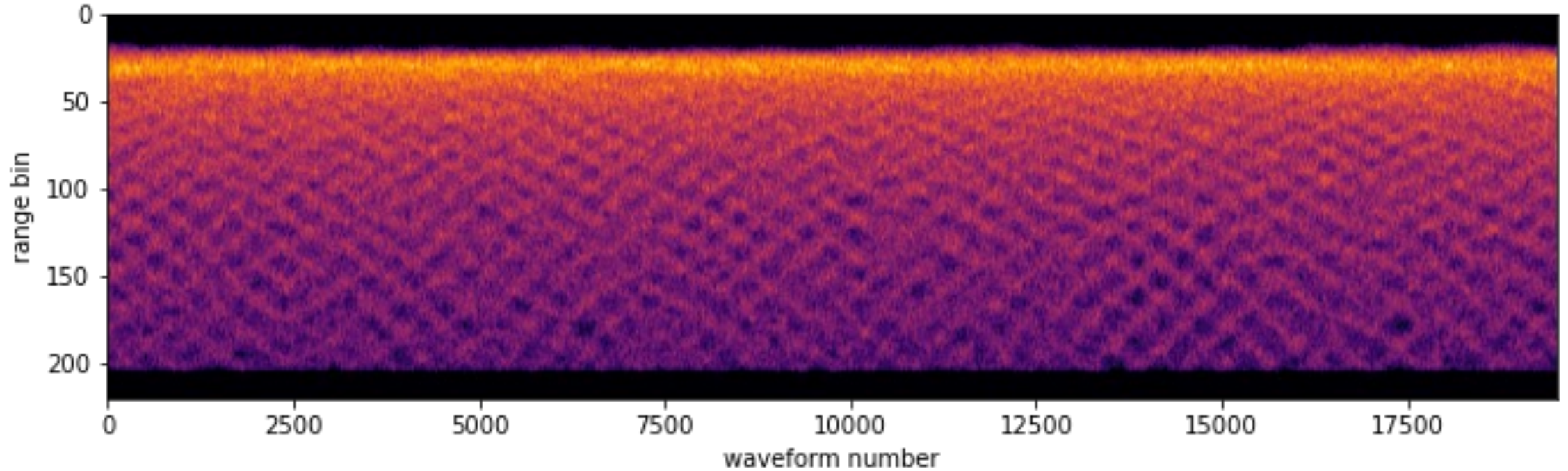


Funded by



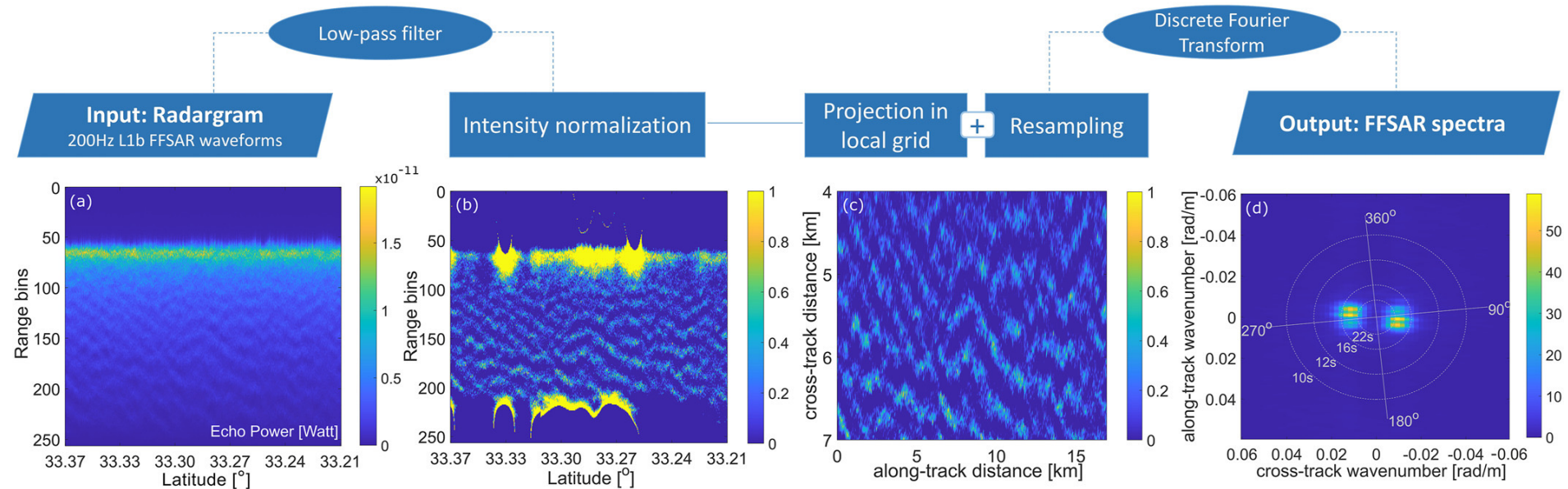


# Waveform tail backscatter modulations



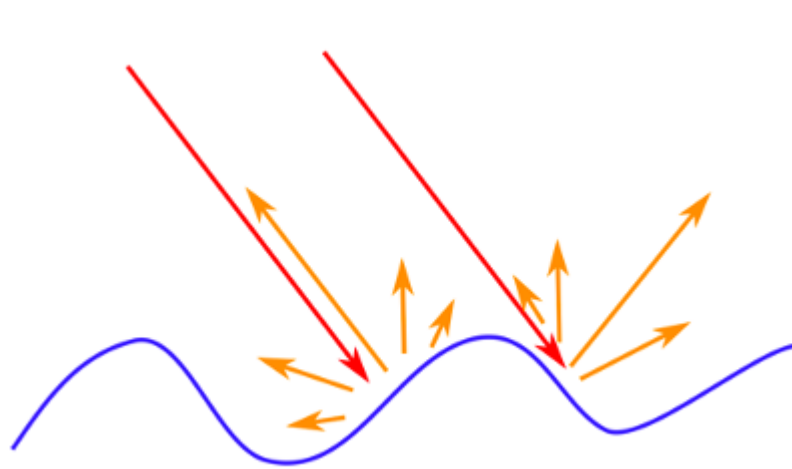
# Altimetry SAR spectrum

- Processing follows Altiparmaki et al. (2022).
- Correction for Earth's curvature.

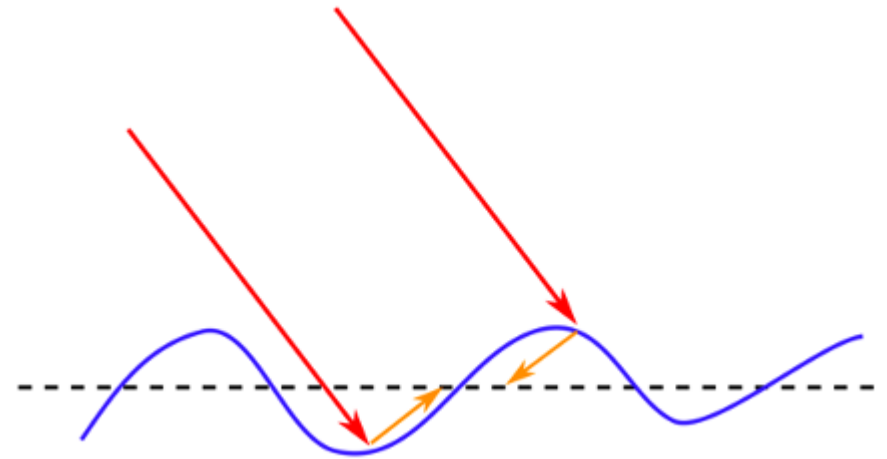




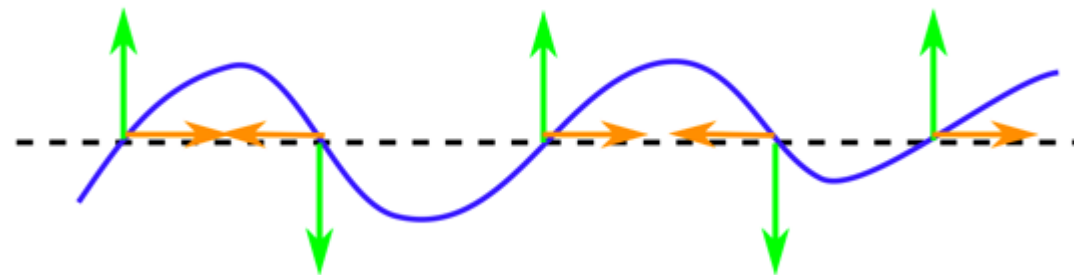
# Modulation mechanisms



Tilt modulation

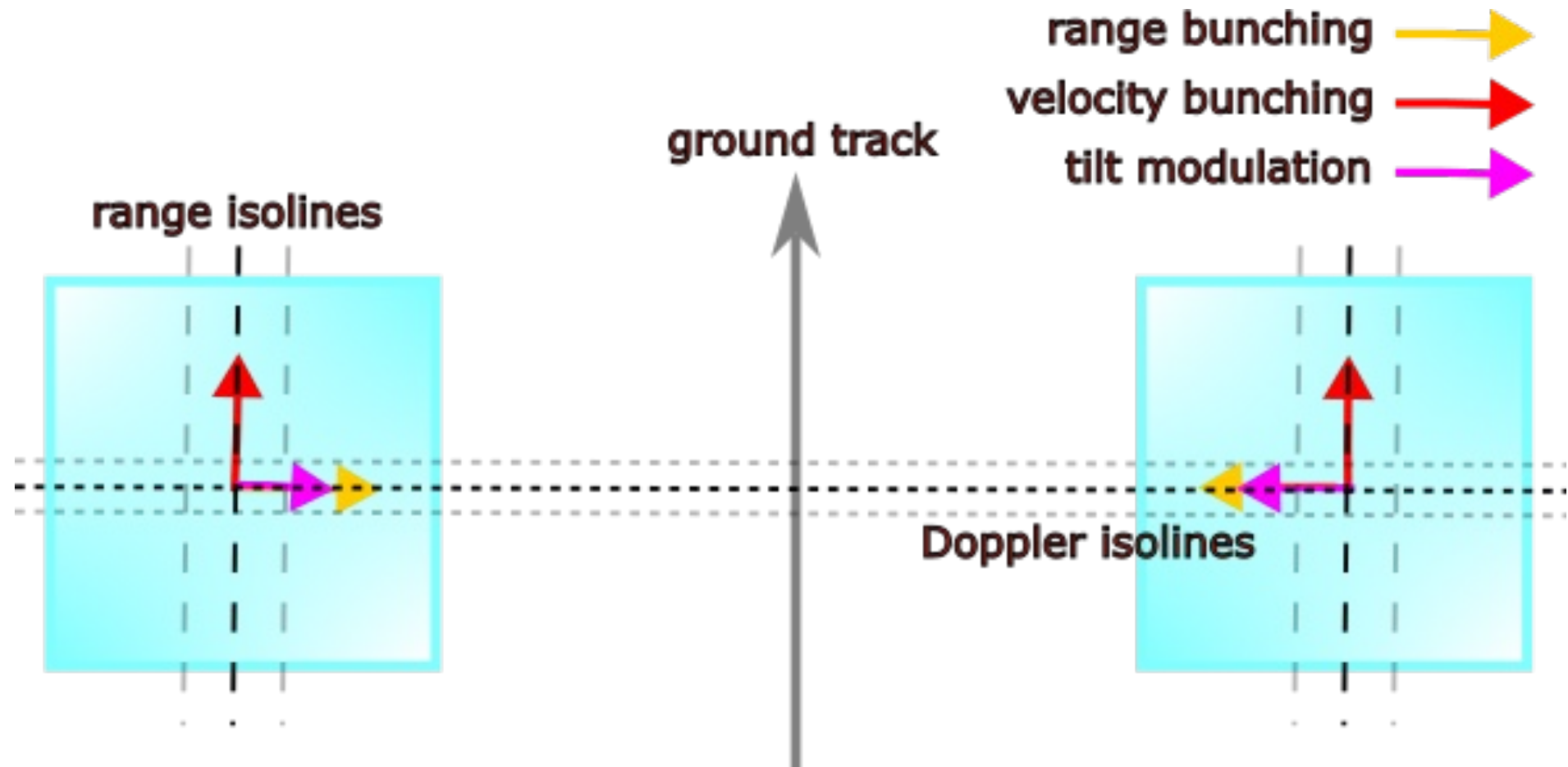


Range bunching



Velocity bunching

# Zero-Doppler geometry



# Closed-form model

Significant wave height      Velocity variance      Range bunching      Velocity bunching      Tilt modulation

$$P(k_x, k_y) = e^{-k_x^2 \rho_{xx}(0,0) - k_y^2 \rho_{yy}(0,0) - k_x k_y (\rho_{xy}(0,0) + \rho_{yx}(0,0))} \iint e^{k_x^2 \rho_{xx} + k_y^2 \rho_{yy} + k_x k_y (\rho_{xy} + \rho_{yx})} (1 + \rho_{II}) e^{-i(k_x x + k_y y)} dx dy.$$

$$\rho_{ab} = \frac{1}{(2\pi)^2} \iint \frac{1}{2} T_a(k_x, k_y) T_b^*(k_x, k_y) S(k_x, k_y) + \frac{1}{2} (T_a(-k_x, -k_y) T_b^*(-k_x, -k_y))^* S(-k_x, -k_y) dk_x dk_y.$$

Transfer functions

$$\lambda_{ct} \propto \pi \sqrt{\rho_{xx}(0,0)} = \pi \sqrt{\frac{\sigma_e^2}{\tan^2(\theta)}} = \pi \frac{\text{SWH}}{4 \tan(\theta)}$$

$$\lambda_c \propto \pi \sqrt{\rho_{yy}(0,0)} = \pi \frac{R_t}{U} \sqrt{\sigma_v^2}$$

Fall-off ellipse!

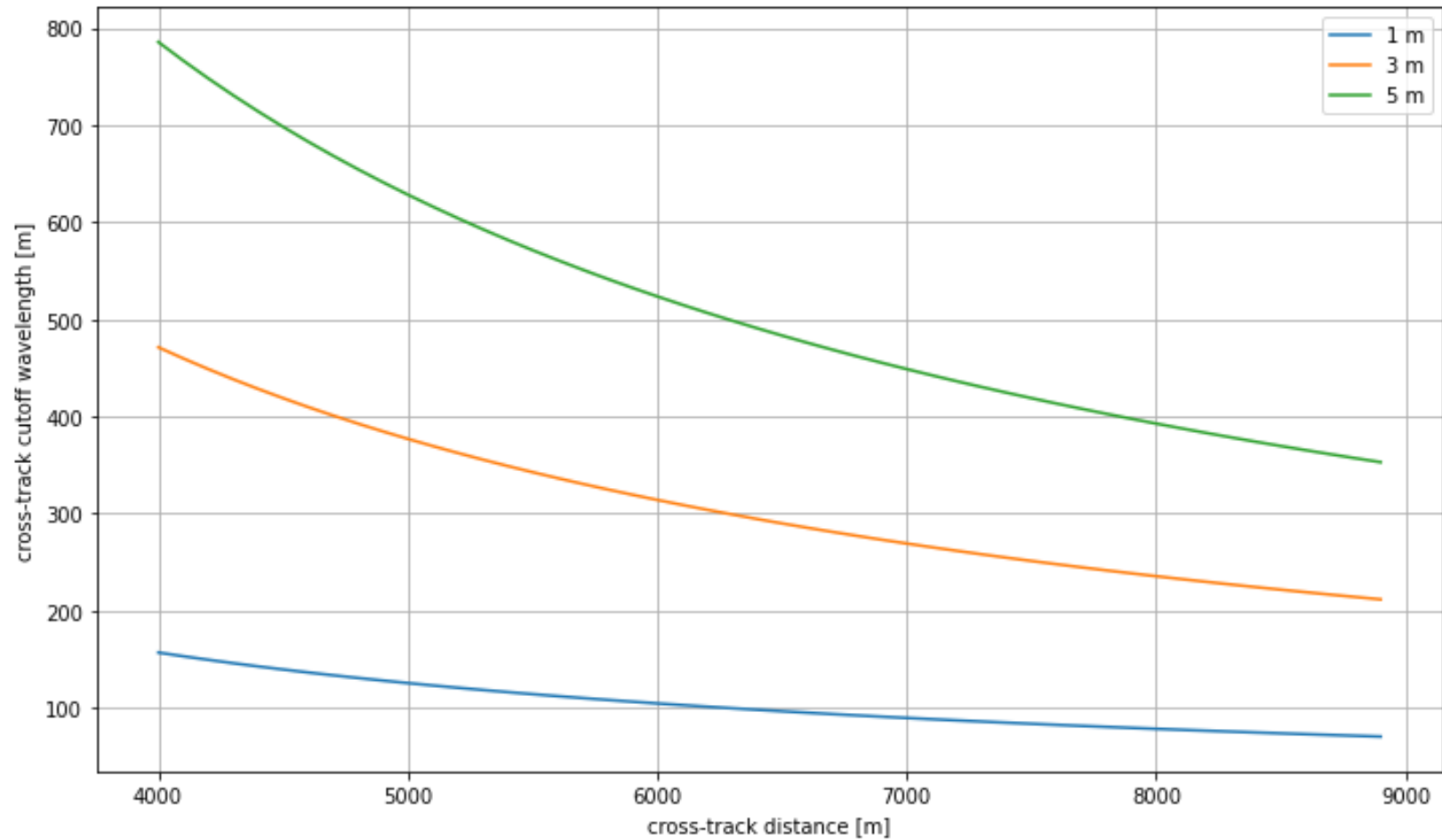
$$T_y = -\frac{R_t}{U}(i\omega).$$

$$T_x = \frac{1}{\tan(\theta)}$$

$$T_I = -ik_x \frac{1}{\sigma_0} \frac{\delta\sigma_0}{\delta\theta}$$

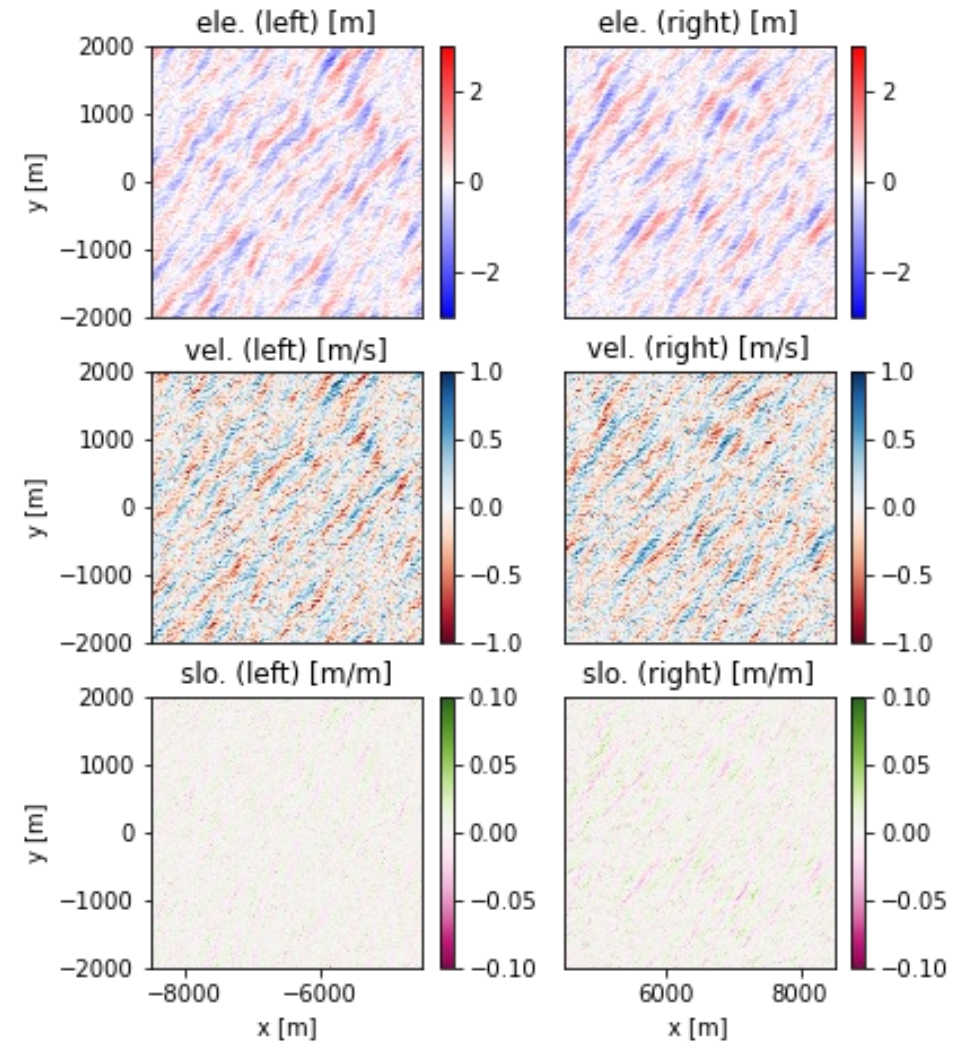
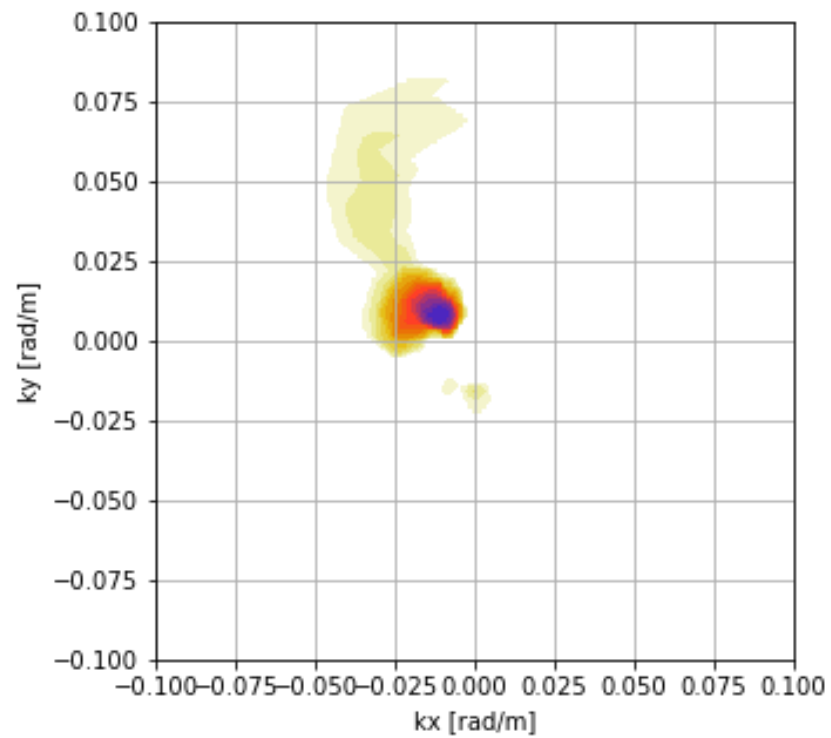
Low incidence angles → highly non-linear range-shift behavior.  
Closed-form not valid for nadir altimeters!!! Only 'pedagogic'.

# Cross-track resolution



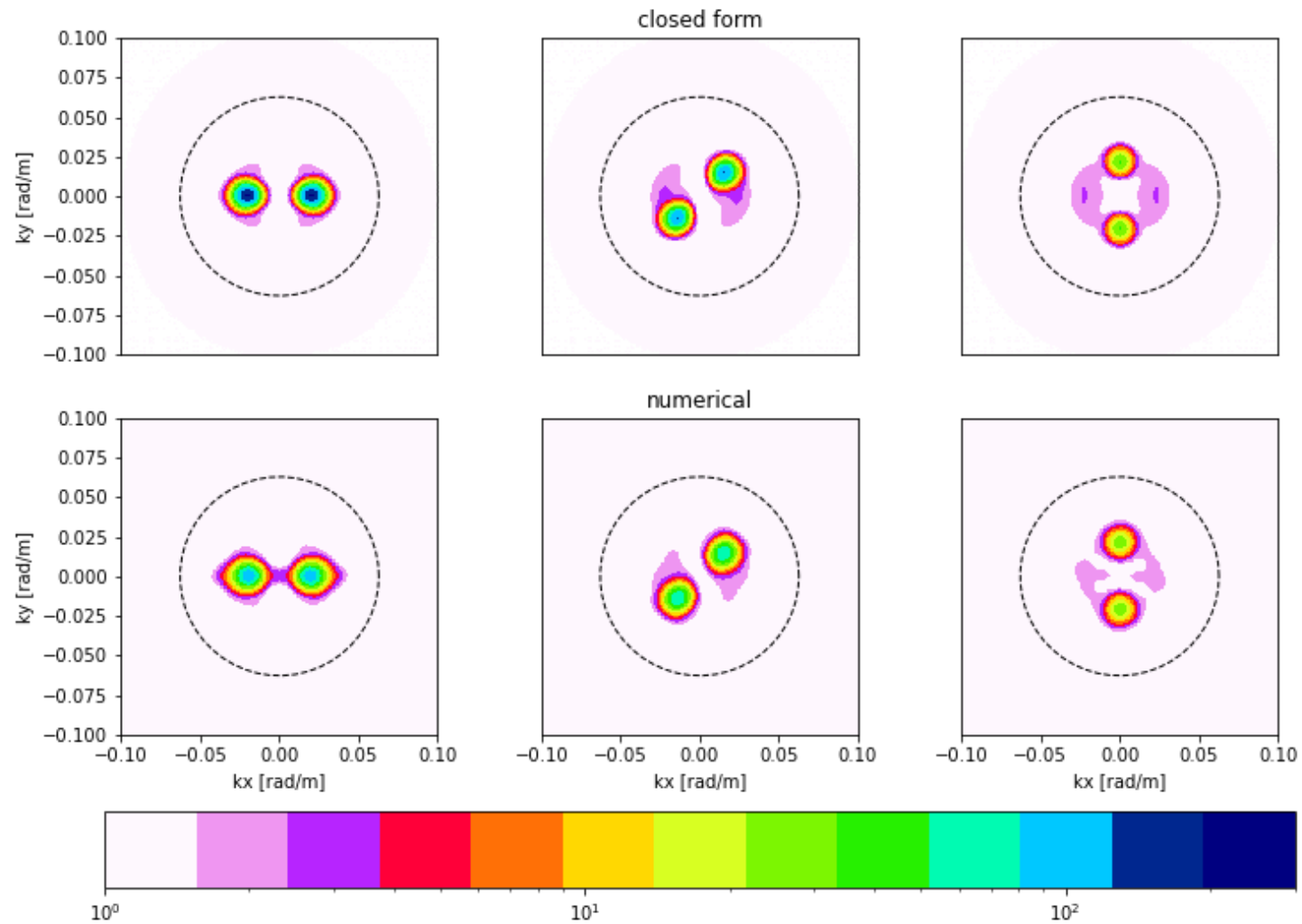


# Numerical model



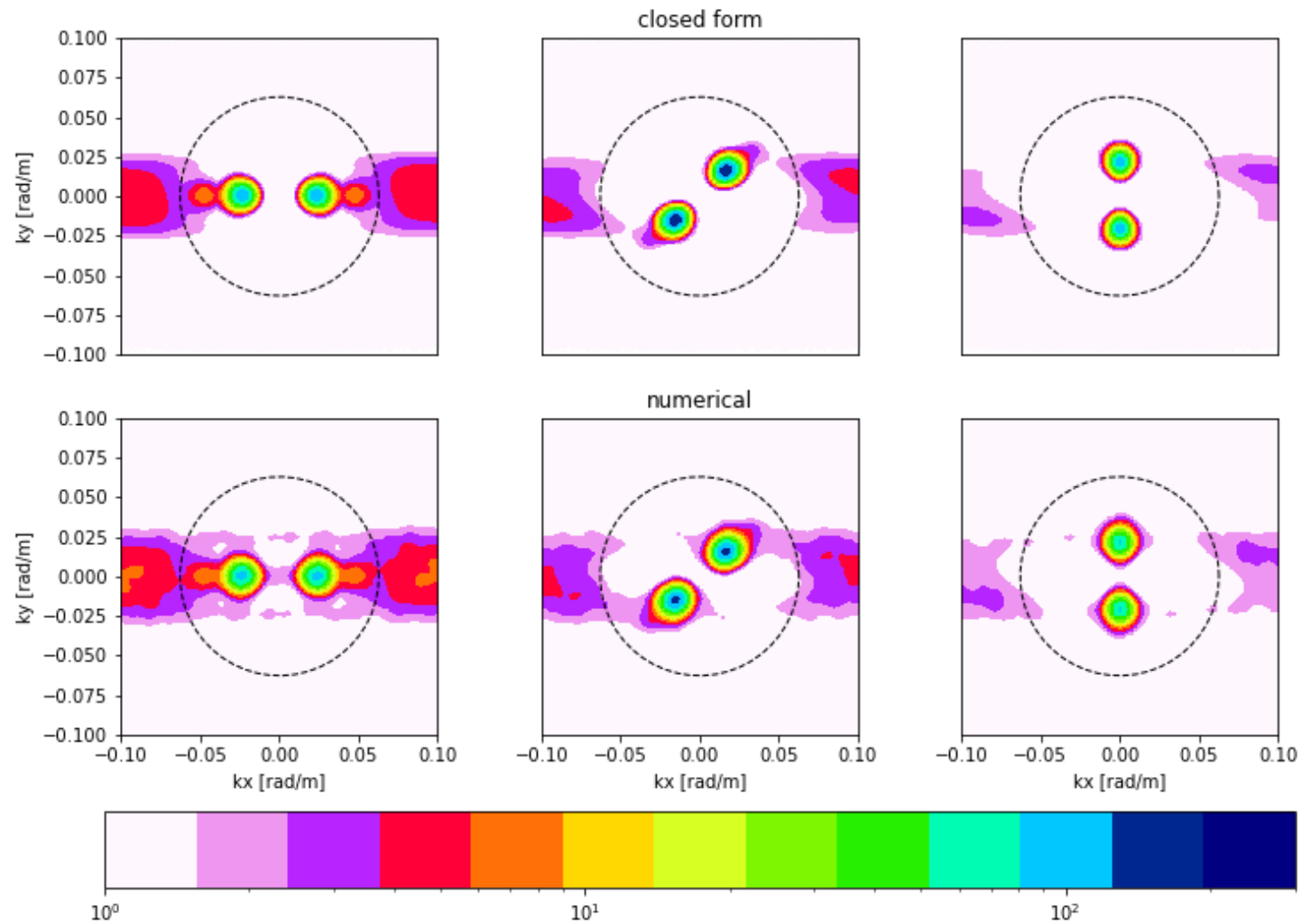


# Numerical vs closed-form: nadir altimeter



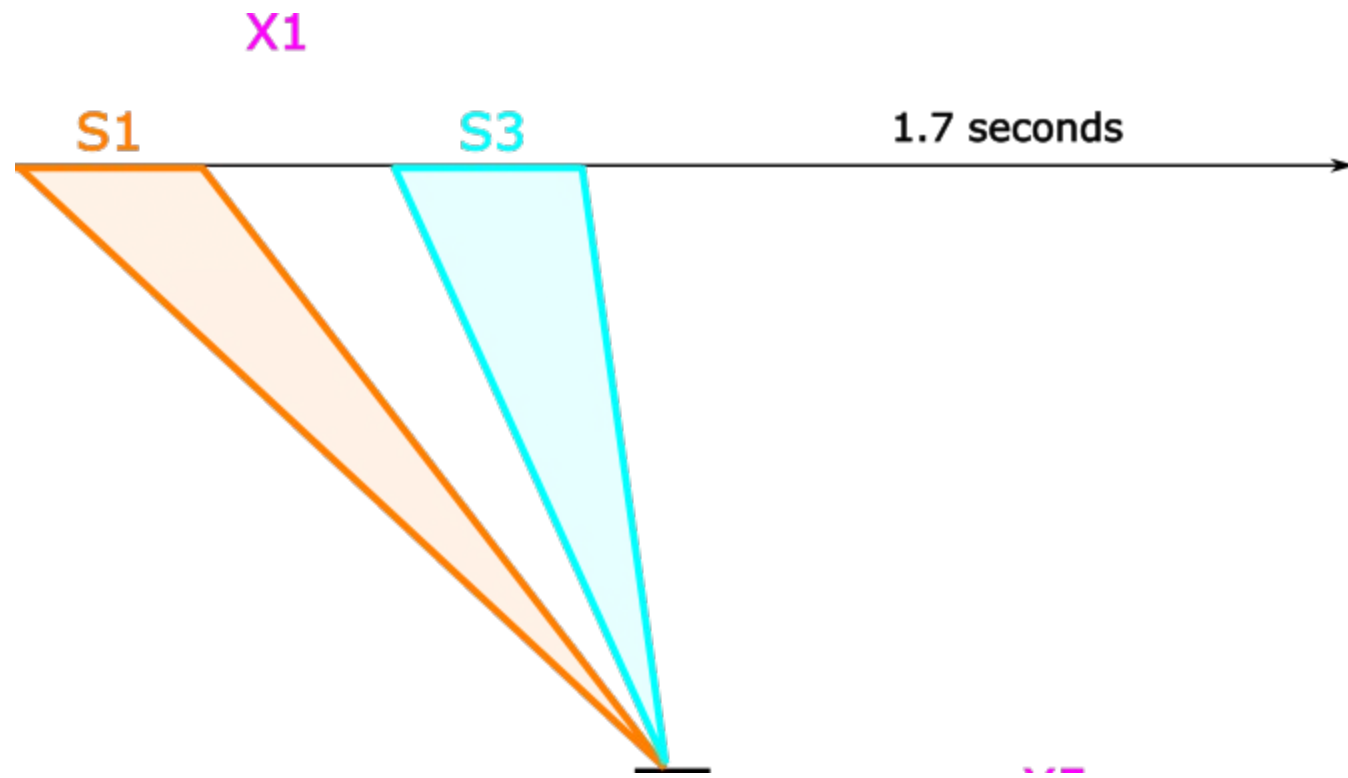
Right side of the ground track only.

# Numerical vs closed-form: swath altimeter

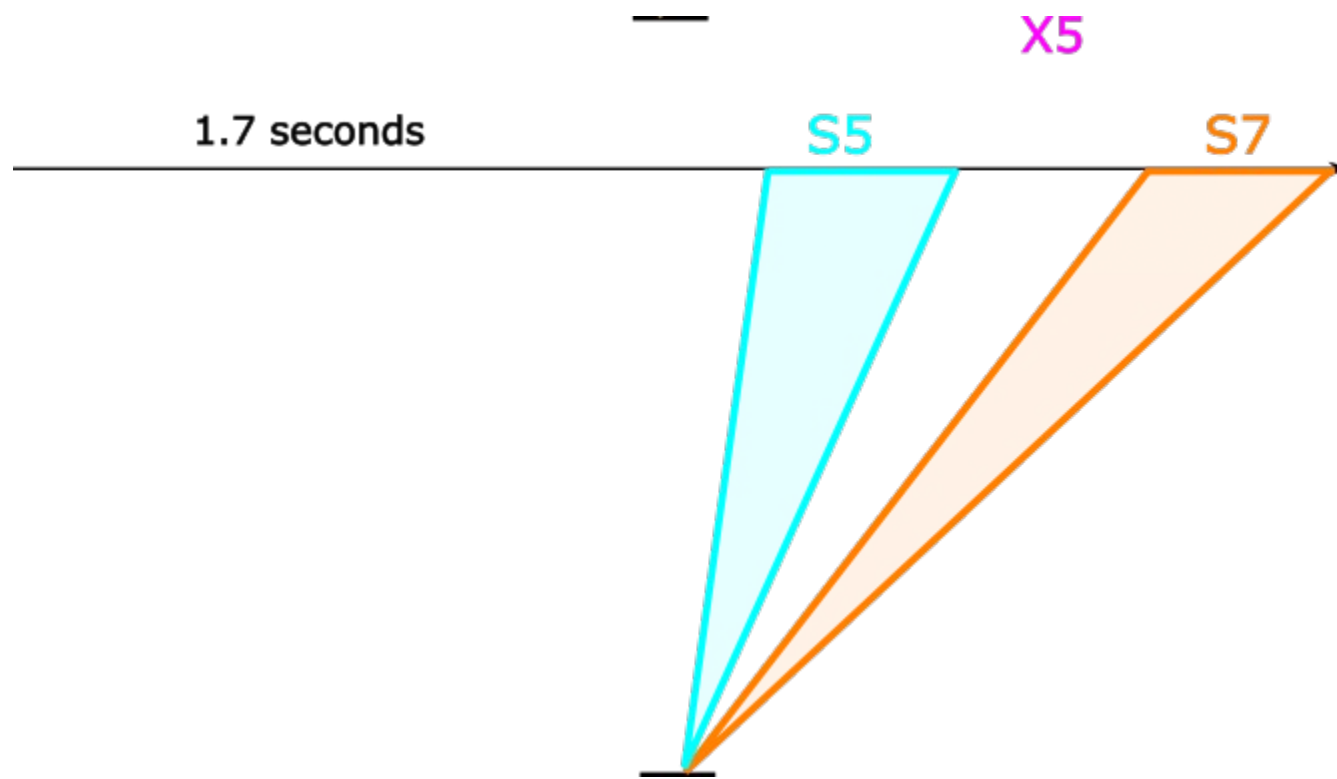




# Sublooking

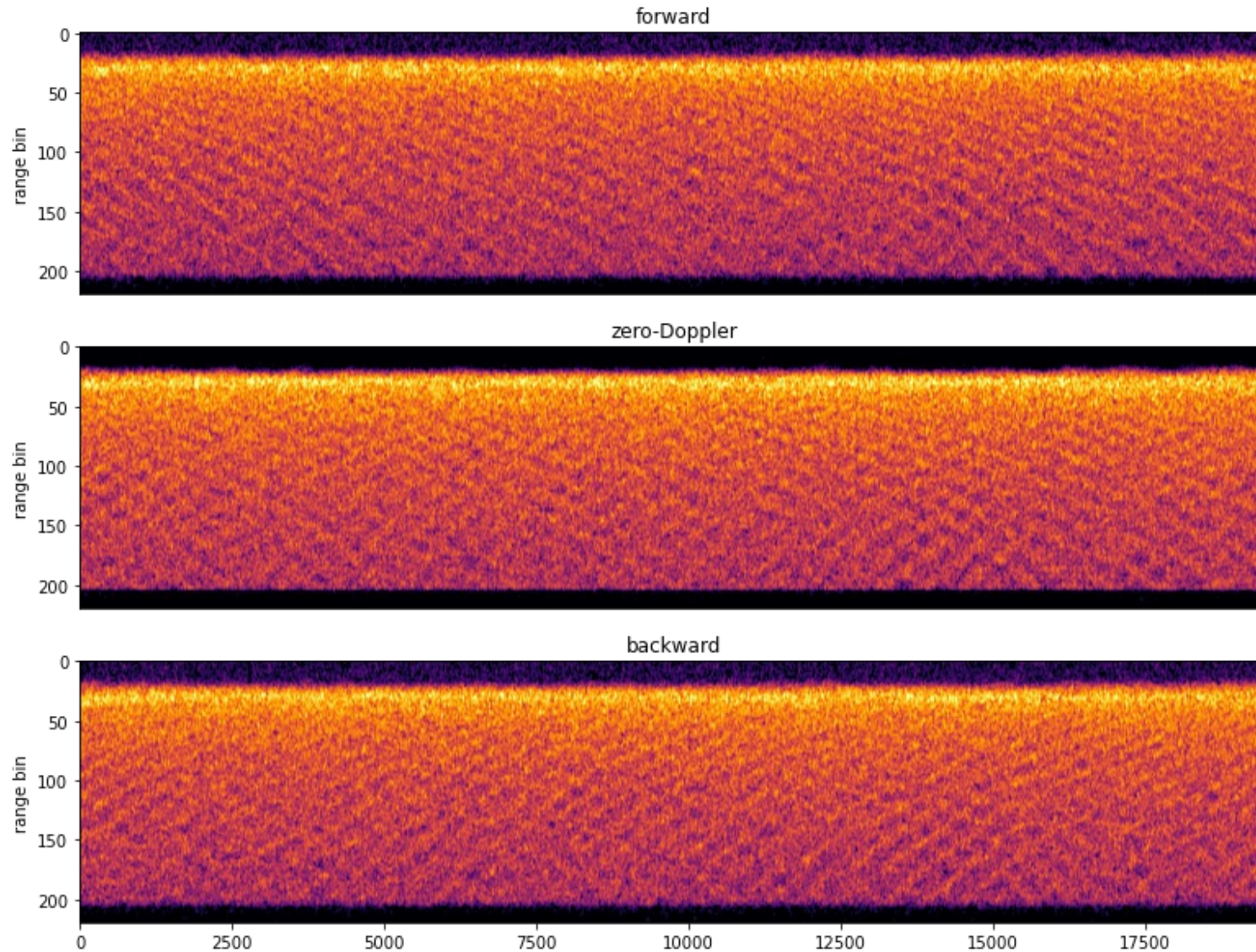


# Sublooking

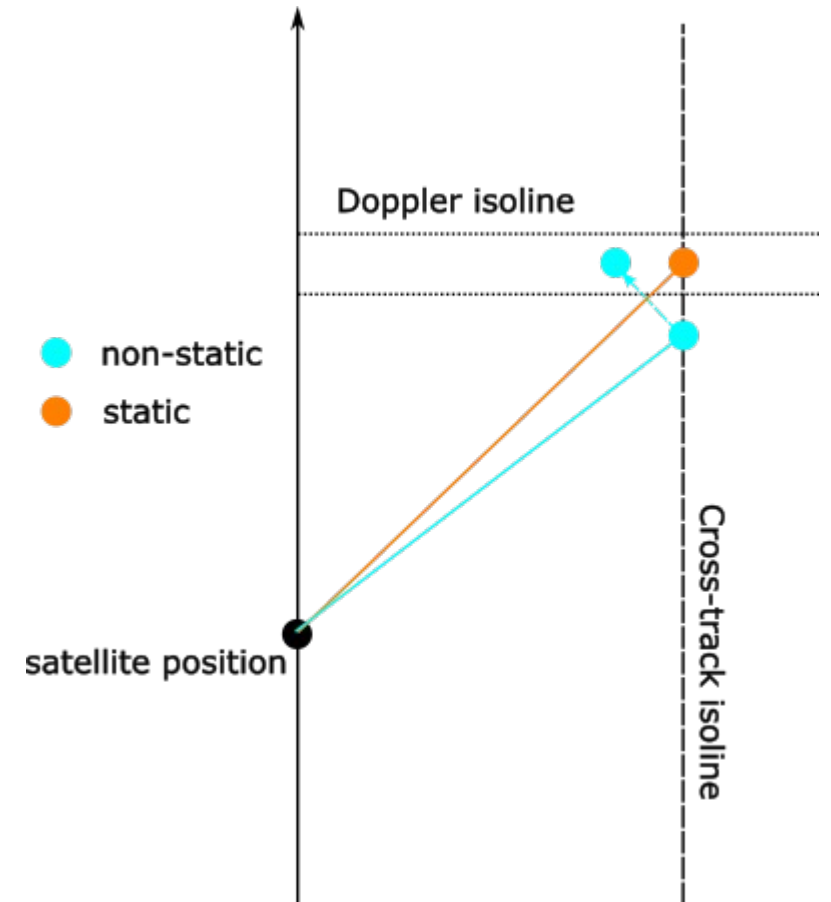
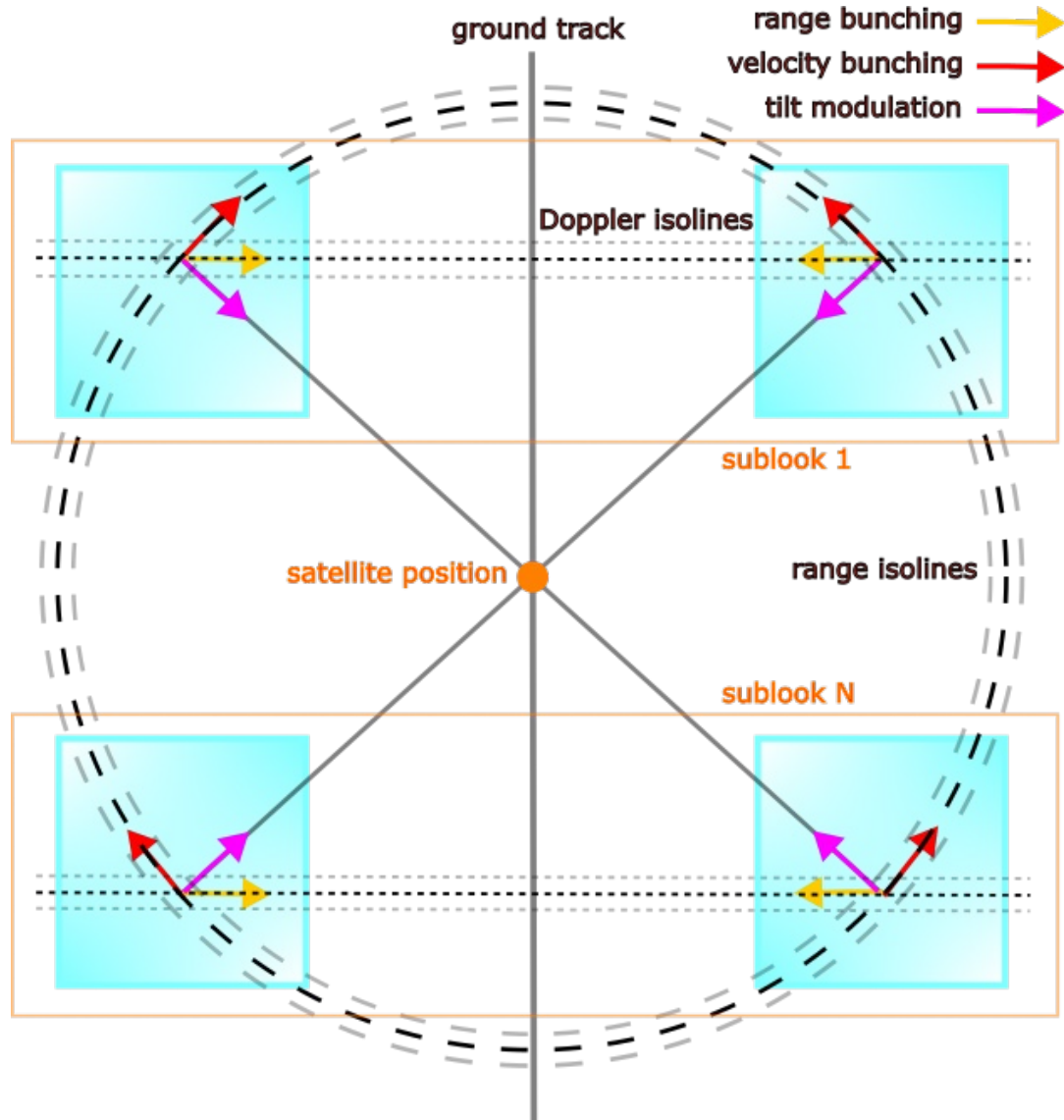




# Cutting the Doppler spectrum

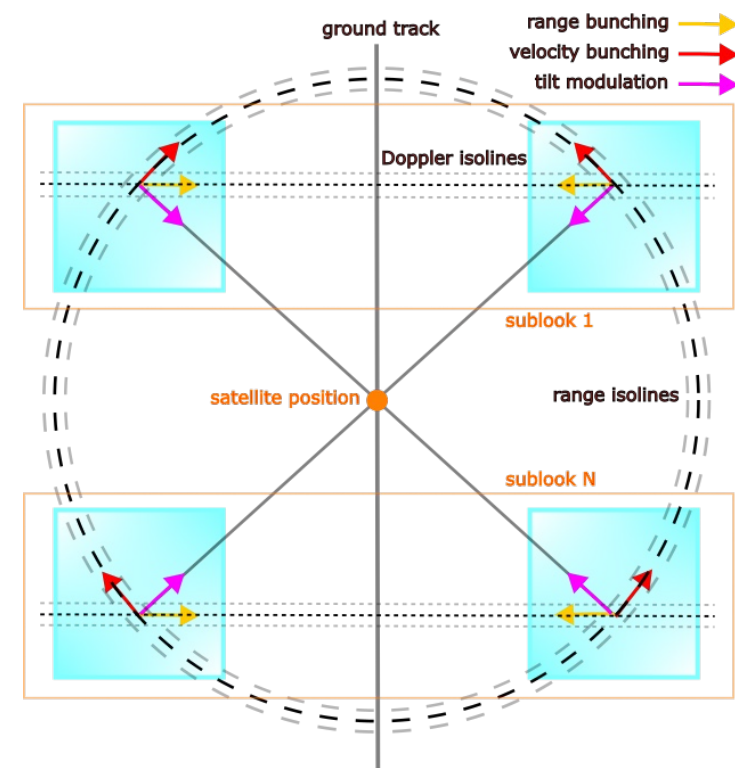
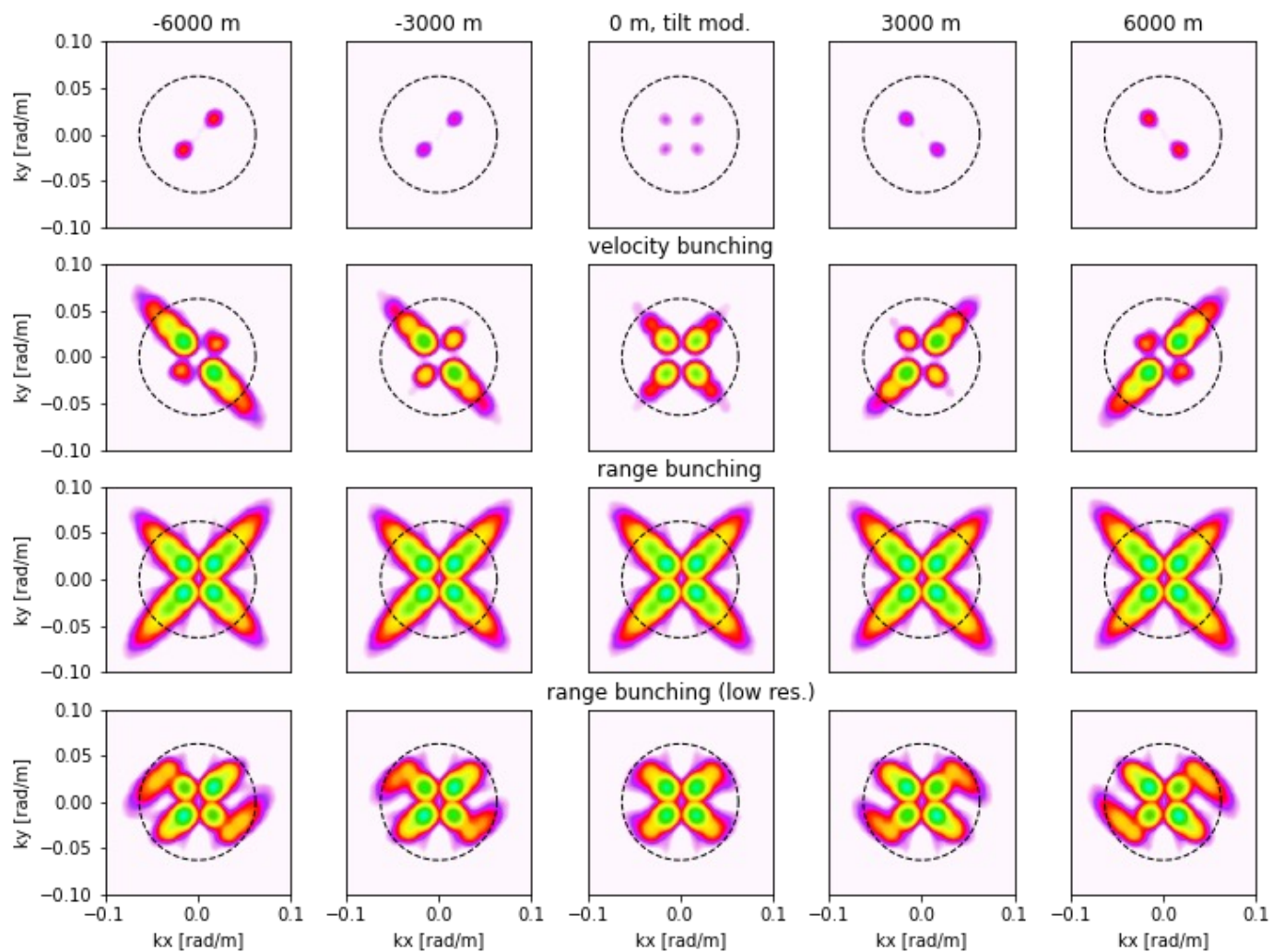


# Non-zero-Doppler geometry

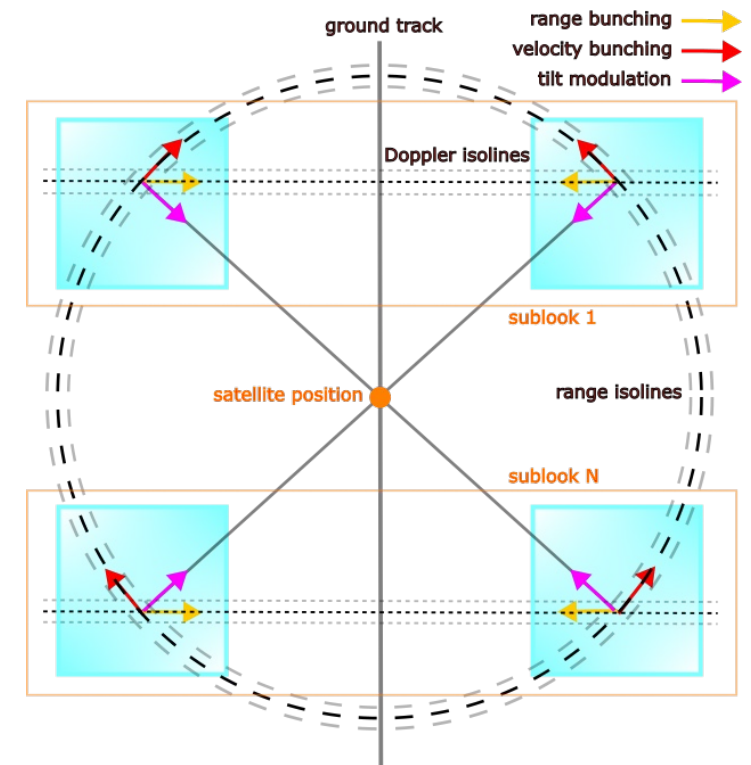
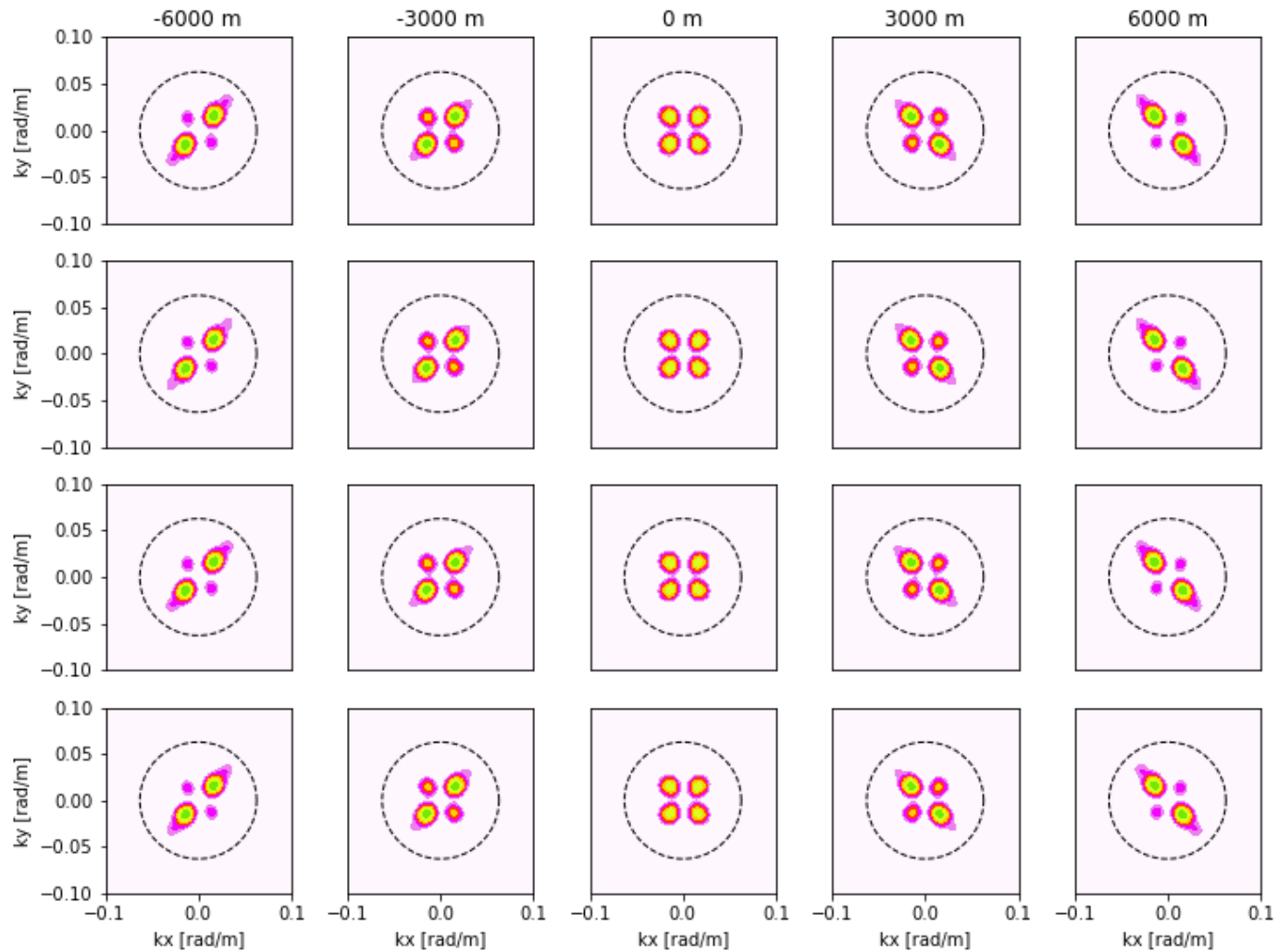




# Individual contributions

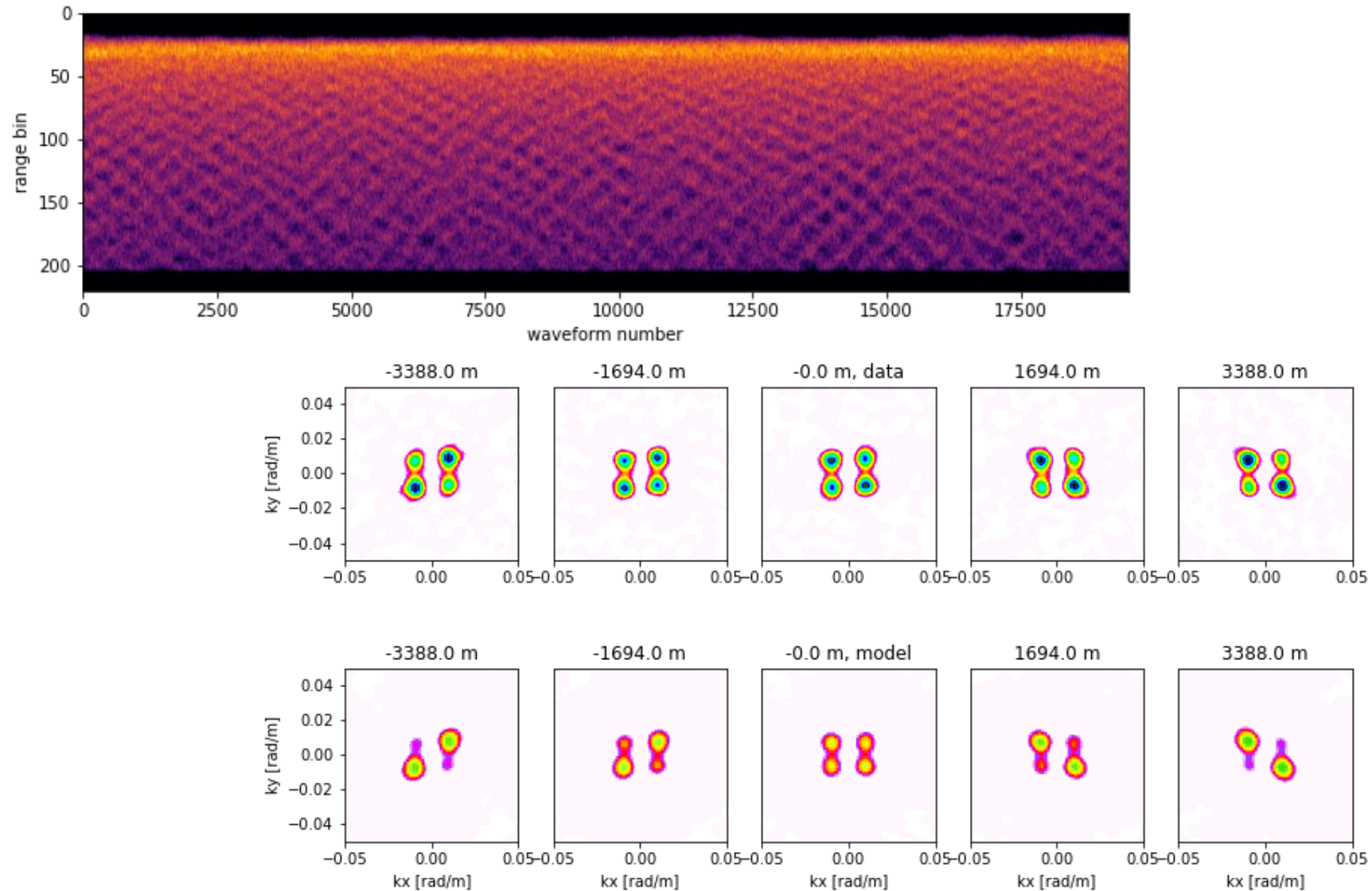


# Wind sea



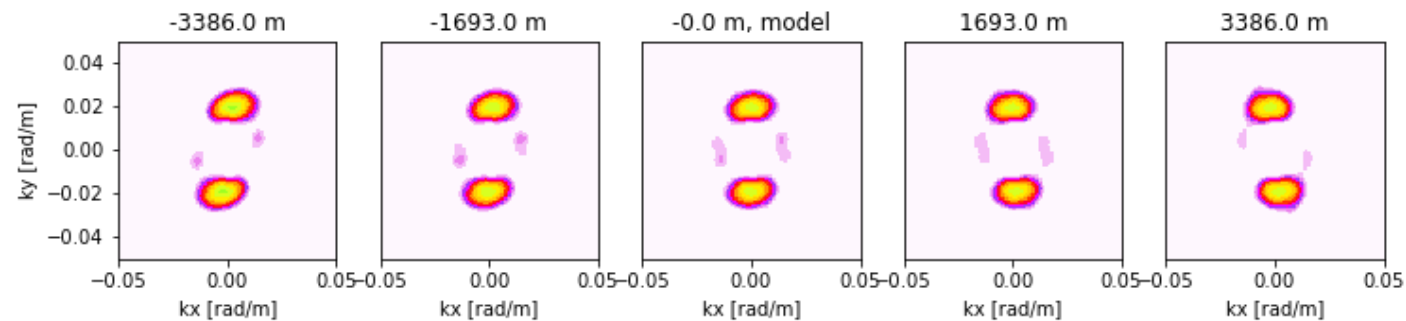
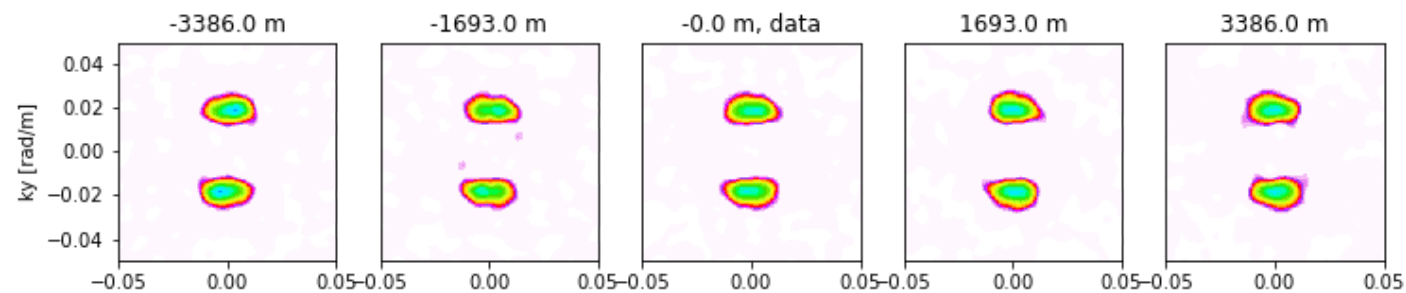
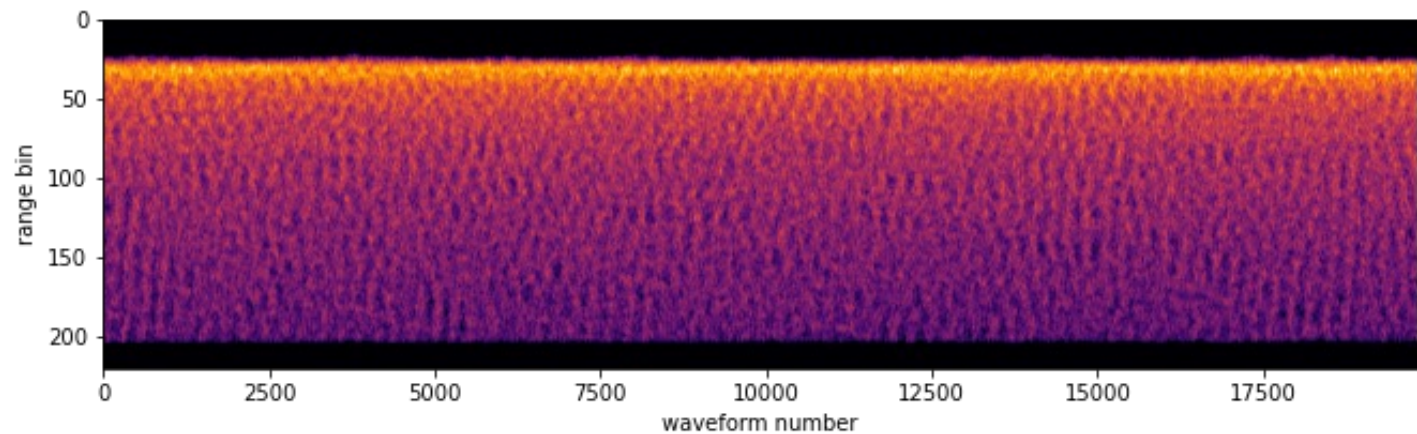


# Real data vs the model

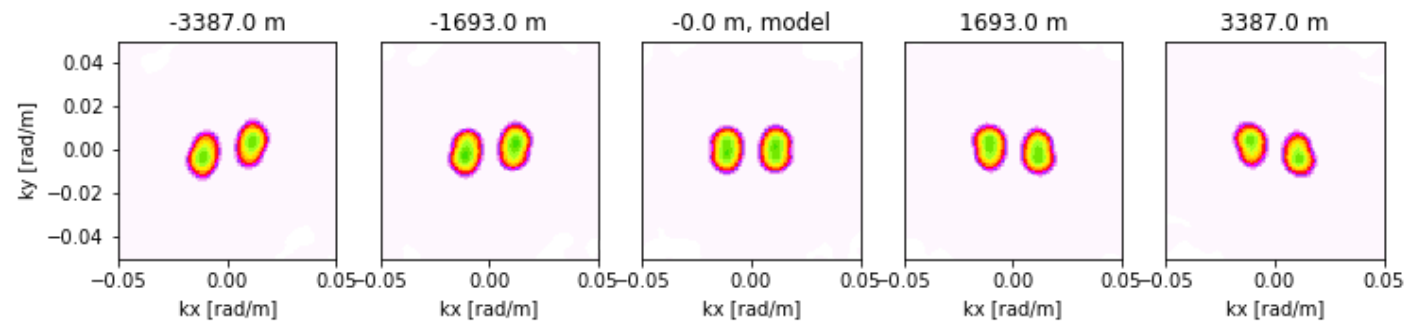
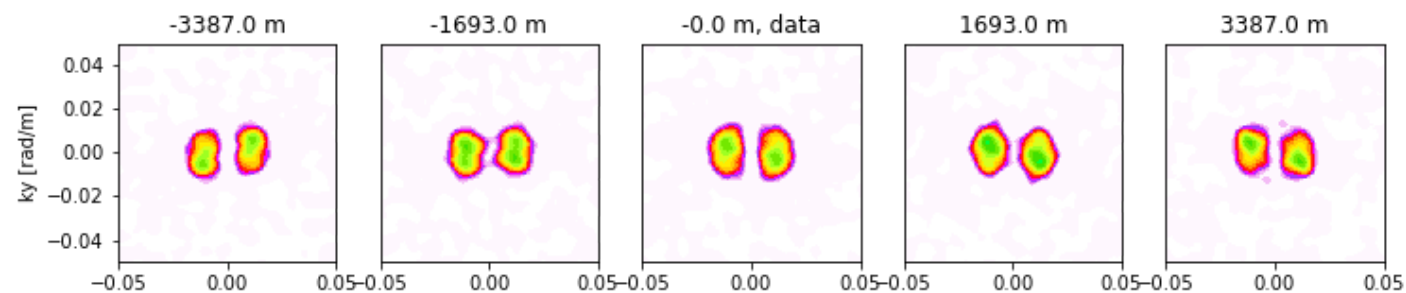
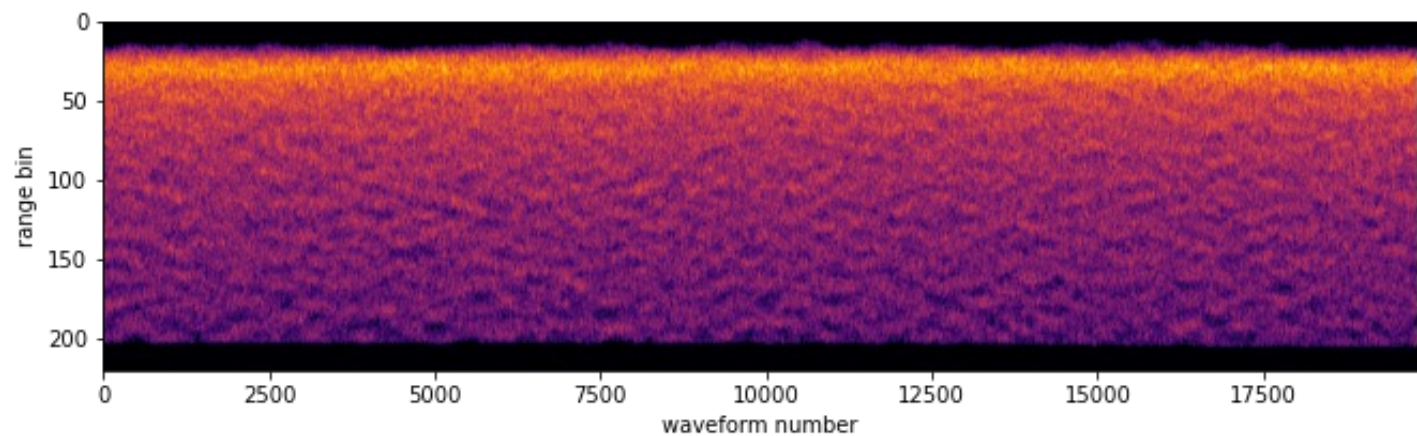




# Along-track swell

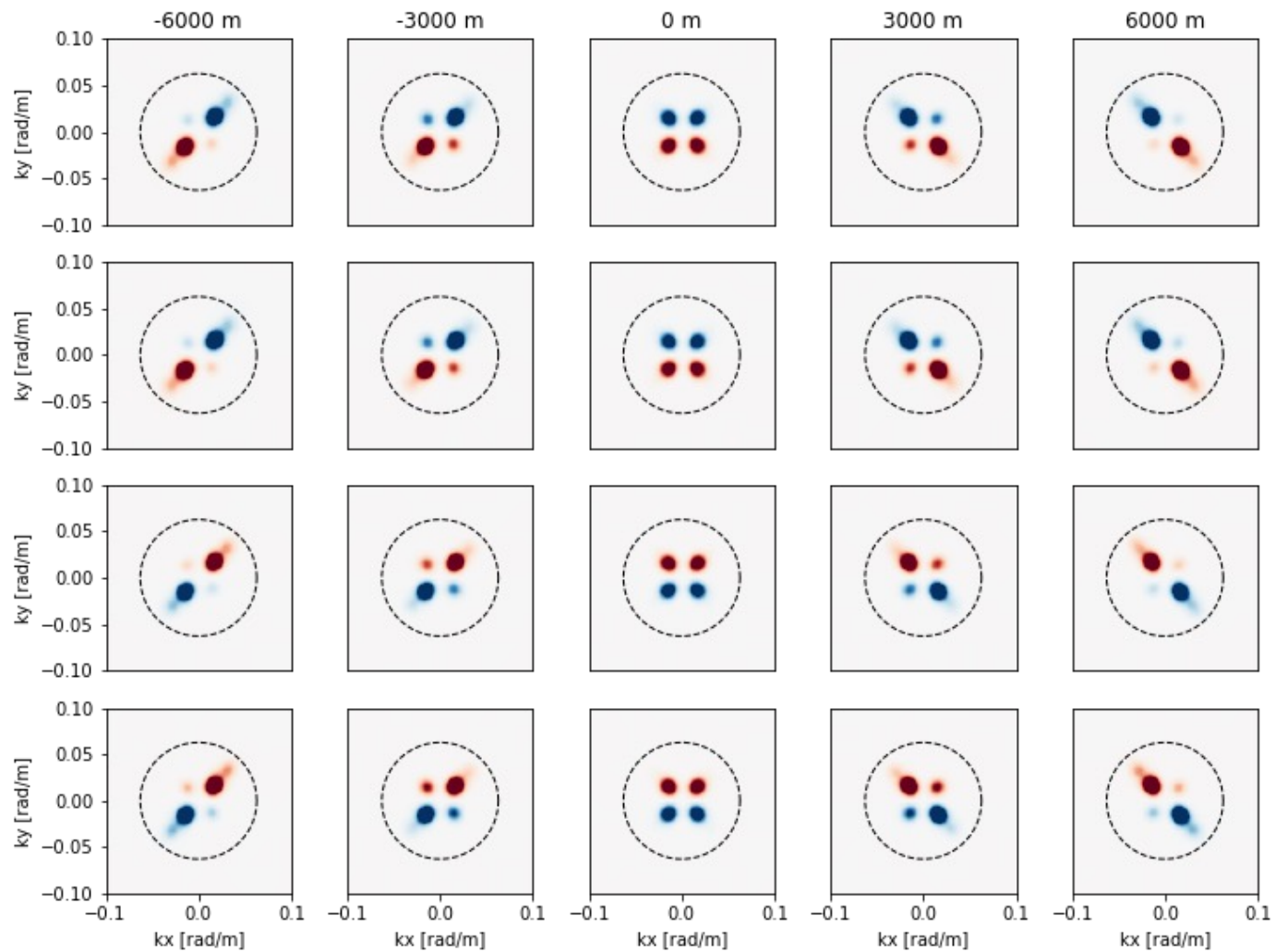


# Across-track swell



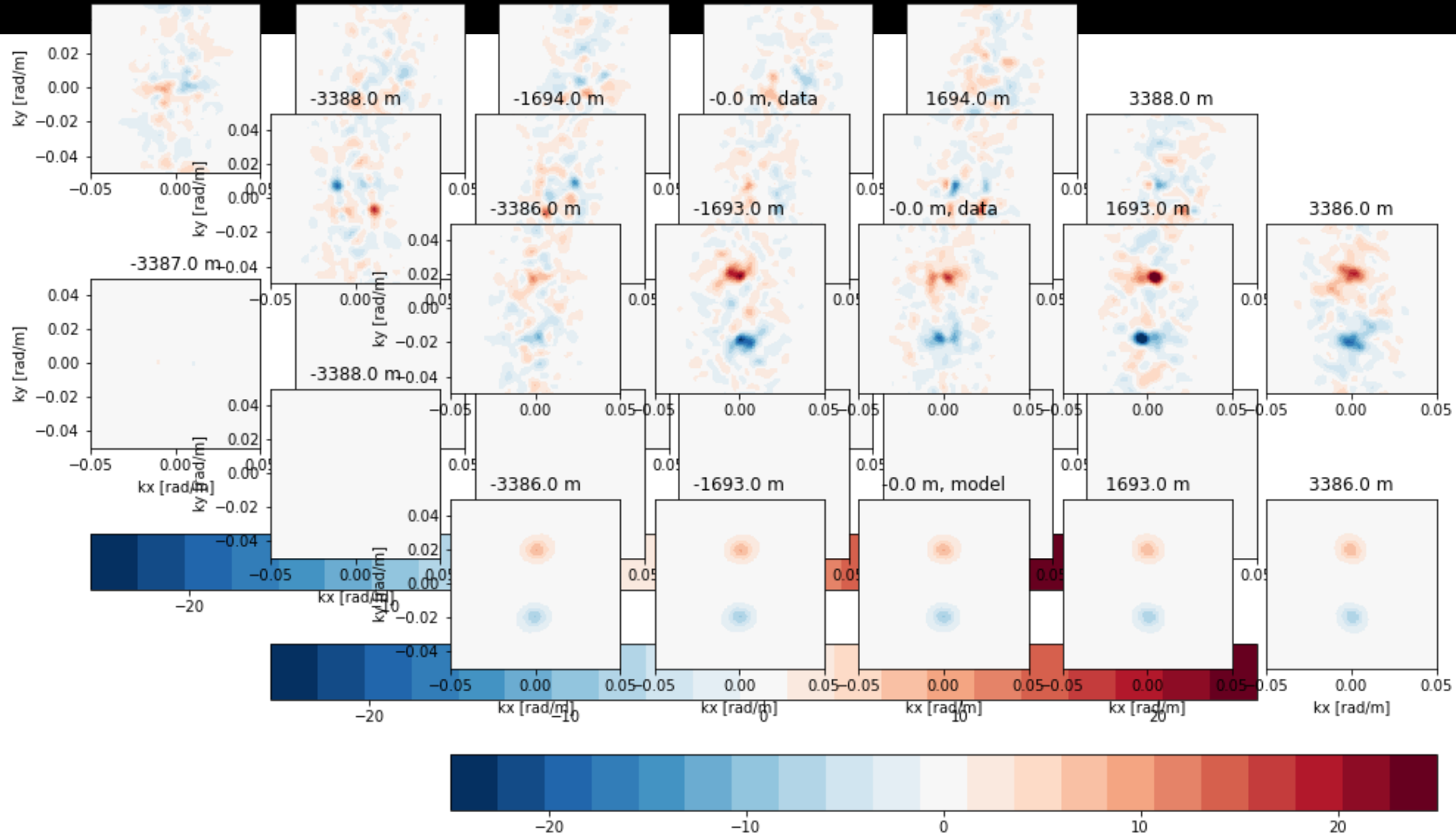


# Imaginary values





# Imaginary values

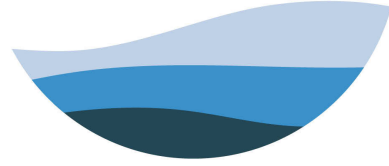


# Summary

- A numerical model is able to reproduce SAR altimetry spectra.
- A closed-form solution is invalid at small incident angles.
- A non-zero-Doppler geometry is likely the cause for Altiparmaki's left-right differences.
- Sublooking might help to retrieve more accurate swell data.
- Swell mean wavelength and energy can be retrieved within the cut-off.
- Direction? Imaginary values are noisy and poorly understood.



**sarwave**



**Thanks!**

[sarwave.org](http://sarwave.org)