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LAKE GEOID AND GRAVITY FROM ALTIMETRY

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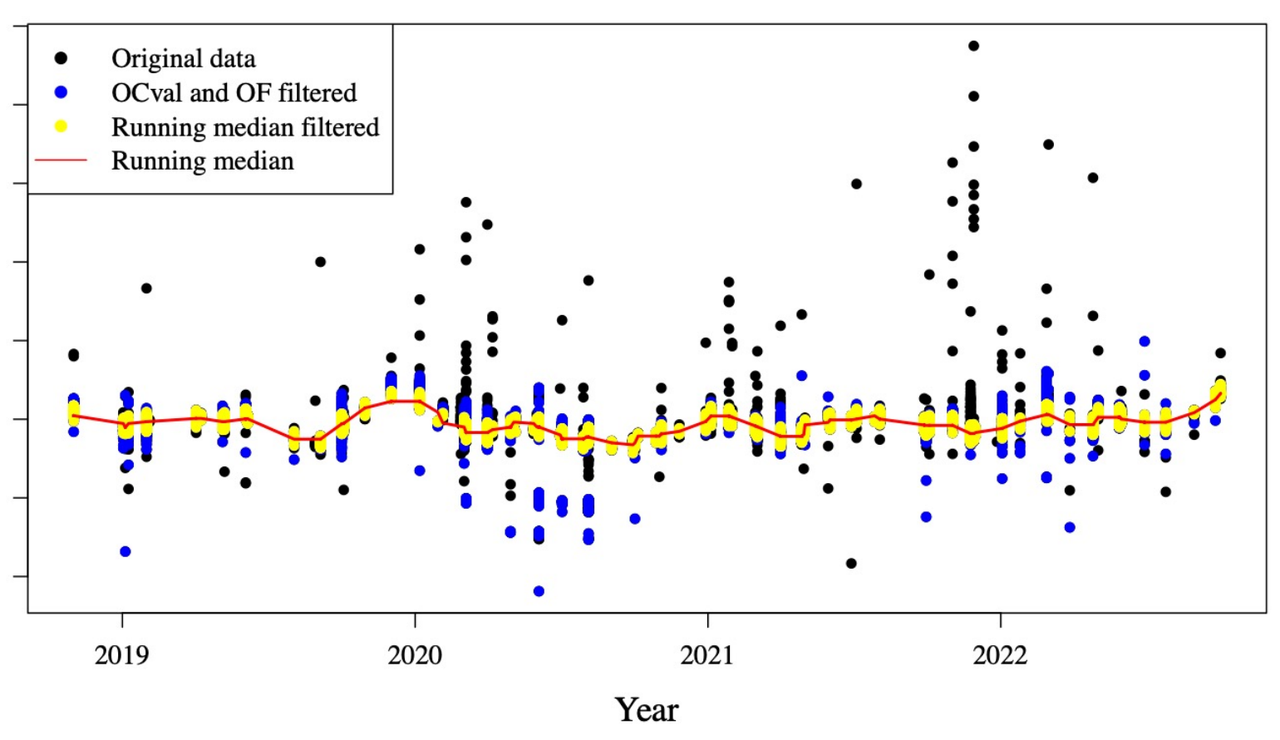
Abstract

The data from NASA's Ice, Cloud, and Land Elevation Satellite-2 (ICESat-2) mission offer a unique opportunity to map rivers and lakes with an unprecedented number of observations and precision in areas where previous radar missions have failed to provide valuable water level estimates. ICESat-2 provides an along-track resolution of the ATL03 product better than 1 meter typically illuminating a circular region with a diameter of 17 meters. With its three pairs of beams located at nadir and 3.3 km to each side, the mission provides exceptional opportunities for inland water studies in areas with mountainous topography. In this study, we evaluate the first attempt to extract gravity anomalies from altimetry over several medium (100-1000 km^2) and large (>1000 km^2) lakes and compare them with conventional radar altimetry to investigate the performance of ICESat-2 for gravity determination. Aerial gravimetry from the GRAV-D project over the United States are utilized as the best estimate of the gravity field over the lakes. We use radar altimetry data from the CryoSat-2 satellite as it has a similar inclination to ICESat-2 giving coverage to within 2 degrees of the poles. We also use radar altimetry measurements from the SARAL satellite for additional comparison. We evaluate the quality of ICESat-2, CryoSat-2, and SARAL for gravity determination by computing gravity from each dataset and comparing it with data from the GRAV-D project over lakes. Gravity determination from altimetry is done using Fast Fourier Techniques (FFT) within a remove-restore geoid-to-gravity approach. The resulting altimetry-derived gravity anomalies are then compared to the EGM2008 geoid over each lake to GRAV-D. 18 lakes with area ranging from 108 km^2 to 82,220 km^2 across the United States were considered. Overall, gravity determination from ICESat-2 provides more reliable estimates than the other two radar altimetry missions. For all considered lakes, the performance of ICESat-2, measured by the standard deviation of the difference between ICESat-2 and GRAV-D, is comparable or better than the EGM2008 estimates over the same lake. Lake Tustumena is the best performing case, in which the standard deviation of the ICESat-2 derived gravity anomaly field is 1.598 standard deviations lower than that of EGM2008, with respect to GRAV-D. Over lake Tahoe, which is surrounded by mountainous terrain, ICESat-2 performs comparably to EGM2008 and captures the clear signal of the gravity field as expected by the lake's bathymetry, whereas CryoSat-2 produces very unstable results. In few cases, CryoSat-2 or SARAL seem to outperform both ICESat-2 and GRAVD-D. While this is seen for lakes Ontario, Huron, and Salton, it should not be taken to be entirely true. This is because for these lakes, CryoSat-2 and SARAL often have groundtracks covering only a part of the lake, resulting in seemingly lower standard deviations. Additionally, it is important to consider that for many of the medium lakes, GRAV-D coverage is sparse, thus it is hard to assert that EGM2008 truly performs better or worse than ICESat-2. Despite this, the method presented here for deriving gravity anomalies from altimetry applied to ICESat-2 laser altimetry data produces results comparable in trend and magnitude to the GRAV-D project.

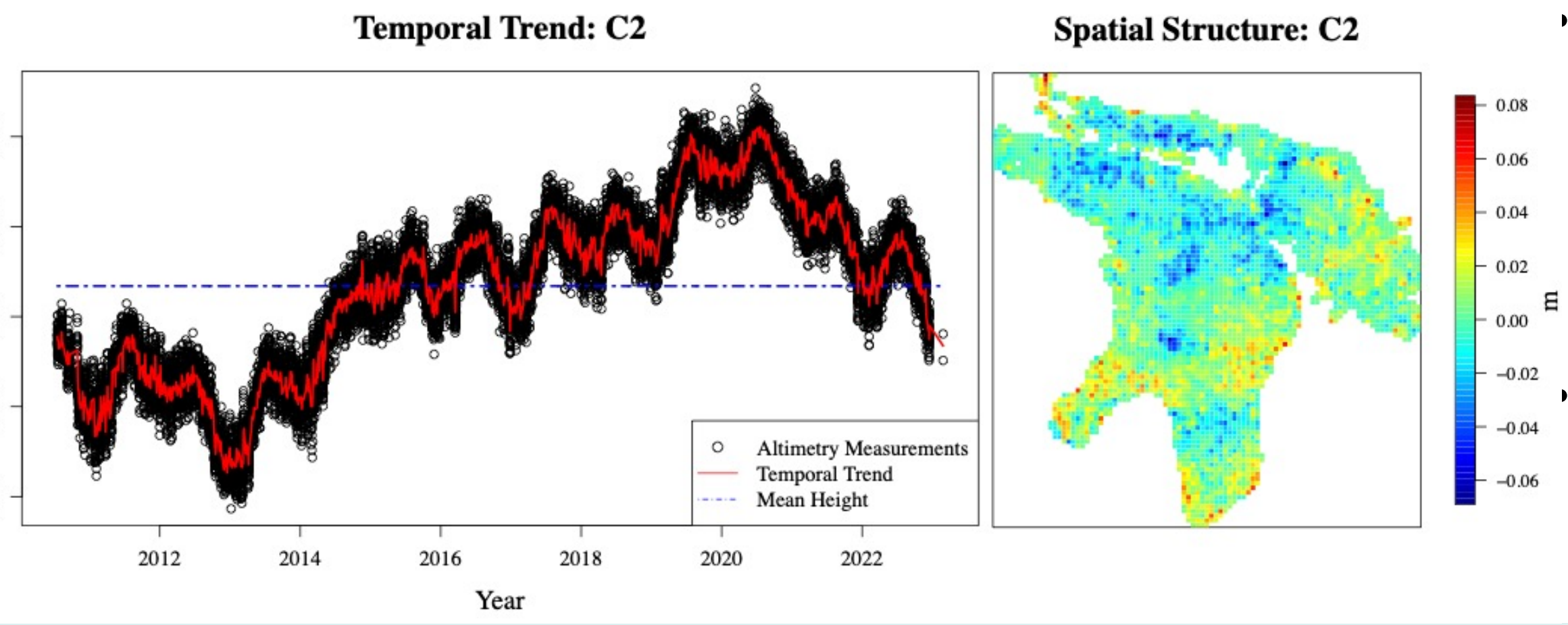


Obtaining the Static Height Field

Outlier Removal: running median filter, data 3.5 median deviations away are removed.

Temporal signal removed and static gravity field extracted via state space model, H as a function of space (s) and time (t) :

$$H_{t,s} = \omega_s + \mu_t + \epsilon_{t,s}, \quad \text{where} \quad \epsilon_{t,s} \sim N(0, \sigma^2_{obs}).$$



- Temporal signal modeled as a *Random Walk* (RW).

$$\mu_t = \mu_{t-1} + \eta_t,$$
$$\text{where} \quad \eta_t \sim N(0, (t-(t-1))\sigma^2_{RW}).$$

- Spatial signal modeled as *Gaussian Markov Random Field* (GMRF).

$$\omega_s \sim N(0, \sigma^2_{\omega} Q^{-1})$$

Static Lake Height to Gravity Approach

Remove-restore geoid-to-gravity approach utilizing Fast Fourier Techniques (FFT) (Andersen & Knudsen (1998)).

- EGM2008 (Pavlis et. Al. (2012)) is first removed from the lake height data.
- The gravity anomalies, Δg, are computed from geoid undulations N and the normal gravity γ.

$$\Delta\hat{g}(u,v) \approx \omega\gamma\tilde{N}(u,v)F(\omega)$$

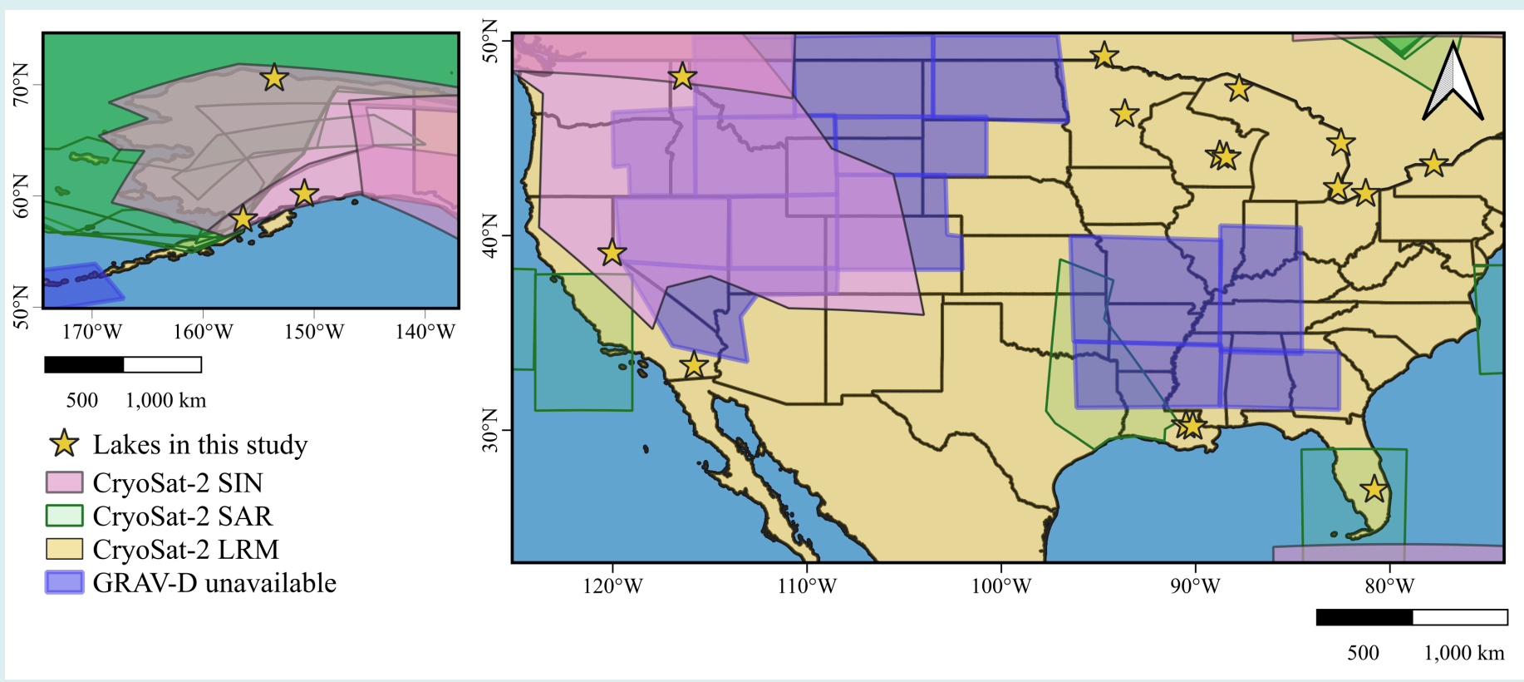
- Weiner filtering $F(\omega)$ for dampening higher order frequencies. A cutoff frequency of 6 km has been determined empirically.

$$F(\omega) = \frac{\omega_c^4}{\omega^4 + \omega_c^4}$$

Results

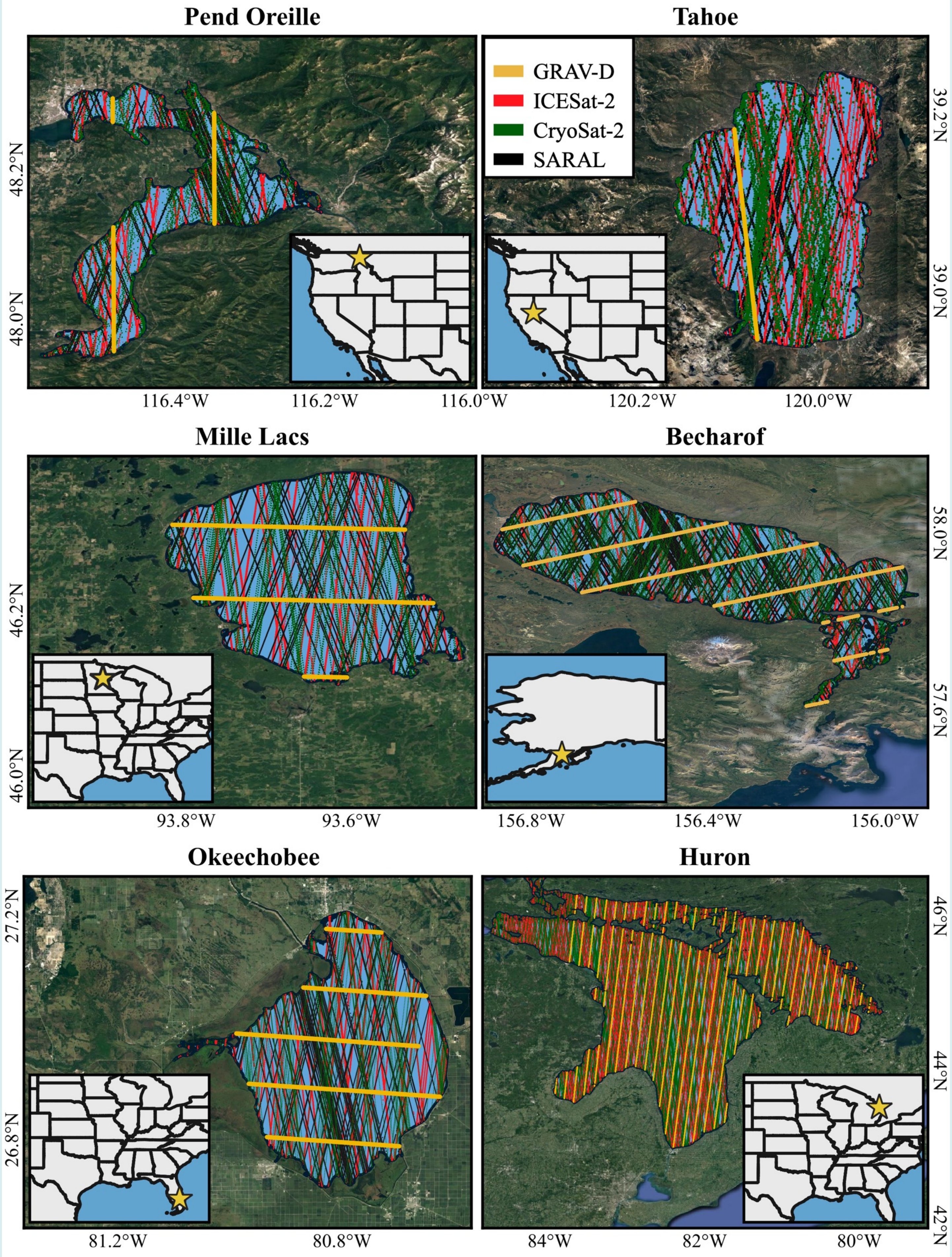
Lake	Area [km^2]	Max Depth [m]	$\sigma_{EGM08-G}$	σ_{I2-G}	σ_{C2-G}	σ_{SA-G}
Butte Des Morts	108	3	0.604	1.128	41.930	4.3110
Maurepas	249	3	0.697	0.574	46.547	8.0670
Tustumena	296	290	5.640	4.042	25.002	15.7520
Pend Orielle	328	351	6.573	13.591	27.262	15.1170
Tahoe	495	501	10.226	15.301	5160.913	12.5660
Mille Lacs	516	13	4.117	3.074	7.650	3.9500
Winnebago	536	6	1.676	10.057	21.322	11.1580
Teshekpuk	857	10	2.494	2.477	2.500	7.7040
Salton	895	16	11.360	11.440	70.355	15.2590
St. Clair	1121	8	4.093	5.128	62.005	6.6810
Becharof	1171	183	5.912	5.888	5.946	6.8037
Okeechobee	1287	4	6.574	6.460	52.658	10.5410
Pontchartrain	1558	20	1.805	1.810	41.189	4.8140
Shoal Lake	3529	64	4.816	4.725	30.662	6.4994
Ontario	19595	244	4.207	4.272	4.305	4.2873
Erie	25711	64	4.070	4.067	3.982	4.0258
Huron	59280	229	3.593	3.623	3.639	3.6597
Superior	82220	406	8.072	9.323	9.291	9.4092

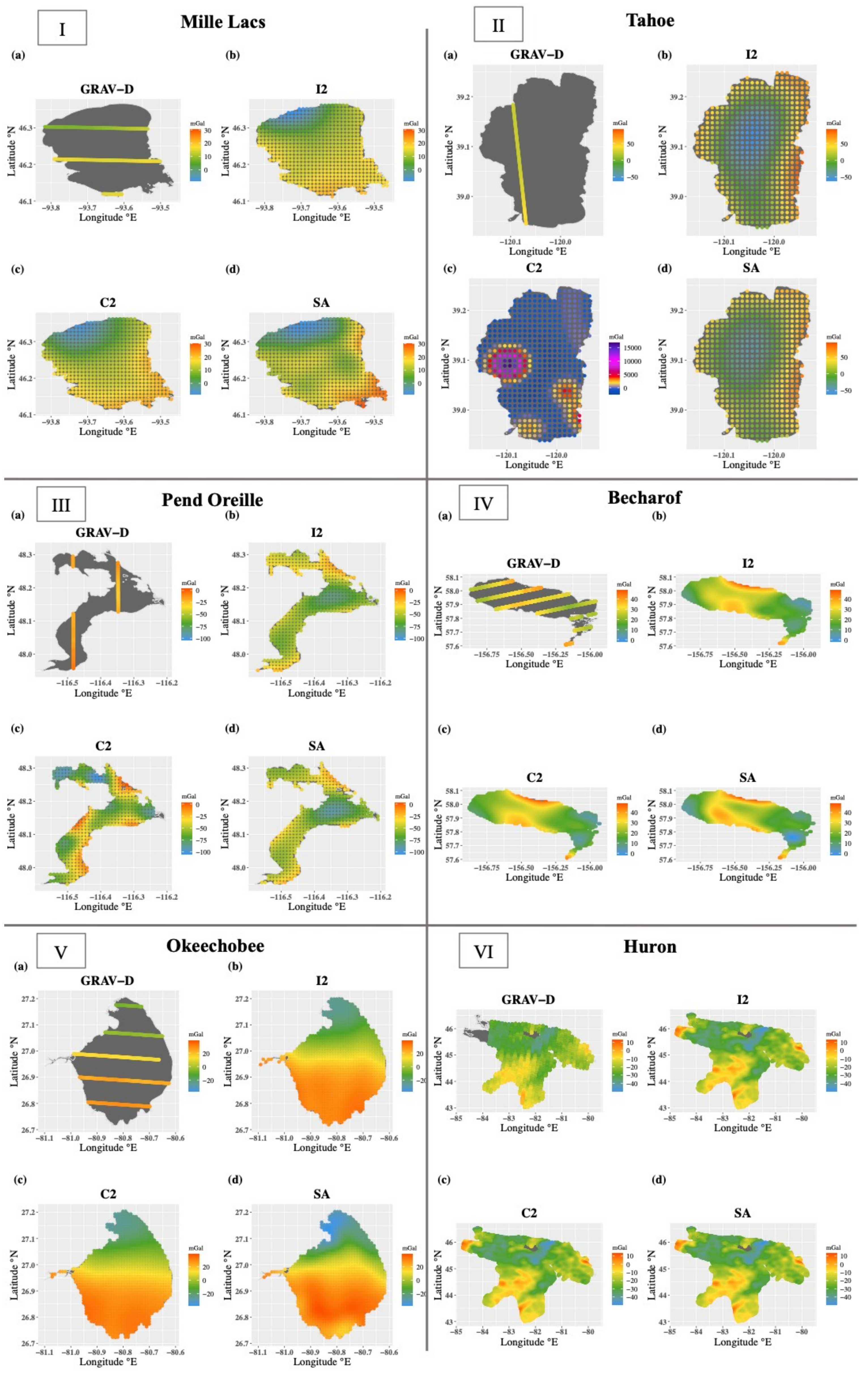
Ground tracks



On the right: Study lake locations, CryoSat-2 Modes and GRAV-D availability.

Below: Ground tracks for 6 of the 18 lakes considered in the study.





References
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