Physically-consistent mapped altimetry on custom grids

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Summary

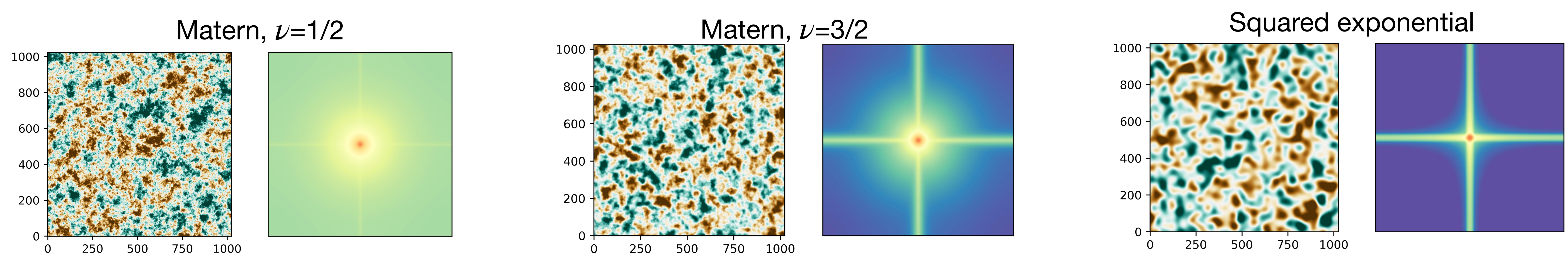
Gridded sea surface height (SSH) products from satellite altimetry are already an integral part of modern oceanography, widely used in both scientific and operational contexts. The well-established CMEMS mapped altimetry has clearly shown the value of a mapped product. However, there are some remaining challenges.

- **Custom grids:** Users often require the SSH field sampled at non-standard locations between grid points. This leads some to re-interpolate mapped products (over-smoothing) or implement their own algorithms (extra work).
- **Improved covariance:** Any mapping or interpolation scheme depends on an explicit or implicit covariance function. The covariance function should accurately reflect the statistical structure of SSH fields. The covariance function imposed by the most common mapped products appears to be based on a single set of North Atlantic observations in 1985. There is ample evidence supporting an update to the covariance function.
- **Published parameters:** Physical parameters from SSH (EKE, spectral slope, correlation scale, phase speed) are often estimated in isolation with ad hod methods. However, we estimate them as part of the mapping algorithm.

We are developing an **open-source**, **reproducible** mapping system, using an **updated covariance** function and publishing **physical parameter estimates**. This project is in an early phase, and we hope to show more results soon.

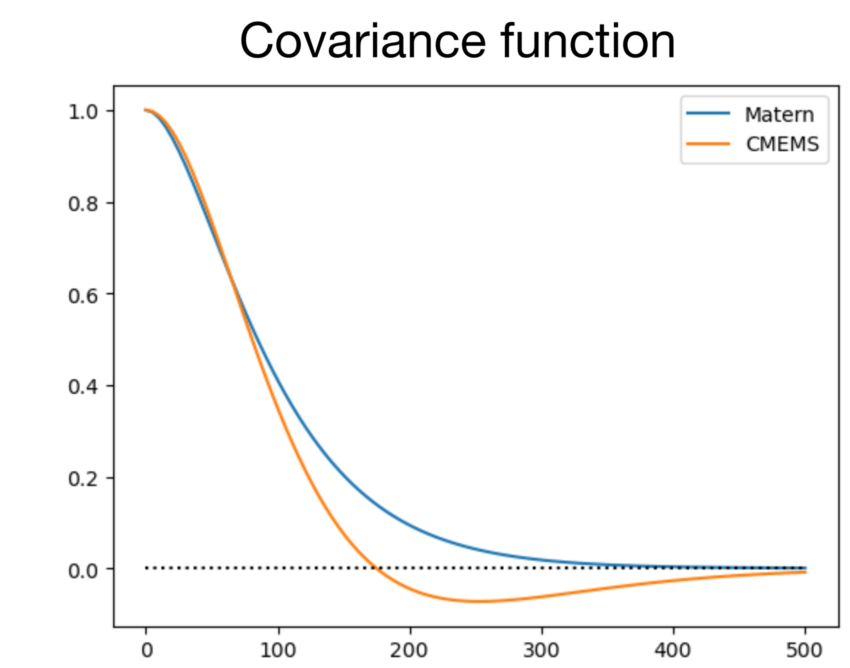
Covariance function impacts mapped result

The covariance function used in Gaussian process regression has a significant impact on the mapped fields. The covariance function influences the smoothness and spectrum of the mapped result. Random samples of three different covariance functions with the same decorrelation scale show dramatically different behavior (left panels below).



The **debiased spatial Whittle likelihood** uses the FFT algorithm to reduce covariance parameter optimization time from $\mathcal{O}(n^3)$ to $\mathcal{O}(n \log n)$. This allows more complex covariance functions and improved covariance parameter estimation. This technique fits the expected periodogram (right panels above) to the observed periodogram, accounting for the sampling pattern.

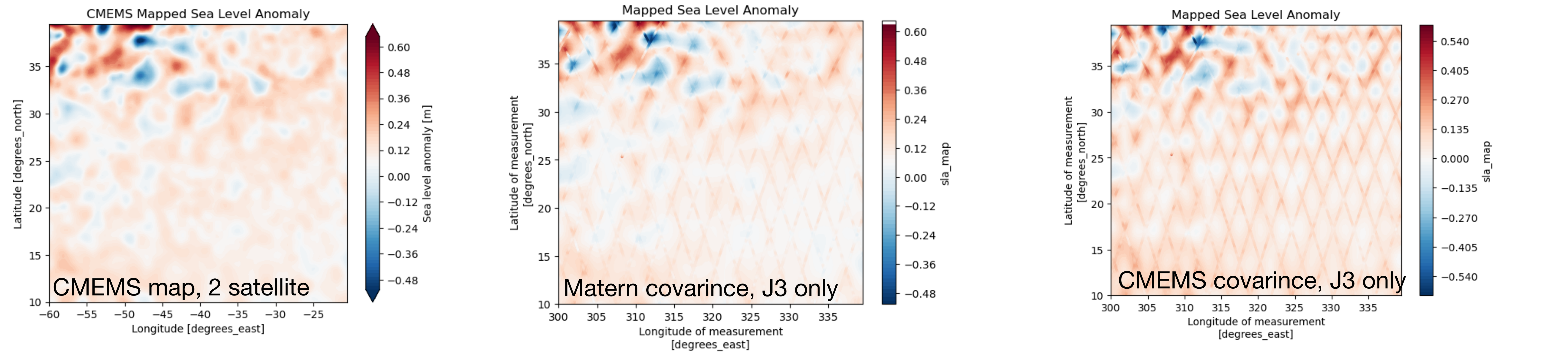
Improved SSH and velocity products



Using Gaussian process regression (GPR, aka optimal interpolation):

$$\tilde{\mathbf{x}} = \sum_{i=1}^{N} \sum_{j=1}^{N} \mathbf{R}_{xx}(\tilde{r}, r_j) \left\{ \mathbf{R}_{xx} + \mathbf{R}_{nn} \right\}_{ji}^{-1} \mathbf{y}$$

Preliminary maps use Jason-3 along-track L3 altimetry from CMEMS.
With only one satellite, nearly match performance of CMEMS gridded SLA with two satellites.
Maps with multiple satellites are in progress.
Mapping performance will be evaluated relative to CMEMS and a new product by Jonathan Lilly.



References

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