Topological analysis of oceanographic time series

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Introduction

Topological tools from nonlinear dynamics are used, for the first time, to analyze oceanographic time series. The procedure called BraMAH (Branched Manifold Analysis through Homologies) enables classifying the different processes at work in time evolving datasets [Charó et al, 2021, 2020]. Results can be used to unveil Lagrangian Coherent Sets (LCSs). The method is applied to time series generated with a simplified model proposed to understand chaotic advection in geophysical flows [Rypina et al, 2007] and to satellite-tracked drifter trajectories previously studied with metric (non topological) tools [Beron-Vera et al, 2020]. Topologically distinct dynamics are found in the weakly communicating flow regions that form basins of attraction for long-time almost-invariant sets on either side of the altimetry-derived barrier. The three main Lagrangian provinces can be associated with three different particle behaviors.

BraMAH Method

The first step is embedding the time series using time delays. This produces a multi-dimensional point cloud whose dimension is computed with a false nearest neighbors algorithm. The points are distributed on a branched manifold whose topological properties have to be determined. The point cloud is used to build a cell complex K, in the sense of algebraic topology. The cell complex is, in turn, used to determine the homology groups $H_i(K)$ $(i = 0, 1, 2)$ and hence the topological class to which the time series belongs.

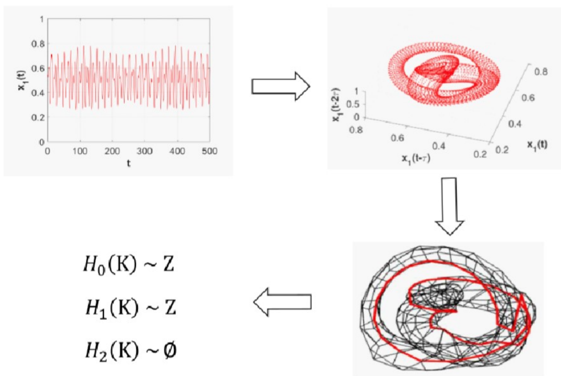

Figure 1: A BraMAH analysis goes from the time series (top left) to the homology groups (bottom left) characterizing the cell complex (bottom right) obtained for the time-delay embedding (top right) used to reconstruct the dynamics.

Streamfunction model

A proof of concept is proposed using time series that are obtained integrating a meandering jet model with the following streamfunction:

$$\psi(x,y,t) = c_3 y - U_0 L \tanh\left(\frac{y}{L}\right) + A_3 U_0 L sech^2\left(\frac{y}{L}\right) \cos(k_3 x) + $$
$$+ A_2 U_0 L sech^2\left(\frac{y}{L}\right) \cos(k_2 x - \sigma_2 t)$$

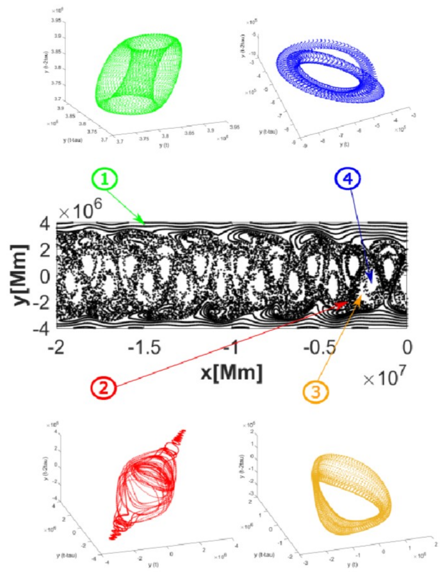

Figure 2: BraMAH applied to Lagrangian time series generated with parameters of Fig 5 in [Rypina et al, 2007]. Four topological classes are obtained: they identify the different types of dynamics within each LCS. Streaklines (in black) are used to visualize the non mixing regions with different topologies. Injections points are placed at (1) p_0= (-1.4e7, 3.8e6) ;(2) p_0= (-0.34e7, -1.7e6); (3) p_0= (-0.24e7, -1.5e6); (4) p_0= (-0.2e7, -0.6e6).

Oceanographic time series

BraMAH is applied to a time-evolving dataset of drifting buoys in the Southwest Atlantic, yielding non-equivalent topologies in the different Lagrangian provinces shown in Figure (3). In order to obtain the point clouds that are shown for each region, a pre-processing including trend removal and Singular Spectrum Analysis is carried out. As expected, the weakly communicating regions in [Beron-Vera et al, 2020] are found to exhibit distinctive dynamical properties, evidenced by the different homologies of the cell complexes approximating the embeddings obtained from the time series in the drifter dataset.

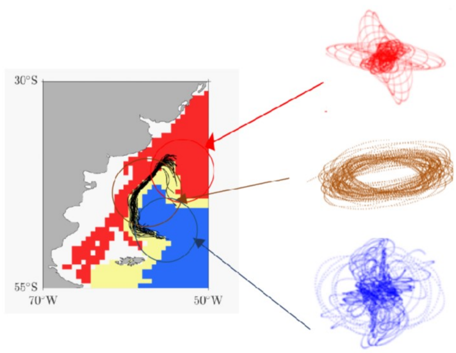

Figure 3: Time-delay embeddings for the trajectories of satellite-tracked drifters. BraMAH confirms that the Lagrangian dynamics differs in the three Lagrangian provinces.

References

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