Physically-consistent mapped altimetry on custom grids

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Summary

Gridded sea surface height (SSH) products from satellite altimetry are already an integral part of modern oceanography, widely used in both scientific and operational contexts. The well-established CMEMS mapped altimetry has clearly shown the value of a mapped product. However, there are some remaining challenges.

- Users often require the SSH field sampled at nonstandard locations between grid points. This leads some to re-interpolate mapped products (over-smoothing) or implement their own algorithms (extra work).
- Any mapping or interpolation scheme depends on an explicit or implicit covariance function. The covariance function should accurately reflect the statistical structure of SSH fields. The covariance function imposed by the most common mapped products appears to be based on a single set of North Atlantic observations in 1985. There is ample evidence supporting an update to the covariance function.
- Physical parameters from SSH (EKE, spectral slope, correlation scale, phase speed) are often estimated in isolation with ad hod methods.

We are developing an **open-source**, **reproducible** mapping system, using an **updated covariance** function and publishing **physical parameter estimates**. This project is in an early phase, and we hope to show more results soon.

Open-source, reproducible mapping system

The primary outcome of this work will be a well-documented and open-source mapping method applicable to past and current generation altimeters, including TOPEX/Poseidon, Jason, and Sentinel missions. We will distribute a mapping package that can be deployed locally or on cloud computing infrastructure, such as Pangeo Cloud, which already has altimetry data prepared for efficient computation.

Regardless of the chosen output grid, it is a near certainty that users will need the data on *some other grid* and will therefore re-interpolate the data. Instead, we will provide a tool to evaluate the GPR value from pre-computed basis functions using simple functions calls:

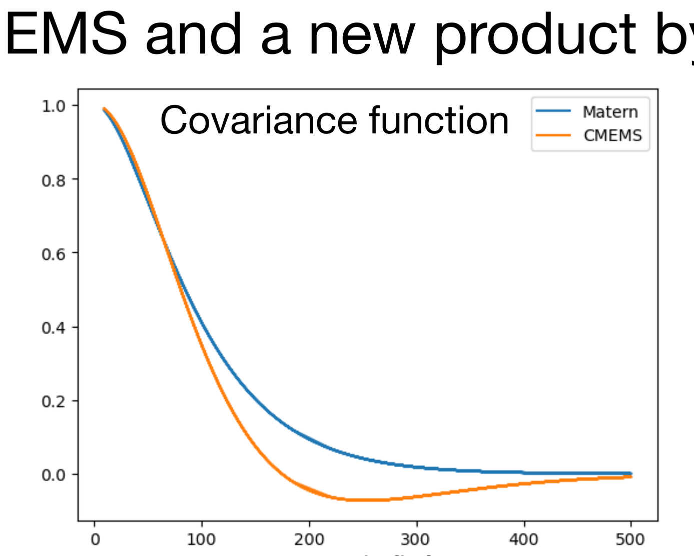
```
[eta, eta_error] = SSH(lat, lon, date);
[eta_x, eta_y] = SSHgradient(lat, lon, date);
```

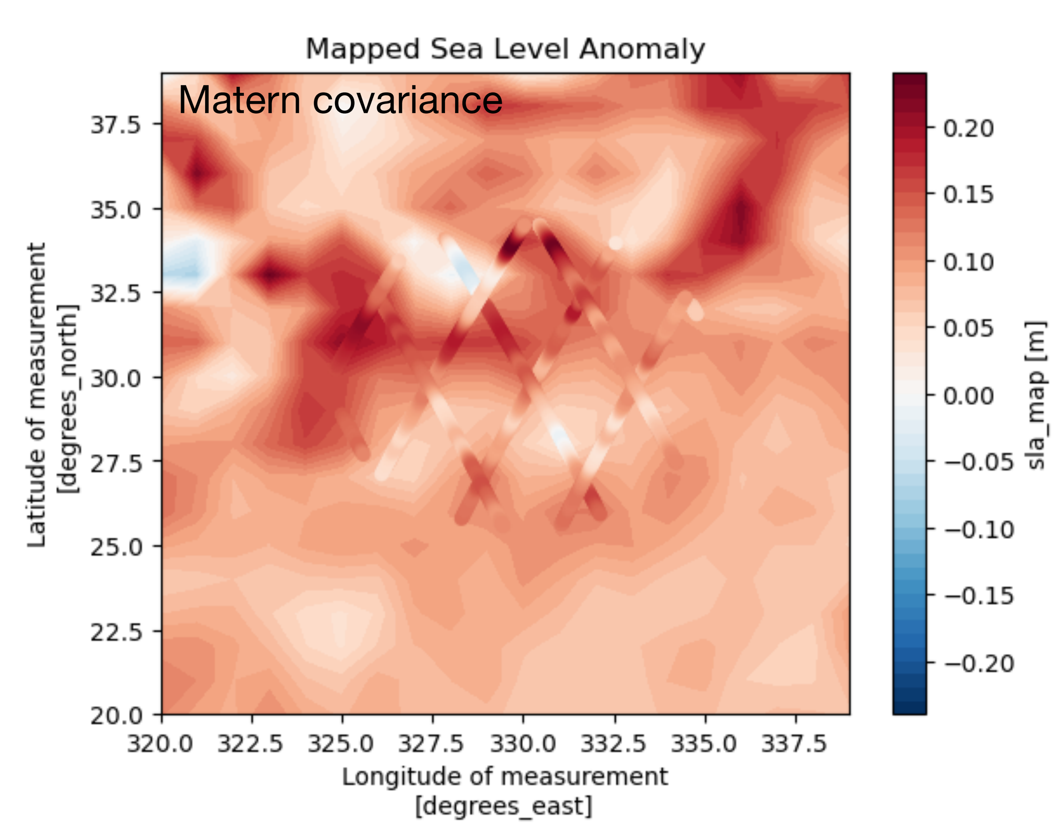
Improved SSH and velocity products

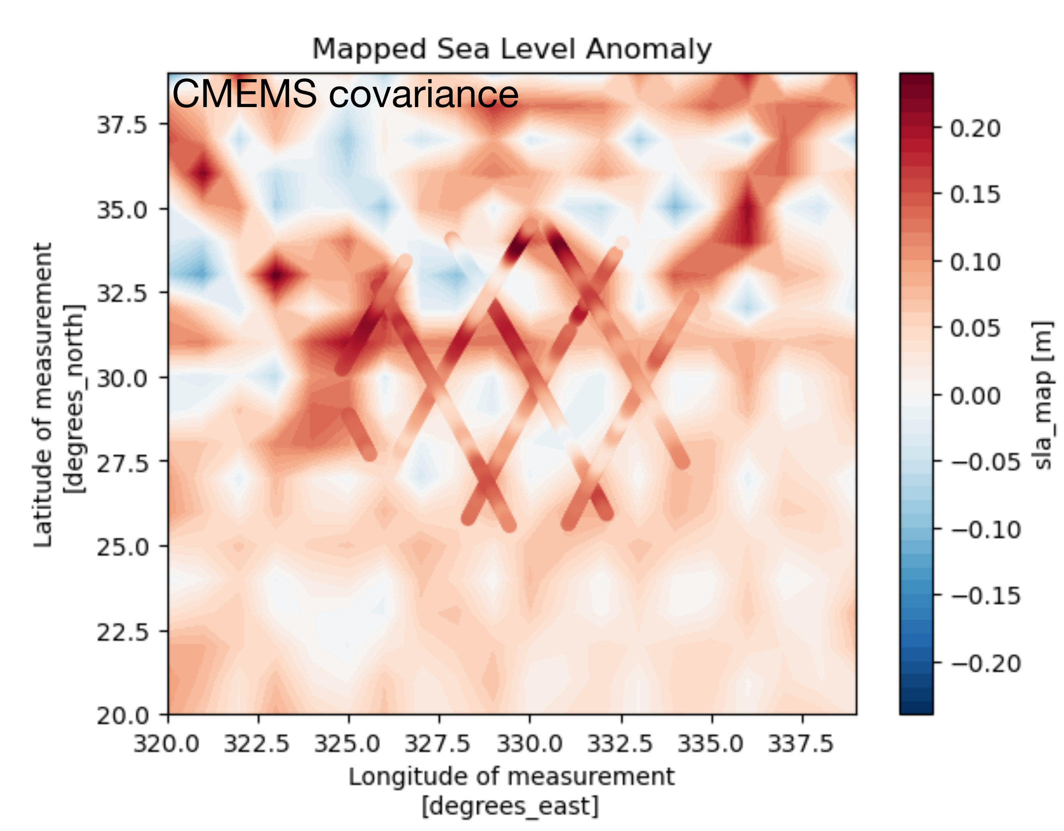
Using Gaussian process regression (GPR, aka optimal interpolation):

$$\tilde{\mathbf{x}} = \sum_{i=1}^{N}\sum_{j=1}^{N} \mathbf{R}_{xx}(\tilde{r}, r_j)\left\{\mathbf{R}_{xx} + \mathbf{R}_{nn}\right\}^{-1}_{ji}\mathbf{y}$$

Preliminary maps use Jason-3 along-track L3 altimetry from CMEMS.
Mapping performance will be evaluated relative to CMEMS and a new product by Jonathan Lilly.







Parameter estimates from parametric covariance function

We use a Matern covariance function

$$R(r) = \frac{2\sigma^2}{\Gamma(\nu)2^\nu}|\lambda r|^\nu \mathcal{K}_\nu(|\lambda r|) \quad \text{(covariance)}$$

which has corresponding spectrum

$$S(\hat{r}) = \frac{A^2}{(\hat{r}^2 + \lambda^2)^{\nu+1/2}} \quad \text{(spectrum)}$$

with

$$r = \sqrt{\left(\frac{\Delta x - c_x \Delta t}{L_x}\right)^2 + \left(\frac{\Delta y - c_y \Delta t}{L_y}\right)^2}$$

Correlation scales Lx, Ly, T and spectral slope (2ν+1) are determined simultaneously by approximate Maximum Likelihood Estimation, optimizing the fit between the parametric covariance function and *all* the available data. This uses the data efficiently and produces a consistent set of parameters.

References

- **J. M. Lilly: A new open-source gridded altimetric product, *OSTST 2022 Forum presentation***
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- Wortham & Wunsch (2014): A multidimensional spectral description of ocean variability, *JPO*