

Determination of the geocentric gravitational constant to monitor the behavior of the Earth



Marie Cherrier¹, Alexandre Couhert^{2,3}, Clément Courde⁴, Pierre Exertier³, Jean-Michel Lemoine^{2,3}, Flavien Mercier^{2,3}, Eléonore Saquet⁵

¹Celad for CLS/CNES, ²Centre National d'Etudes Spatiales, ³GET-Université de Toulouse (CNES, CNRS, IRD, UPS), ⁴Géoazur – Université Côte d'Azur (CNRS, Observatoire de la Côte d'Azur, IRD), ⁵Collecte Localisation Satellites

INTRODUCTION

The geocentric gravitational coefficient is defined by the product of the Earth's universal gravitational constant G and its mass M (GM). The last official determination of the gravitational coefficient ($GM = 398600,4415 \pm 0,0008 \text{ km}^3/\text{sec}^2$) dates back to 1992. Regarding the third law of Kepler, GM is directly proportional to the semi-major axis (or altitude) of the orbiting object. Thus, the given uncertainty on the value of GM refers to an offset of $\pm 2 \text{ cm}$ on high orbiting GNSS satellites, which will then transfer to the altimetry missions relying on GNSS receivers through Precision Orbit Determination (POD) errors. Thus, the purpose of this study is to reassess the uncertainty of the Earth's GM value using Satellite Laser Ranging (SLR) towards eight specific spherical geodetic satellites, simultaneously estimating laser station and satellite biases.

METHOD

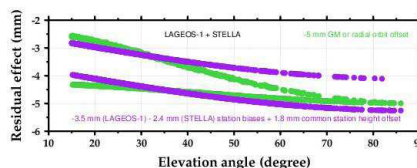
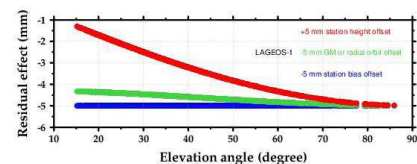
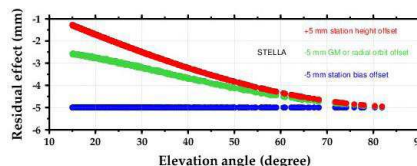
To refine the GM value, we wanted to consider the bias of the laser stations, as well as the satellites biases (signature effects), over a period from 2004 to 2020. Both types of biases are the main sources of uncertainties in the determination of GM . To do so, we used zero-signature targets and a privileged geometrical configuration of satellites at extreme altitudes. We relied on the SLR technique for the orbit determinations to estimate several parameters.

Parameters	Spacing
Dynamical parameters	
GM	Yearly/Constant
Stokes coefficients of low degrees ($C_{00}, C_{21}, S_{21}, C_{22}$ and S_{22})	Monthly
Empirical forces (Constant along-track + Periodic along/cross-track accelerations)	MEOs Weekly LEOs Daily
Measurement parameters	
Laser range biases	Yearly (station & satellite specific)
Laser station heights	Monthly
Geocenter coordinates and equatorial rotations	Monthly

Interest of a LEO/MEO combination

We know that GM , station biases and heights are correlated to each other. Regarding the two first right-hand side figures, one can observe a coupling between station biases and GM for a MEO satellite (LAGEOS-1). Likewise, one can observe a coupling between GM and station heights for a LEO satellite (Stella). Depending on the satellite altitude, the joint observation of one of these three parameters is delicate with one satellite because artificial combinations could appear between them, distorting the estimation of GM .

For decoupling these three parameters, a combination of LEO and MEO satellites seems to be a better approach. Indeed, regarding the third right hand side figure, one can see that by combining Stella and LAGEOS-1, the curves representing GM , station biases and heights are decorrelated from each other.



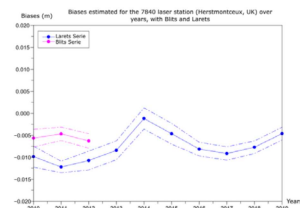
Interest of zero-signature targets

Zero-signature targets are said to be without signature effects because when the laser beam reaches the surface of the satellite, there are no multiple reflections in the direction of the laser station but one. The satellite signature effect is thus minimized. We had recourse to laser measurements realized on Blits and Larets, who both are LEO satellites. For all satellites we adjust a bias per station and per satellite to avoid these signature effects.

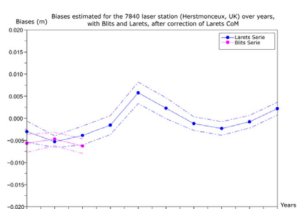
SATELLITES PROBLEMS AND RESULTS

Larets Center of Mass correction

When estimating laser range biases with a zero-signature satellite, the biases estimated must be consistent with the biases estimated for the same station with another zero-signature target. We can see on the figure below that Blits and Larets biases are not consistent. There is a gap of about 7 mm. This difference seems to come from an incorrect Center of Mass (CoM) correction. The value given on the ILRS website is 56,2 mm. A CoM computed by Sosnica et al. (2015) gives a value of 63,1 mm for Larets.

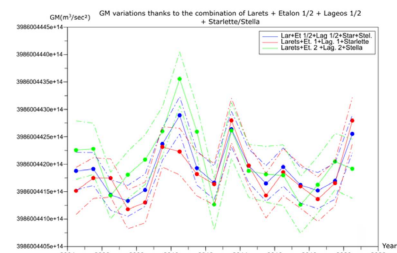


We corrected our CoM and obtained consistent biases between Blits and Larets.



Final estimate of GM

We made two independent estimations, by combining Larets, Starlette, LAGEOS-1, Etalon-1 on the one hand; and Larets, Stella, LAGEOS-2, Etalon-2 on the other hand. For the first combination, we obtained an estimate of $GM = 398600,4418 \text{ km}^3/\text{sec}^2$. For the second one, we obtained a value of $GM = 398600,4420 \text{ km}^3/\text{sec}^2$.



Our final estimate of GM was made using seven geodetic satellites at different altitudes, based on the principle of the MEO/LEO combination.

The estimated value retained with this configuration is $GM = 398600,4419 \pm 0,0002 \text{ km}^3/\text{sec}^2$. It is given with an estimation uncertainty of 0,5 ppb, based on the formal covariance at 3σ and considering a measurement noise of 2 cm over the station network. Furthermore, the uncertainty is consistent with the differences observed between the two previous independent estimations.

