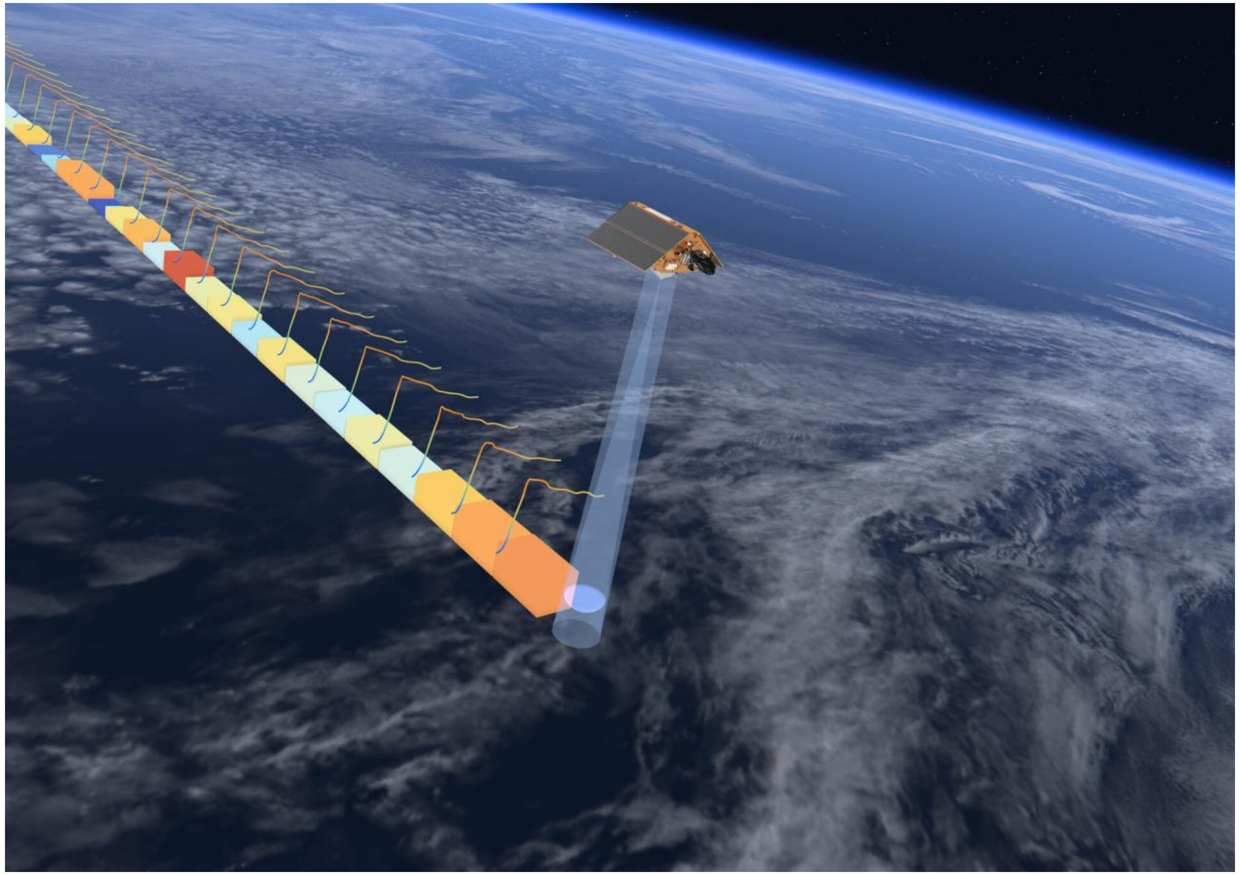
FastAdaptive: a new, optimal, unbiased, and computationally efficient retracking solution for the analysis of conventional altimetry data



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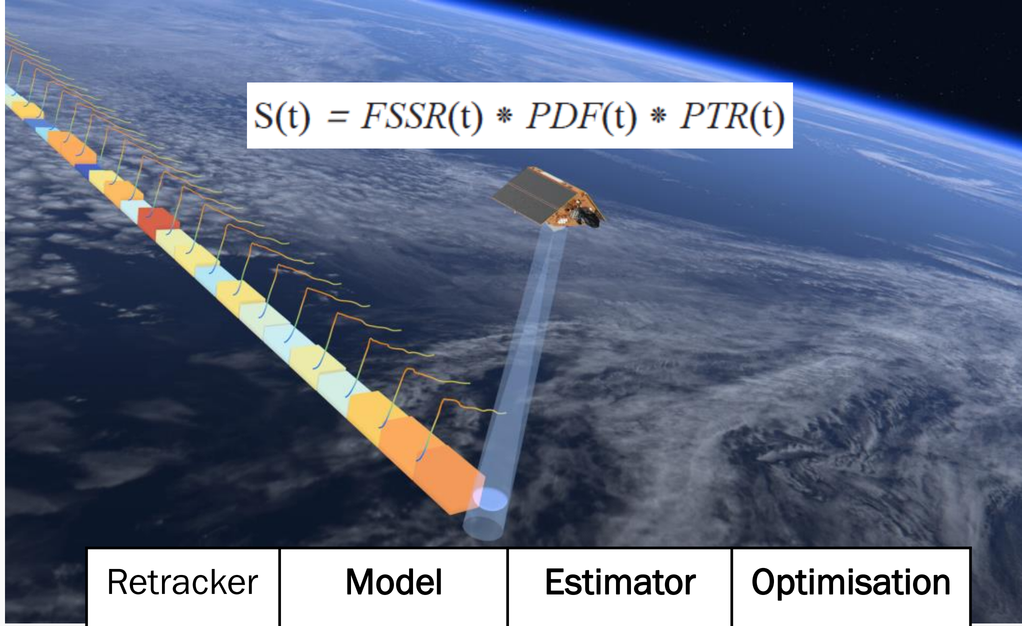
Outline

- Context and motivation
- FastAdaptive solution formalism
- Results on simulations
- Preliminary results on Jason-3 data
- Conclusions and perspectives

Context and motivation



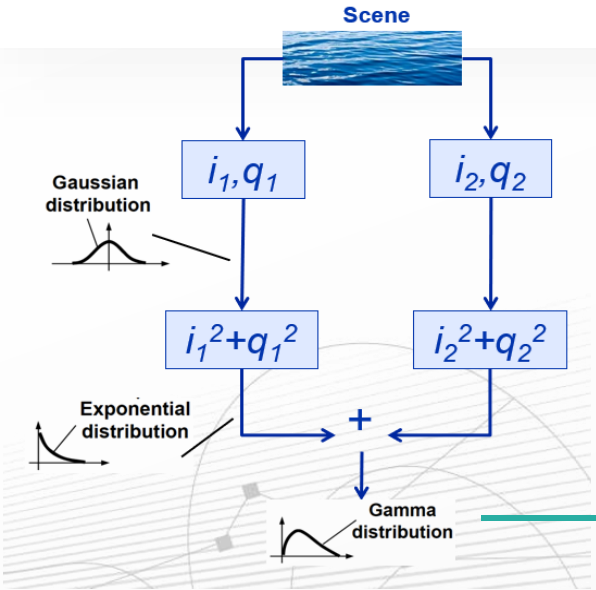
- The continuous improvement of retracking algorithms for *optimal and efficient estimation of the geophysical parameters* is a key goal that becomes more critical with the increase of the amount of data and the high demanding requirements in terms of data quality and resolution
- The **baseline retracking solution** currently implemented in the ground segments for conventional altimetry is the **MLE4** solution which is computationally **fast** ☺ but yields a **biased** and **sub-optimal** parameter estimation ☹ that needs Look Up Table correction afterwards
- Lots of efforts have been done to improve this solution, leading to the development of the **Adaptive** retracker which yields an **unbiased** and **optimal** parameter estimation ☺ but is computationally **slow** ☹
- Ideally: **optimal**, **unbiased** and computationally **efficient** retracking solution

FastAdaptive retracking

Retracker	**Model**	**Estimator**	**Optimisation**
MLE4	• Brown Model • Gaussian PTR	Ordinary Least Squared (not an MLE!)	Newton-Raphson
Adaptive	• Adaptive model • Real PTR	True Maximum Likelihood	Nelder-Mead

see Fanny Piras's presentation for more details

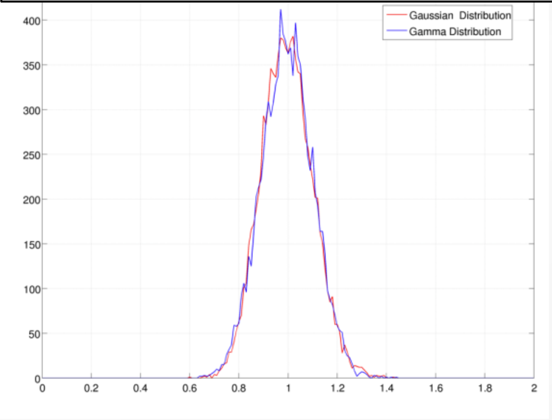
FastAdaptive: formalism



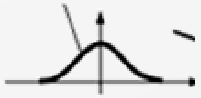
The observed radar waveform is a backscattered radar wave signal corrupted by a multiplicative speckle noise whose distribution is described by a Gamma function

On board averaging of N echos to reduce noise. For N big enough, the Gamma distribution converge to a Gaussian distribution (central limit theorem).

Example of Gaussian vs Gamma distribution for N=90



Gaussian distribution



Under this assumption, the averaged waveform, as a function of the range gate k and model parameters θ, can be modelled as:

Noise

$$S(k) = \boxed{s_t(k,\theta)}[1+\boxed{n(k)}] = s_t(k,\theta) + \boxed{n'(k)} \longrightarrow \sim \mathcal{N}(0, s_t^2(k,\theta)/L)$$

Theoretical signal (=the model)

$$\sim \mathcal{N}(0, 1/L)$$

L<=N is the number of **independent** summed echos (ENL)

FastAdaptive: formalism

- With this formalism, the <u>likelihood function</u> (join probability of the measurements **y** given the model $s_t(\theta)$) is a multi variate Gaussian distribution with mean s_t and noise covariance matrix C (see e.g. [Rodriguez 1998], [Halimi et al 2015]) :

$$\mathcal{L}(\mathbf{y}|\theta) = \prod_i \mathcal{L}(\mathbf{y}_i|\theta) \sim \prod_i \frac{1}{\sqrt{2\pi det(\mathbf{C})}} exp(-\frac{1}{2}(\mathbf{y}_i - \mathbf{s}_t(\theta))^T \mathbf{C}^{-1}(\mathbf{y}_i - \mathbf{s}_t(\theta)))$$

$$\mathbf{C} \equiv \mathbf{C}(\theta, L)$$

diagonal matrix with elements

$$C_{k_n k_n} = s_t^2(k_n, \theta)/L$$

- Maximum Likelihood Estimator (MLE) $\hat{\theta}_{MLE} = argmin(-2ln\mathcal{L})$

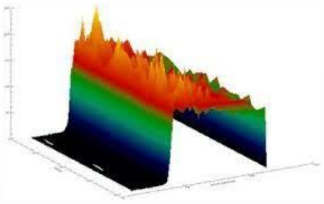
$$-2ln\mathcal{L} = (\mathbf{y} - \mathbf{s}_t(\theta))^T \mathbf{C}^{-1}(\theta)(\mathbf{y} - \mathbf{s}_t(\theta)) + ln[det(\mathbf{C})] + const \underset{L>>1}{\sim} (\mathbf{y} - \mathbf{s}_t(\theta))^T \mathbf{C}^{-1}(\theta)(\mathbf{y} - \mathbf{s}_t(\theta)) + const$$

$$\hat{\theta}_{WLS} = argmin[(\mathbf{y} - \mathbf{s}_t(\theta))^T \mathbf{C}^{-1}(\theta)(\mathbf{y} - \mathbf{s}_t(\theta))]$$

Take away message:

- The MLE is equivalent to a Weighted Least Square (WLS) estimator with weights defined as the inverse of the noise variance of the measurements
- <u>Key advantage</u>: the system of equations can be resolved using the Levenberg-Marquardt method (therefore no need of Nelder-Mead), ensuring a much faster computation, while preserving optimality
- Different formalism with respect to the Adaptive retracker

FastAdaptive solution summary



Model

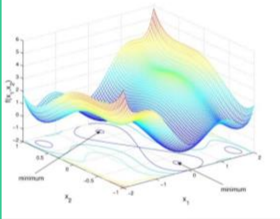
4 parameters Adaptive Model

$\theta = [\mathrm{SWH},\ \mathrm{epoch},\ \mathrm{Amplitude},\ \gamma]$

convolved with the real PTR

Estimator

Weighted Least Square



Optimisation

Levenberg-Marquardt minimization

- WLS based solutions have been explored in altimetry data analysis: e.g.[Sandwell et al. 2005], [Halimi et al. 2013], [Garcia et al. 2014], [Mangilli et al 2022]

see poster SC4 2022-12 on WLS solution applied for Lake Ice Thickness retrievals (LRM and SAR data)

- Weights (noise variance) estimation: analytical expression $C_{k_n k_n} = s_t^2(k_n, \theta)/L$. Possibilities:
  - Iterative estimation from initialisation values → Current implementation
  - Include the model dependent noise variance in the fit function

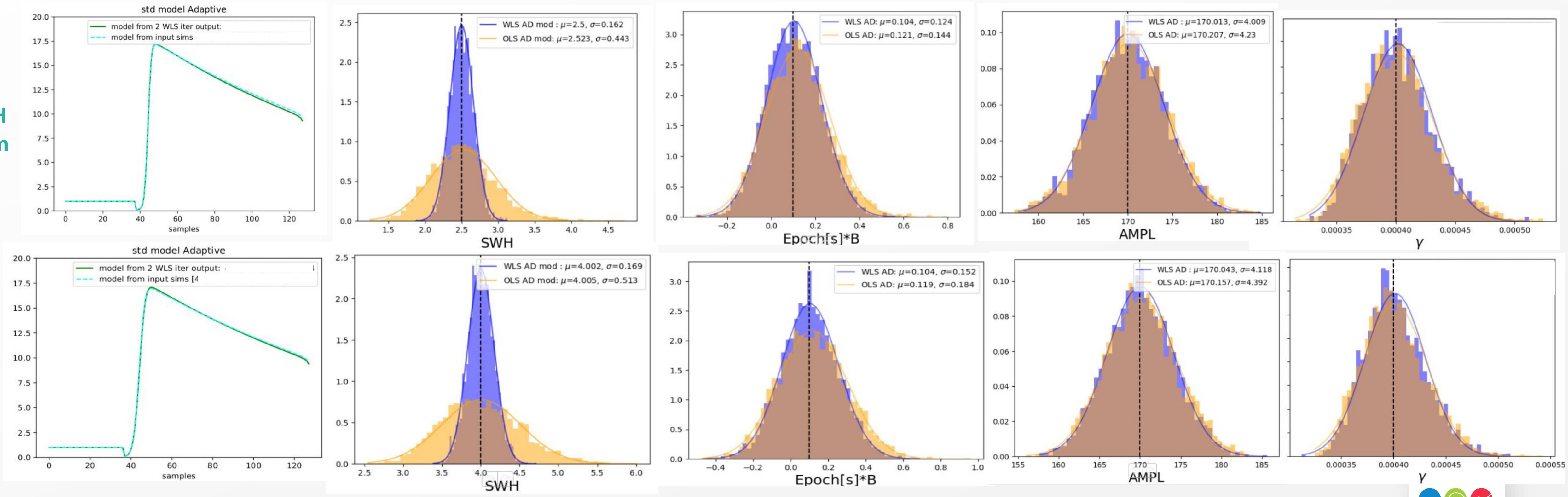
FastAdaptive results on simulations

- Jason-3 like simulations generated with the Adaptive model with sinc2 PTR and a multiplicative speckle noise with a gamma distribution and constant ENL=90. 3000 sims for different sets with SWH inputs, from 1m to 6 m.
- Comparison of results obtained with the FastAdaptive and Ordinary Least Square (same model, no weights) solutions

WLS Weights | SWH | EPOCH | Ampl | Gamma

SWH 2.5m

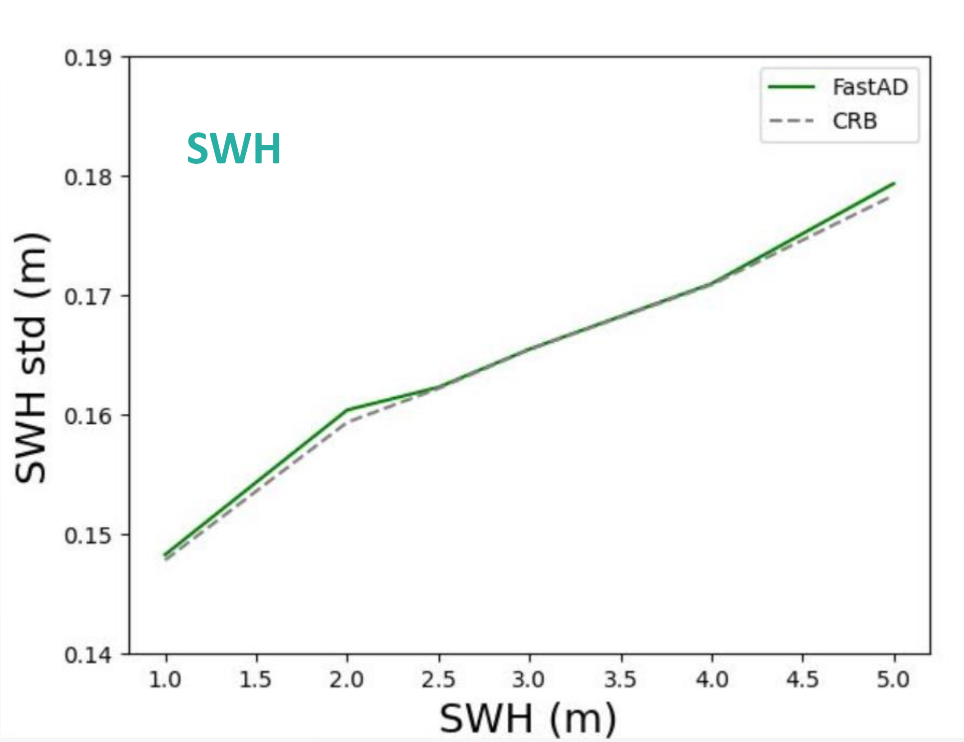
4m

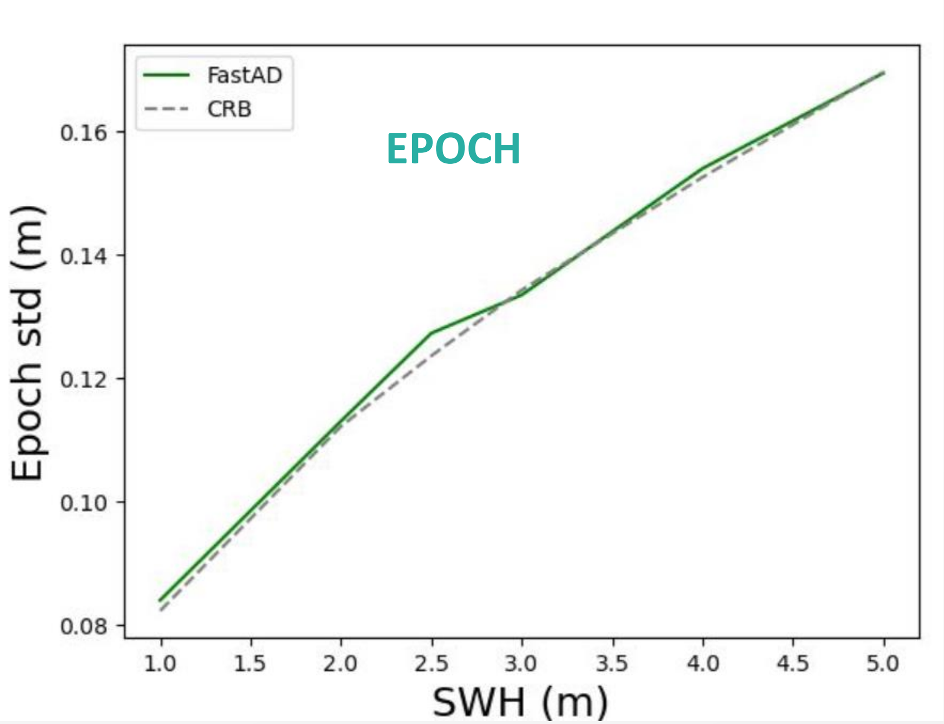


<u>FastAdaptive: **unbiased results** and significant improvement of the parameter estimation accuracy for all the simulation sets</u>

FastAdaptive results on simulations

Optimality: comparison with **Cramer-Rao bounds (CRB)** = theoretical bounds of minimal parameter variance





<u>FastAdaptive: parameter estimation compatible with CRB, thus optimal</u>

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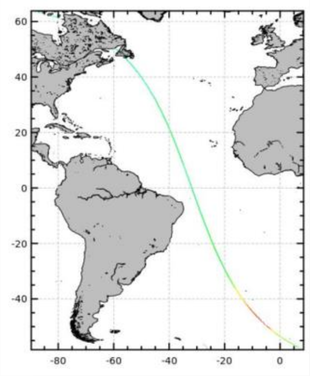
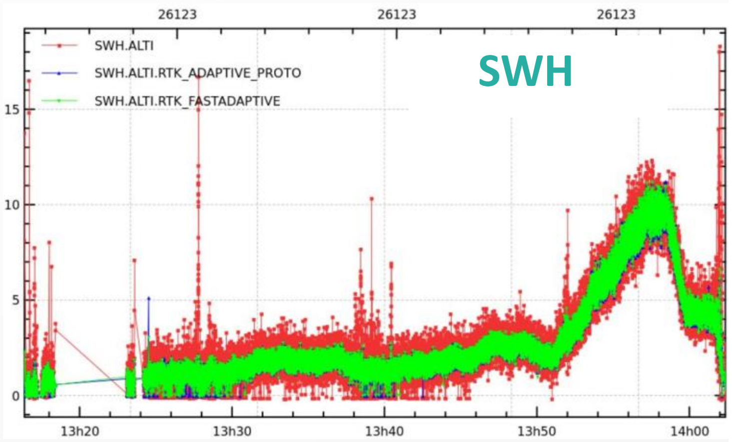
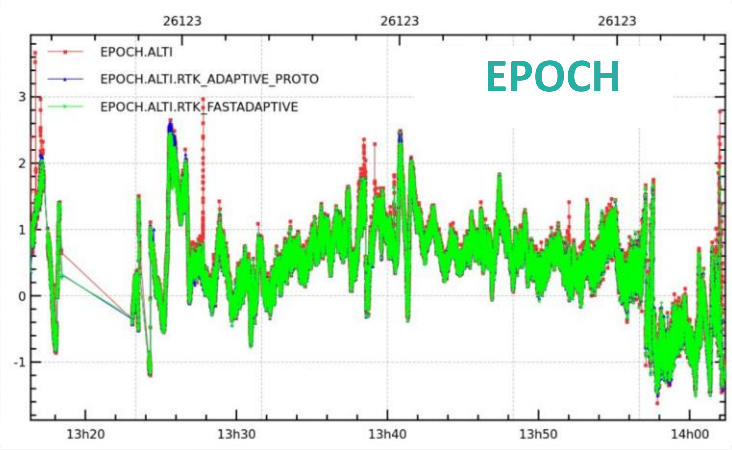
FastAdaptive results on Jason-3 data

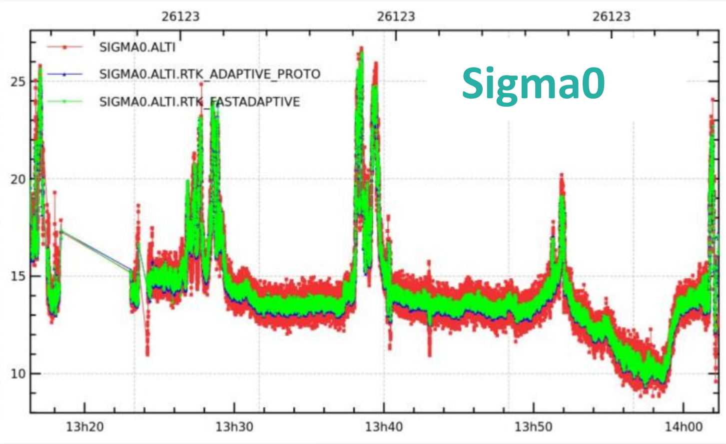
- Jason-3 waveform data for cycle 199 (July 2021) are analysed with the FastAdaptive retracker and the results are compared with the Adaptive (and MLE4) retracker outputs. The numerical PTR is used for each pass.
- Data selection over ocean (some successful tests also on peaky waveforms)
- The **FastAdaptive execution time** is in the order of **~3 times the Real Time** for a standard run without parallelisation, therefore significantly faster than the Adaptive retracker (the <u>main computational cost</u> is due to the <u>PTR convolution</u> with a factor of 64 oversampling and not to the optimisation step)



J3 c 199 pass 150

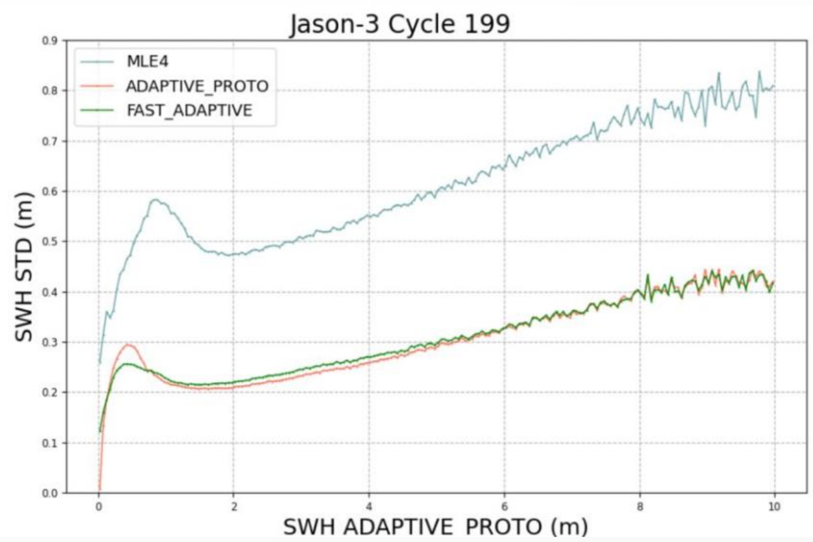


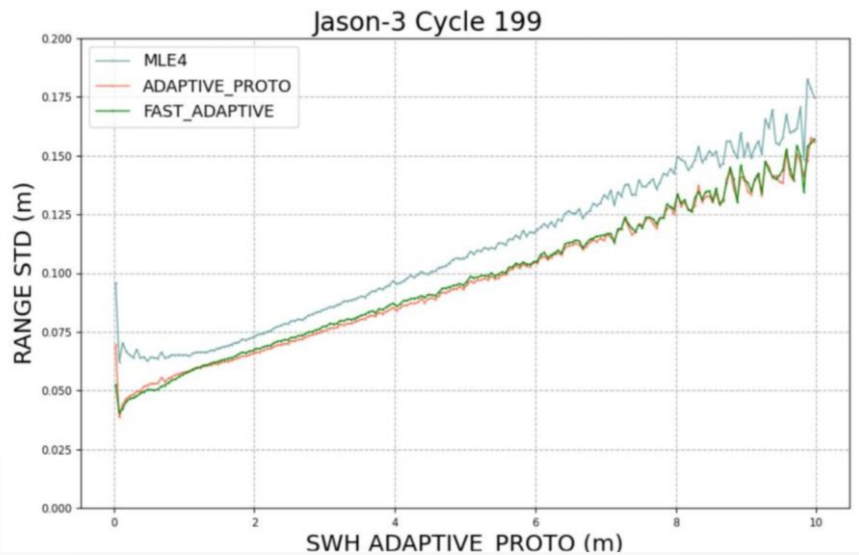


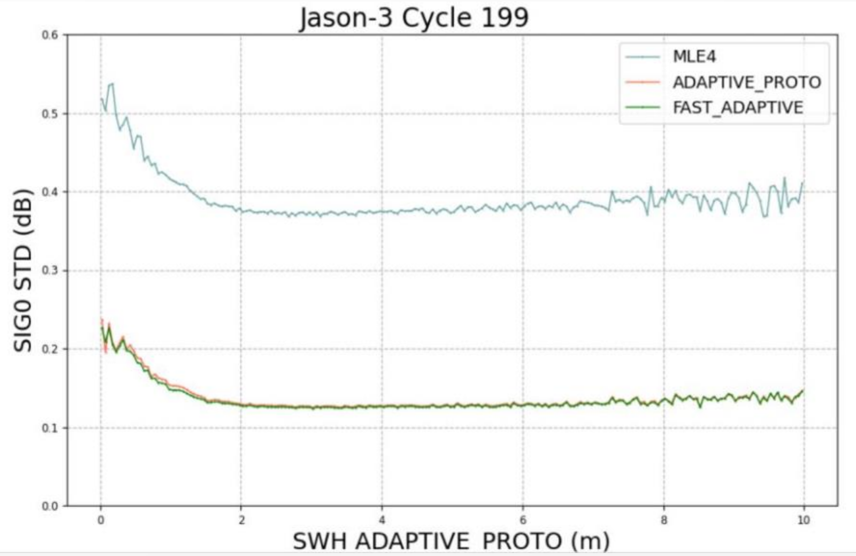


<u>**FastAdaptive results compatible with Adaptive results. Optimality preserved (significant noise reduction with respect to MLE4).**</u>

FastAdaptive results on J3 data: parameter errors at 1 Hz



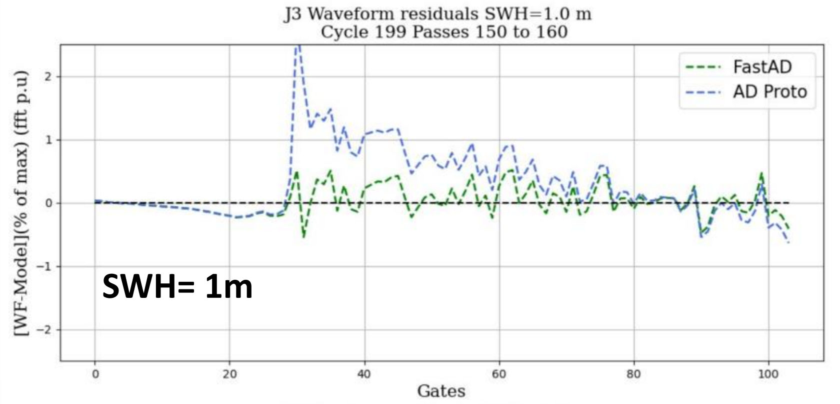


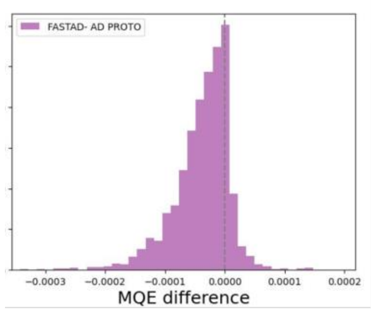


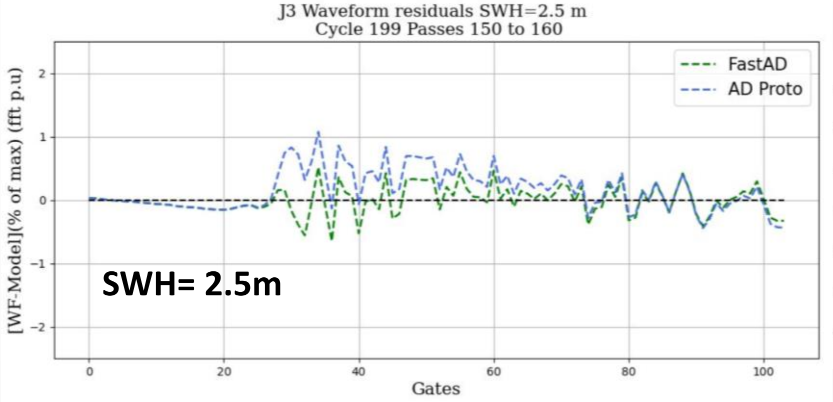
<u>The FastAdaptive matches the high performances of the Adaptive retracker for all the parameters</u>

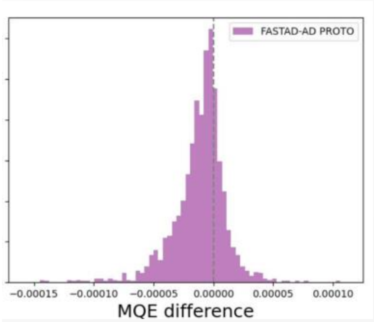
FastAdaptive results on J3 data: waveform residuals

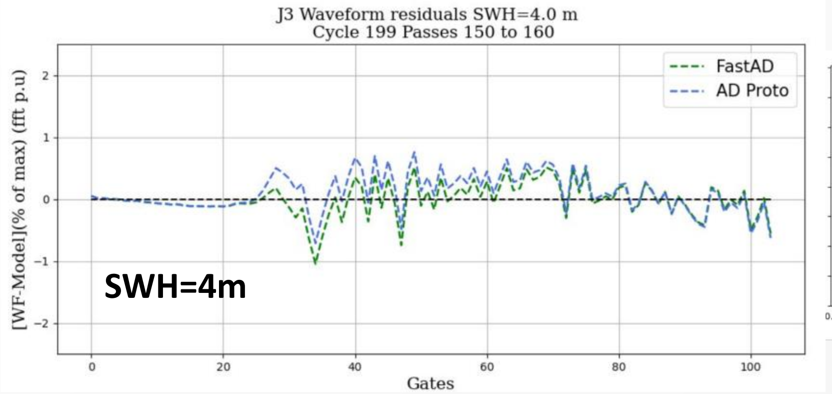
$$Res(k) = WF_{data}(k) - WF_{model}(k, \hat{\theta})$$

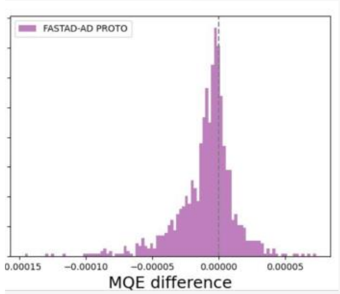


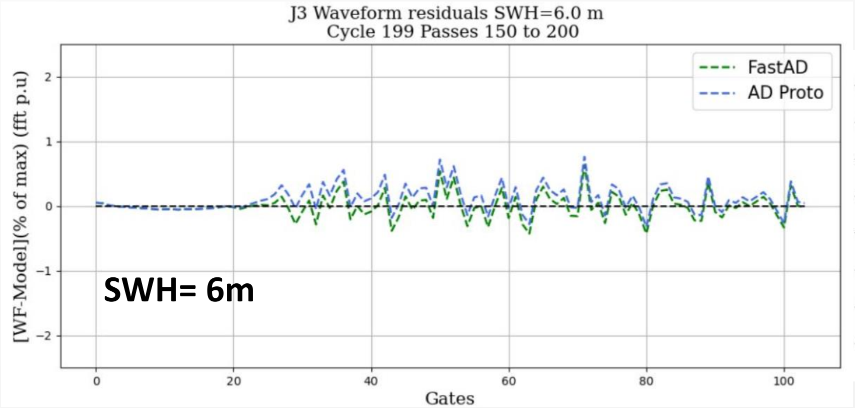


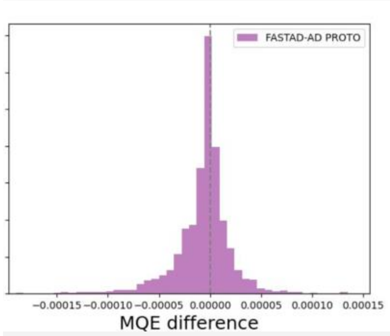












- Overall, consistent residuals between FastAdaptive and Adaptive for different SWH.
- Some small differences are seen in particular around the leading edge for SWH=1m with slightly smaller MQE for the FastAdaptive. More investigation ongoing

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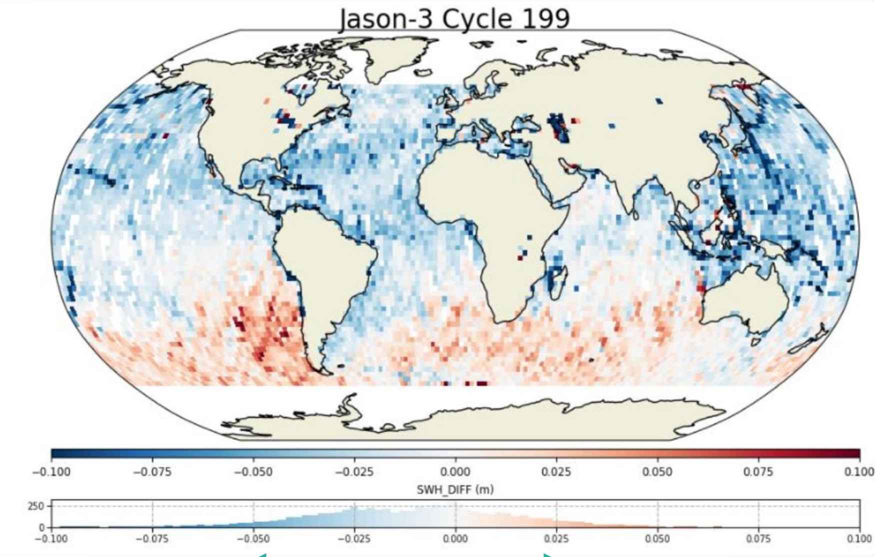
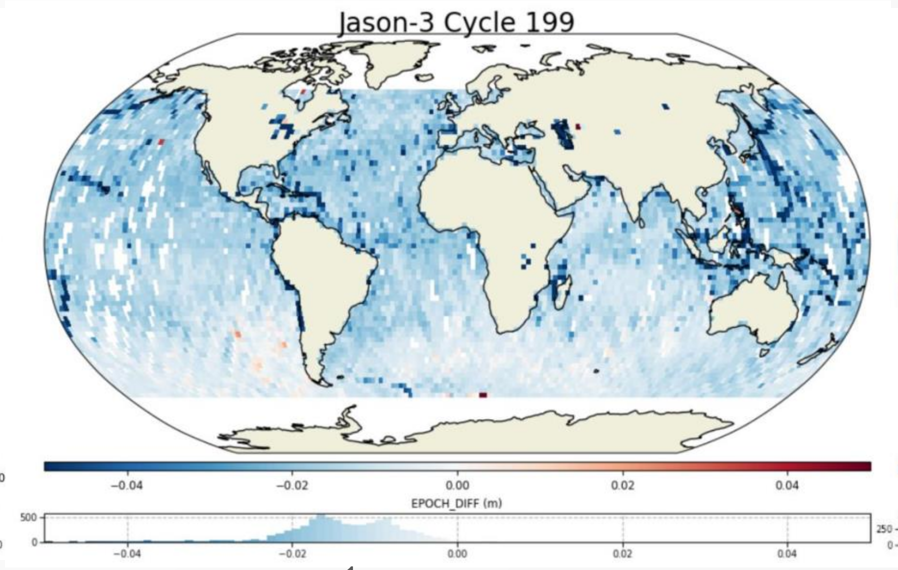
FastAdaptive vs Adaptive maps difference

SWH



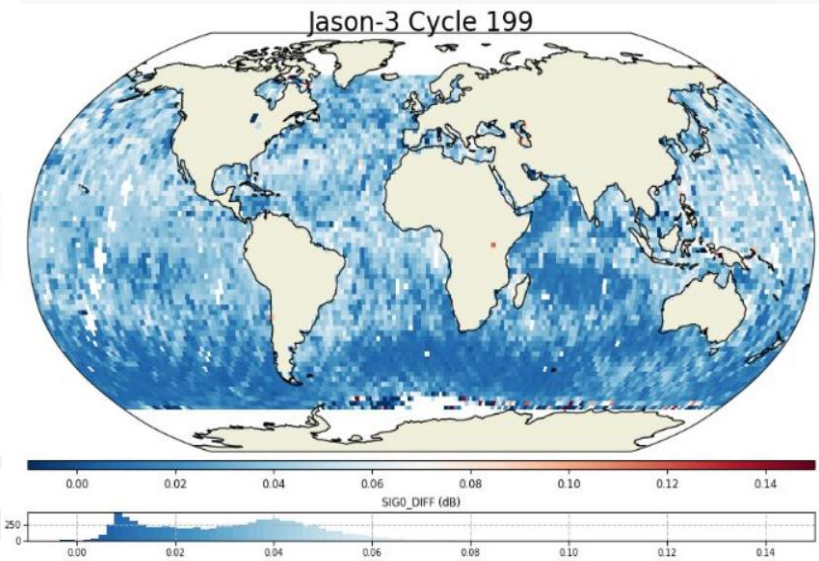
-2.5 cm +2.5 cm

EPOCH



-1 cm

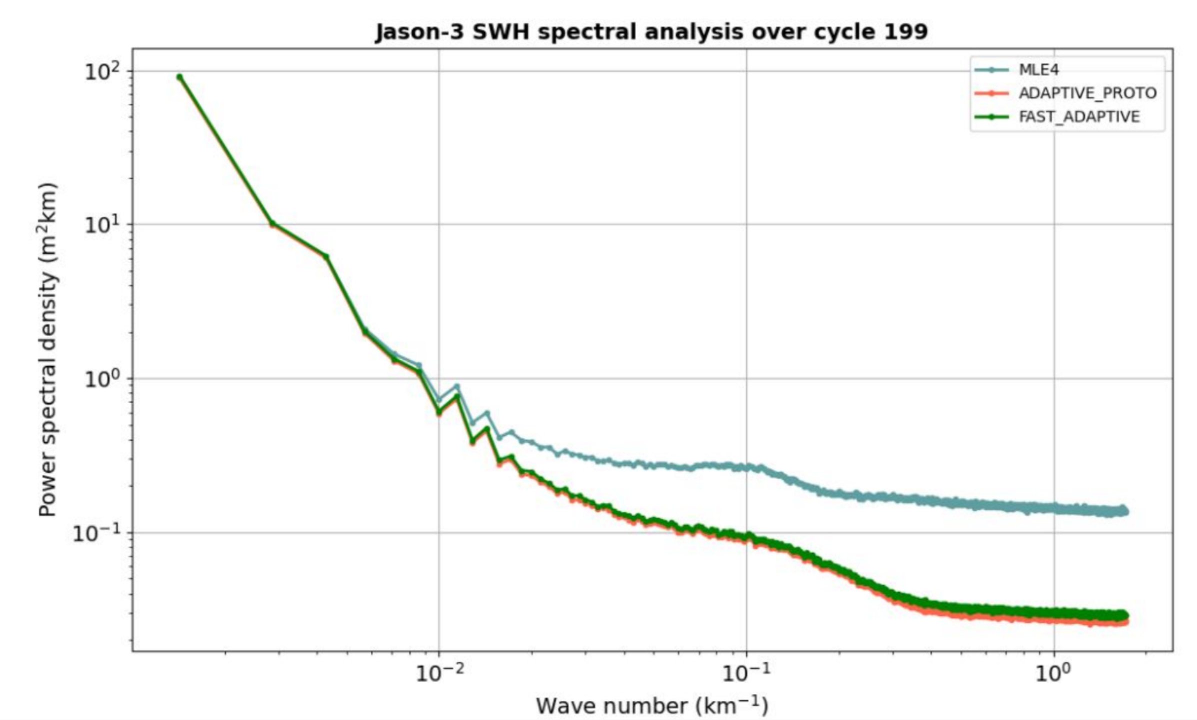
Sigma0

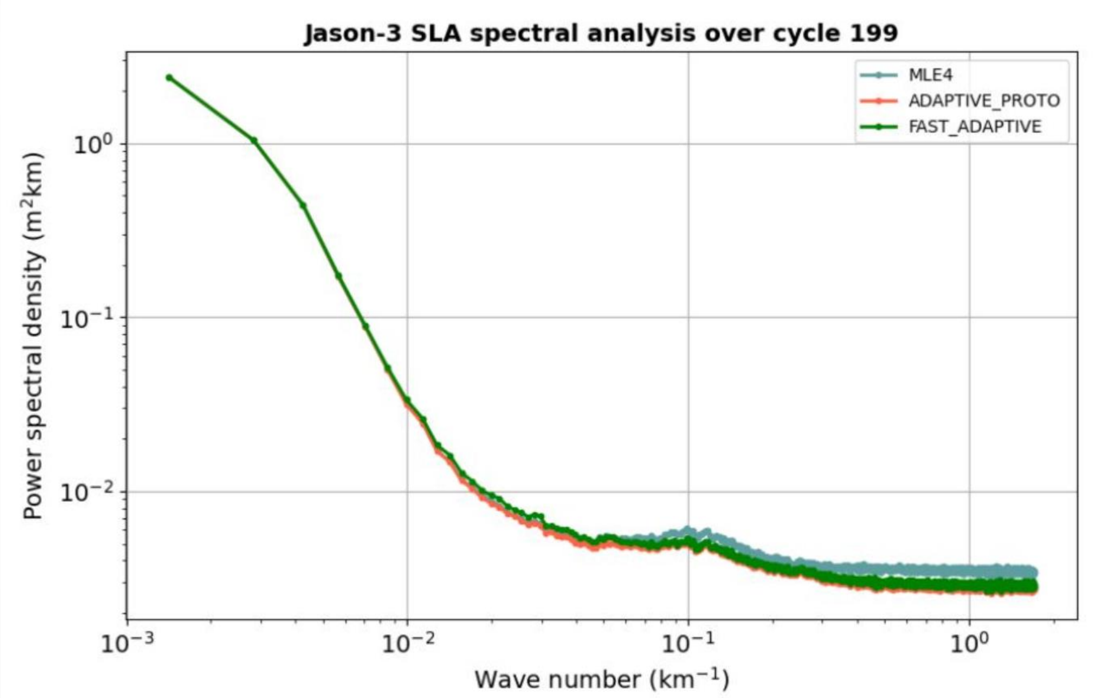


0.02 dB

- Overall, the maps difference between the FastAdaptive and the Adaptive retracker over 1 data cycle shows **compatible results** between the two retrackers. More extensive CalVal analysis foreseen
- The **significant noise reduction** on the SWH estimation of both solutions allows to measure the impact of geophysical signals (oceanic trenches?) on the LRM waveform shape (e.g. SWH) for the first time (similar signatures seen in doppler processing [Moreau et al 2021])
- More investigation underway

FastAdaptive results on J3 data: SWH and SLA spectra



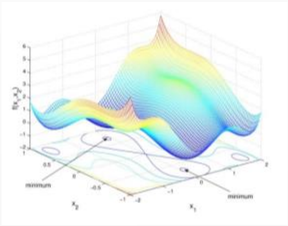


Very good performances of the FastAdaptive, comparable with the Adaptive, for both SWH and SLA

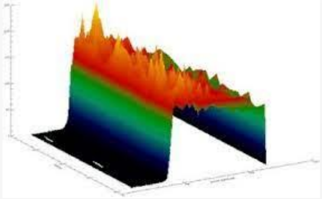
Conclusions and perspectives

	MLE4	Adaptive	Fast Adaptive
Optimal parameter estimation (noise reduction)	✗	✓	✓
CPU time	✓	✗	✓
GMSL stability (PTR)	✗	✓	✓
Multi-surface processing	✗	✓	✓

Jason-3 GDRF



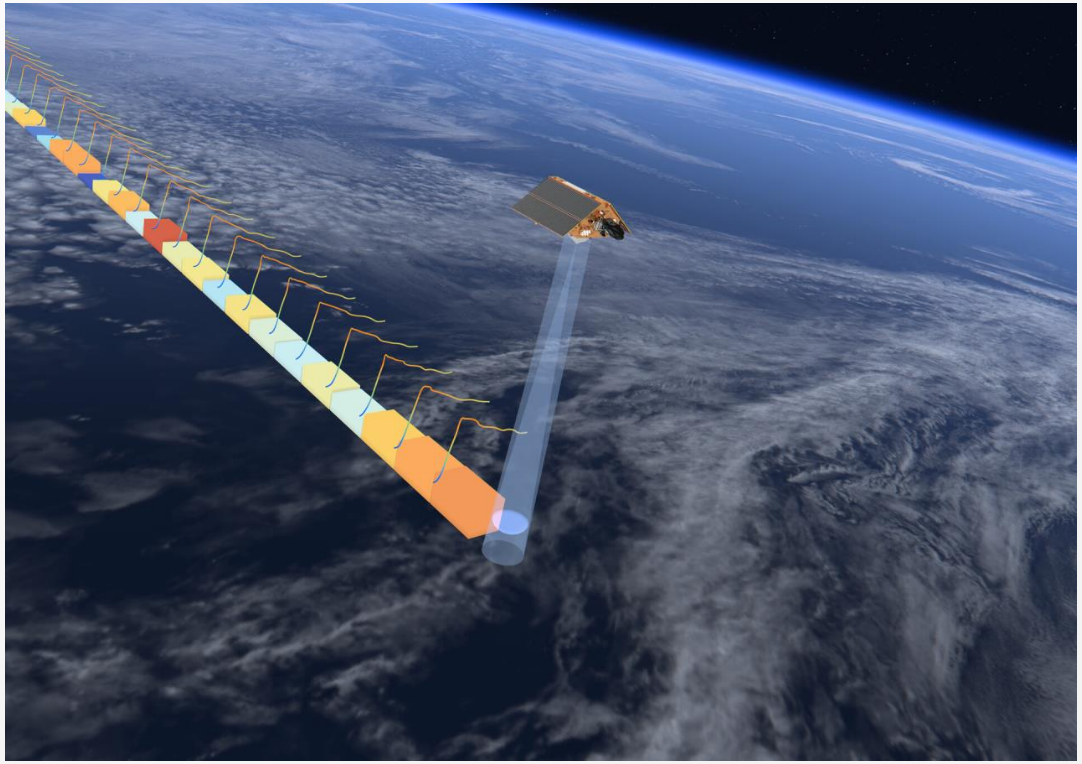
Estimator & Optimisation



Model

- **First results obtained with the FastAdaptive are promising!** More extensive tests on Jason-3 data for validation (also on different surfaces)
- **Next steps:** application to S6A-LRM data and validation (data during S6 and J3 tandem phase)
- **Perspectives:** Could be used also for future LRM mission (SWOT-nadir, S3C/D PLRM) and adapted for SARM data

Thank you!



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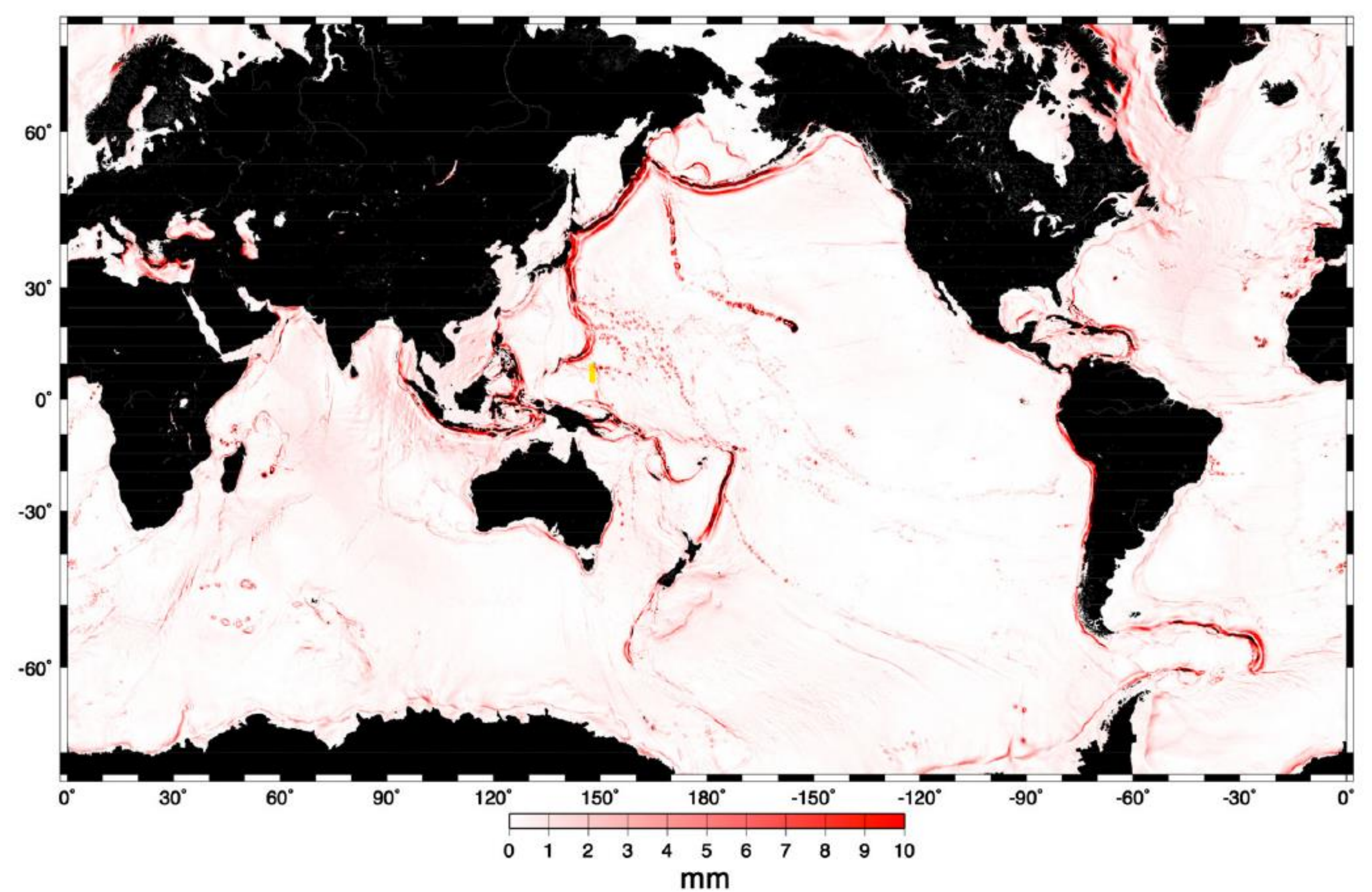
Oceanic Trenches map



Smith et al. 2014

Empirical ENL for Jason-3