



# Waves mean square slope (*mss*) estimation from CFOSAT/SWIM measurements

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# Outline

1. Definitions and disambiguation of *mss*
2. SWIM data used for *mss* inversion
3. *mss* estimation using Geometrical Optics (GO)
4. Second order models for *mss* inversion

# The mean square slope ( $mss$ )

The mean square slope is an important ocean sea surface statistic. It is mainly governed by the **short wind waves** which are those supporting the wind stress,

It is known that  $mss$  increases with wind speed [Cox & Munk, 1954]

→ Strongly dependent on wind speed, slightly non-isotropic

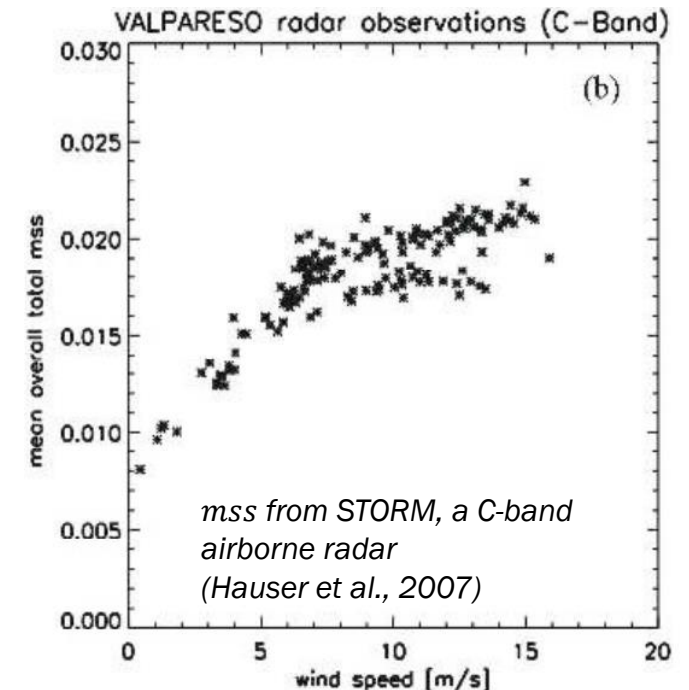
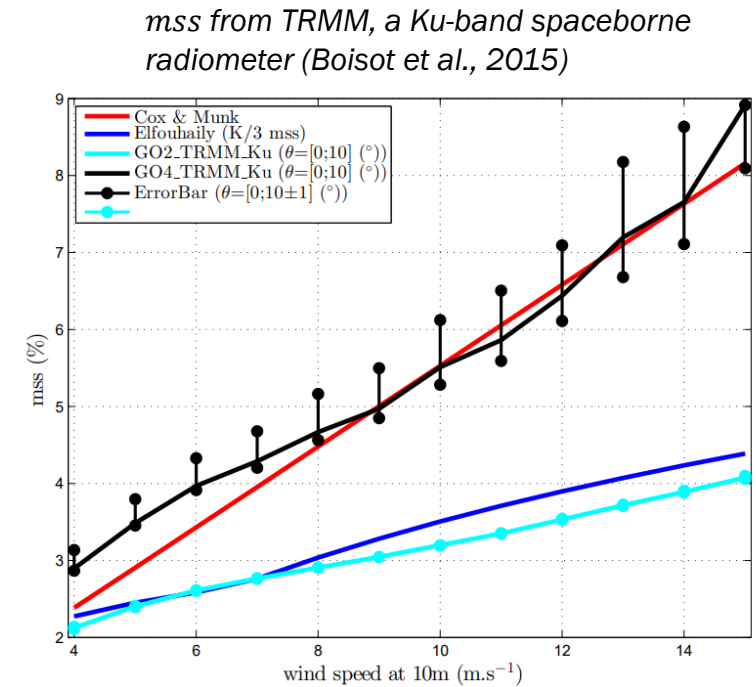
Following the Geometrical Optics (GO) theory, the  $mss$  can be analyzed from the variation of the radar cross-section ( $\sigma^0$ ) with incidence angle ( $\theta$ ) at near nadir incidences.

Several examples of  $mss$  estimation from radar data can be found on the literature

→ From Ku- and C-band airborne [Jackson et al., 1992; Walsh et al., 1998; Hauser et al., 2007]

→ From satellite observations in Ku- and Ka-band [Freilich, 2000, Boisot et al., 2015; Nouguier et al., 2016]

→ A recent study investigated the  $mss$  from CFOSAT/SWIM data [Karaev et al., 2021]



# Definitions and models from the literature

## Definitions:

- Filtered  $mss$  from radar observations (GO model)

[Jackson et al. (1992); Chapron et al, 2000, Hauser et al. (2008); Freilich and Vanhoff (2003); Chu et al. (2012), Boisot et al. (2015); F. Nouguier et al, 2016, Chen et al 2018, etc... )

- ›  $mss_{shape}$  : Only wavelength which are « seen by the radar” ( $> \sim 3 * \lambda_{radar}$ ) contribute

- Total (unfiltered)  $mss$ :  $mss_T$  : all wavelengths are considered

- › Optical measurements (Cox and Munk 1954),
- › Radar measurements using higher order backscattering models

- Dependence on wind direction

[Cox & Munk, 1954 , D. Hauser, 2007, Chu et al, 2012, Chen et al, 2018, etc]

- ›  $mss_{upwind}$  and  $mss_{crosswind}$  are  $mss$  along and across the wind direction
- ›  $mss_{total} = mss_{crosswind} + mss_{upwind}$
- ›  $mss_{omni} = \frac{1}{2} mss_{totale}$  (also called  $mss_{iso}$ )

## Empirical models considered here:

- Cox & Munk [1954] : From optical observations of sun glitter

Clean-Sea : All wave wavelengths are observed ( $mss_T$ )

$$mss_{up,clean} = 0.00316U_{10}$$

$$mss_{cx,clean} = 0.00192U_{10} + 0.003$$

$$\begin{aligned} mss_{CM,clean} &= mss_{up,clean} + mss_{cx,clean} \\ &= 0.00512U_{10} + 0.003 \end{aligned}$$

Slick-Sea : Small waves are filtered out (equivalent to  $mss_{shape}$  with  $\lambda_{min} = 0.38 \text{ m}$ )

$$mss_{up,slick} = 0.00078U_{10} + 0.005$$

$$mss_{cx,slick} = 0.00084U_{10} + 0.003$$

$$mss_{CM,slick} = 0.00156U_{10} + 0.008$$

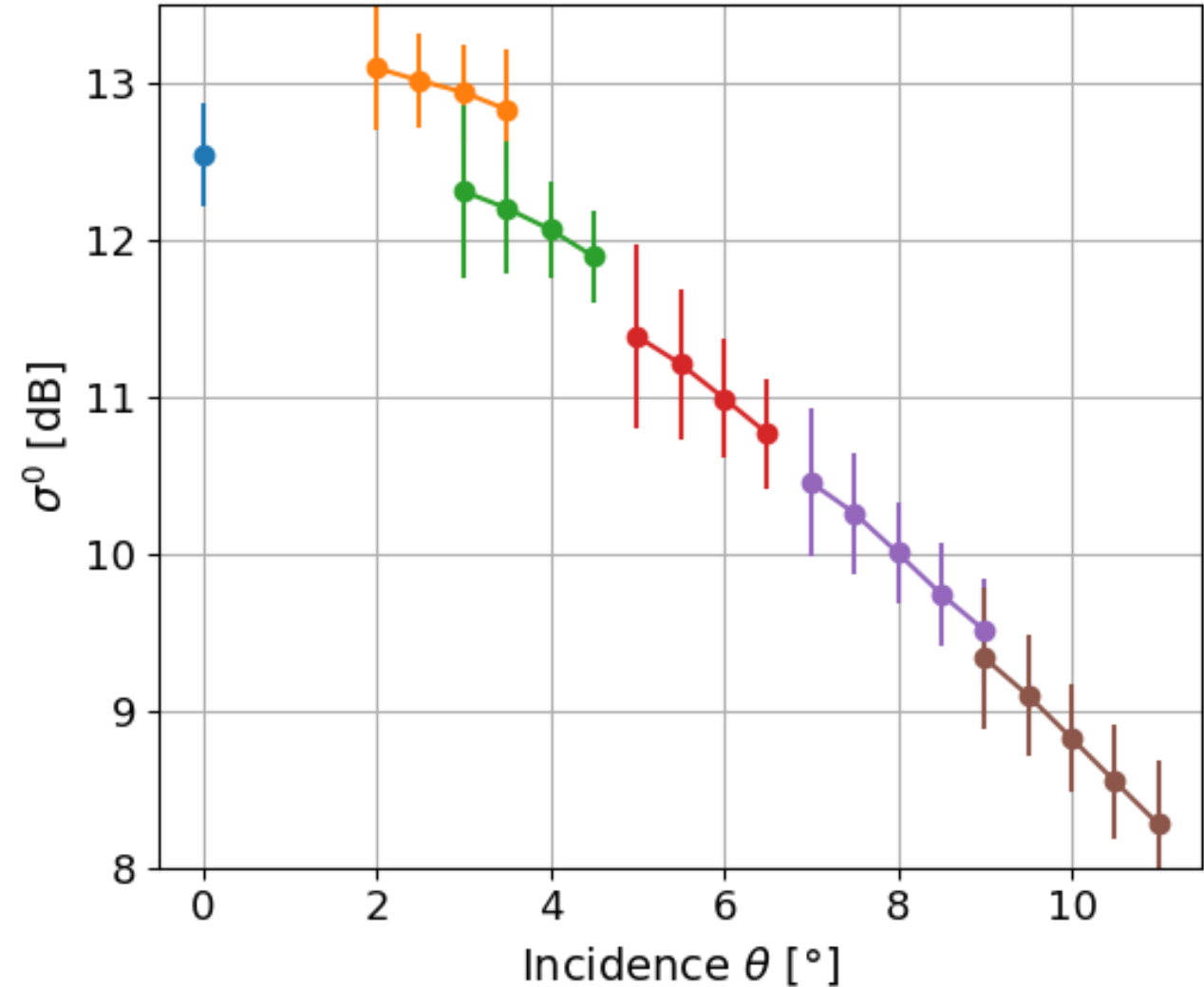
- Freilich & Vanhoff [2003] : from TRMM (Ku-band) observations ( $mss_{shape}$ )

$$mss_{FV} = 0.0016U_{10} + 0.016$$

# SWIM data for mss estimation $mss$

## Data used:

- SWIM L2  $\sigma^0$  « *mini-profiles* »,
  - 12 (one per azimuth)  $\sigma^0$  mini-profiles per box
  - Nadir to  $\sim 11^\circ$  incidence angles
  - Up to 9  $\sigma^0$  values per off-nadir beam
  - Beam  $2^\circ$  is not used in the inversion because of important inter-beam bias observed
- Data are from cycle 63 (15/01/2021 – 28/01/2021), with little to no anomalies due to microcuts
- Version V6.0 of AWWAIS processing
- Sea-ice and land data are filtered out



# Analytical models – GO

Jackson [1981]:  $\sigma^0(\theta)\cos^4(\theta) = |\mathcal{R}|^2 p(\tan(\theta))$  with  $p$  the probability density of wave slopes

**GO (or GO2) model:** Assumes a Gaussian distribution of sea surface slopes

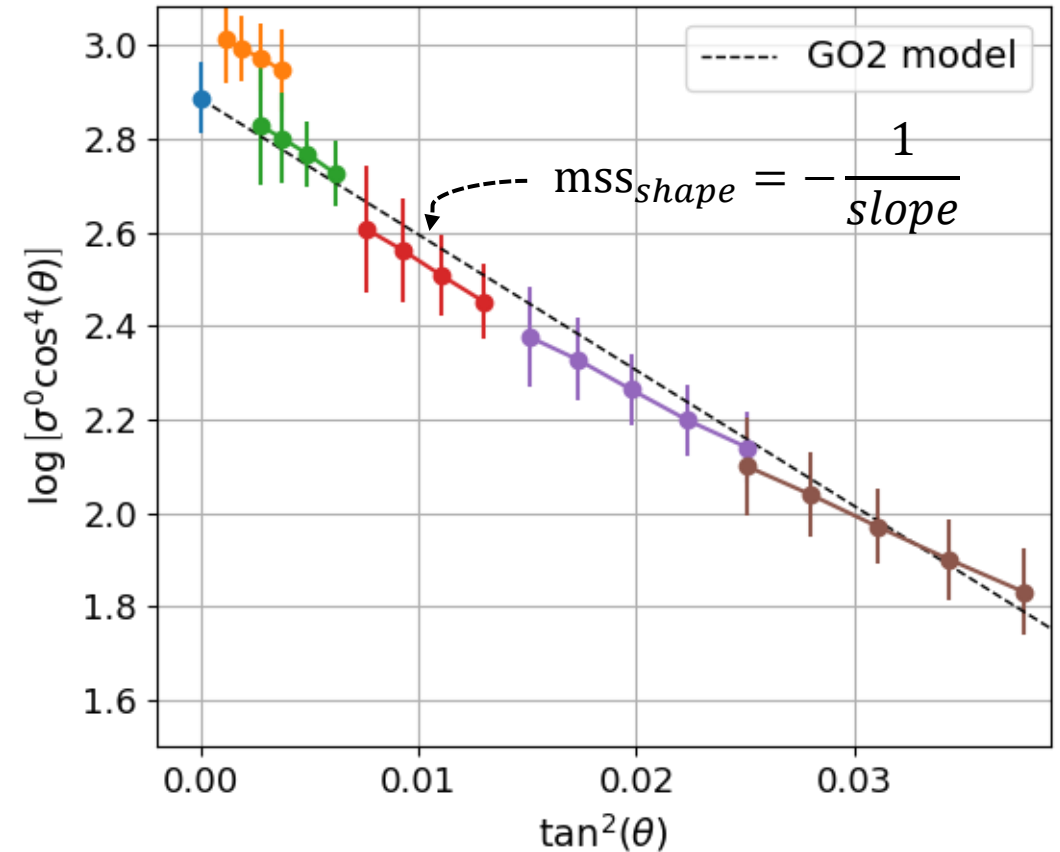
› Directional:

$$\sigma_{\text{GO2,dir}}^0(\theta, \phi) = \frac{|\mathcal{R}|^2}{\sqrt{mss_{\text{crosswind}} mss_{\text{upwind}}}} \cos^4 \theta \exp\left(-\frac{\tan^2 \theta}{2 mss_{\text{shape}}(\phi)}\right)$$

› Omnidirectional:

$$\sigma_{\text{GO2,omni}}^0(\theta; mss_{\text{shape}}) = \frac{|\mathcal{R}|^2}{mss_{\text{shape}} \cos^4 \theta} \exp\left(-\frac{\tan^2 \theta}{mss_{\text{shape}}}\right)$$

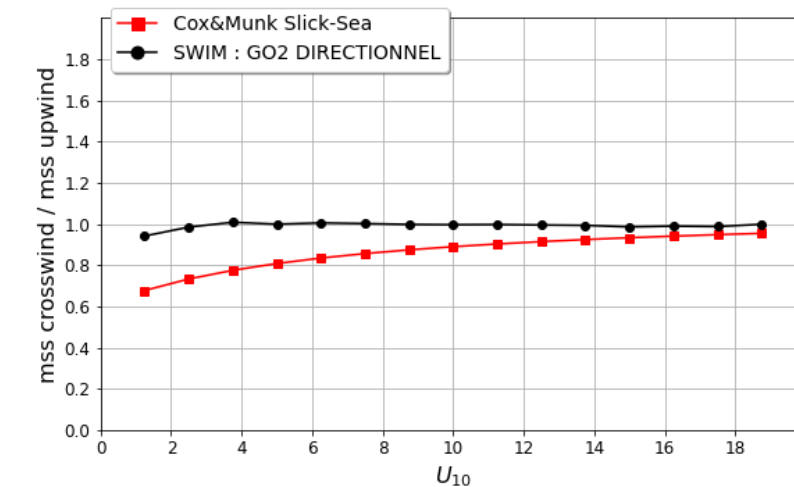
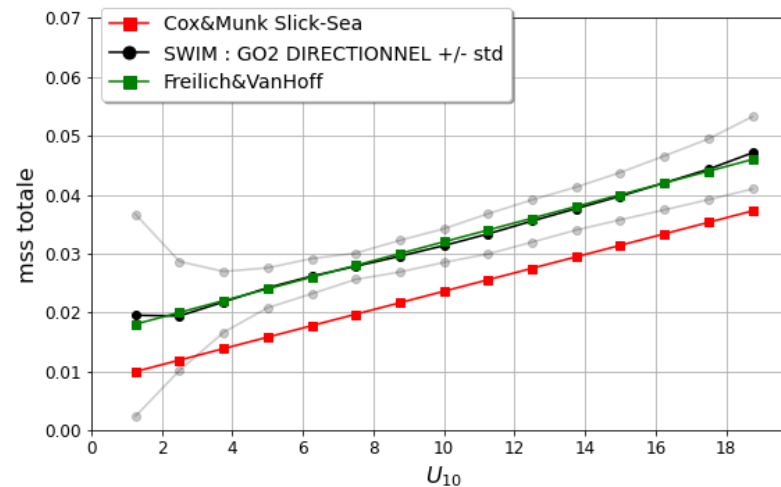
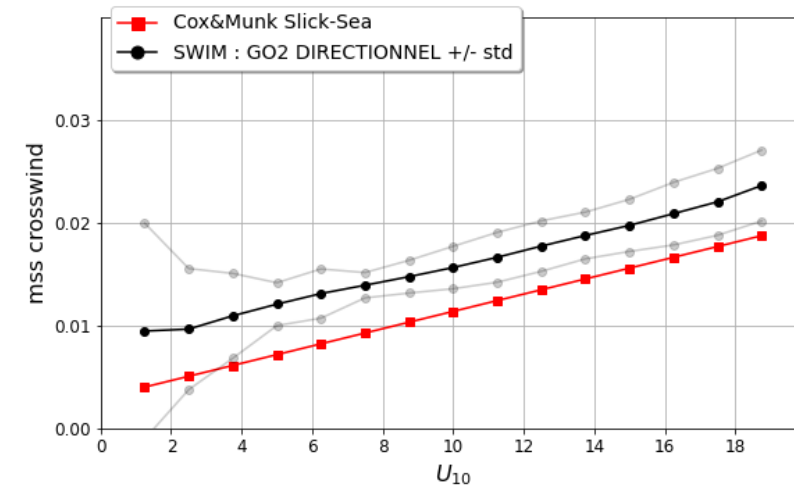
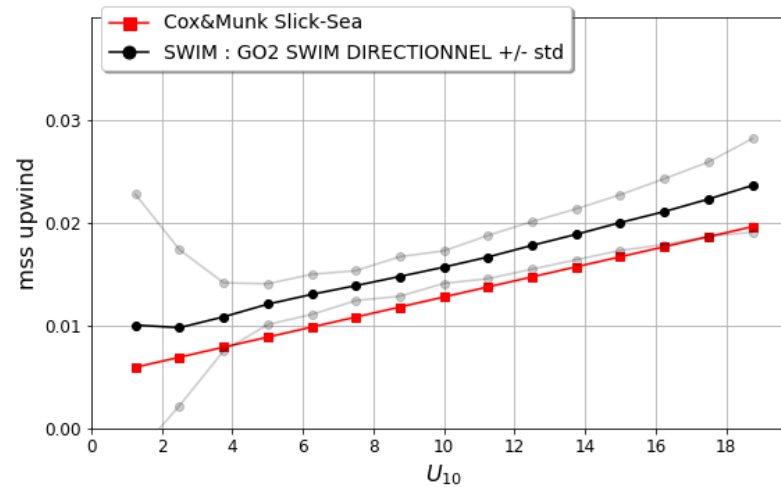
with,  $\sigma_{\text{omni}}^0(\theta) = \frac{1}{2\pi} \int \sigma_{\text{swim}}^0(\theta, \phi) d\phi$



$mss_{\text{shape}}$  can simply be estimated with a linear regression of  $\log(\sigma^0(\theta)\cos^4(\theta))$  vs  $\tan^2(\theta)$

# Analytical models – GO

- From SWIM  $\sigma^0(\phi)$  measurements and co-located ECMWF wind, we compute  $mss_{shape}$  in up- and cross-wind directions, and total  $mss_{shape}$
- Differences from Cox & Munk-slick can be explained by the different cutoff freq.  
 → C&M  $\lambda_{min} \approx 30$  cm (see Wu et al, 1972)  
 → Ku band  $\lambda_{min} \approx 3$ -6
- There are no measurable difference between up- and cross-wind  $mss_{shape}$  measured with SWIM
- Remarkable agreement between total  $mss_{shape}$  from SWIM and Freilich & Vanhoff model (same  $\lambda_{min}$ )



# Analytical models – Second order models

The G02 model provides the filtered  $mss$  ( $mss_{shape}$ ) under a Gaussian assumption of the surface. In the literature, the approaches are used to obtain the **unfiltered**  $mss_T$  from near nadir radar observations:

- Assume a Student distribution for probability density of wave slopes for wave (*Guimbard, 2010*)
- Use a higher order development of the geophysical approach (G04, *Boisot et al., 2015*)

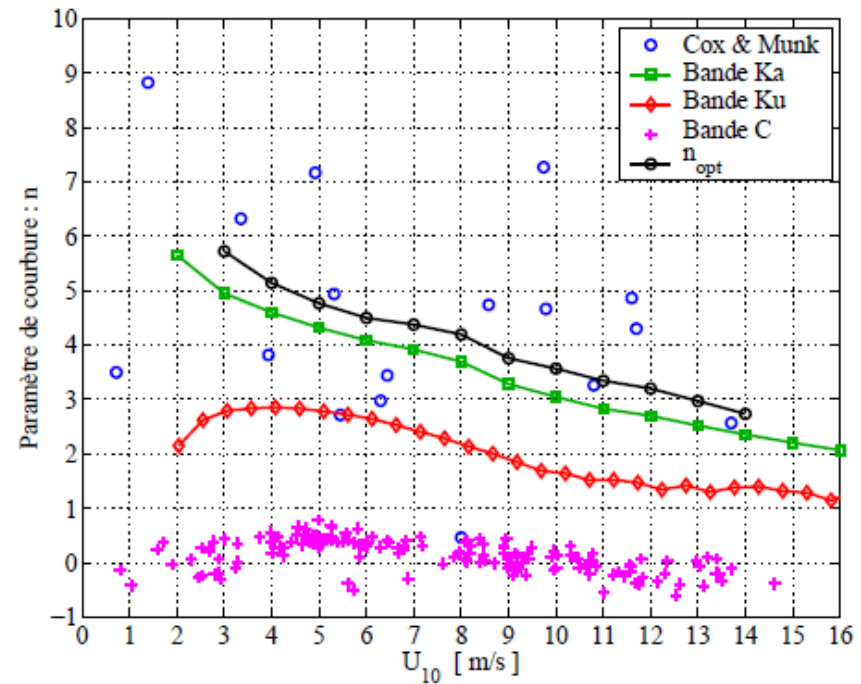
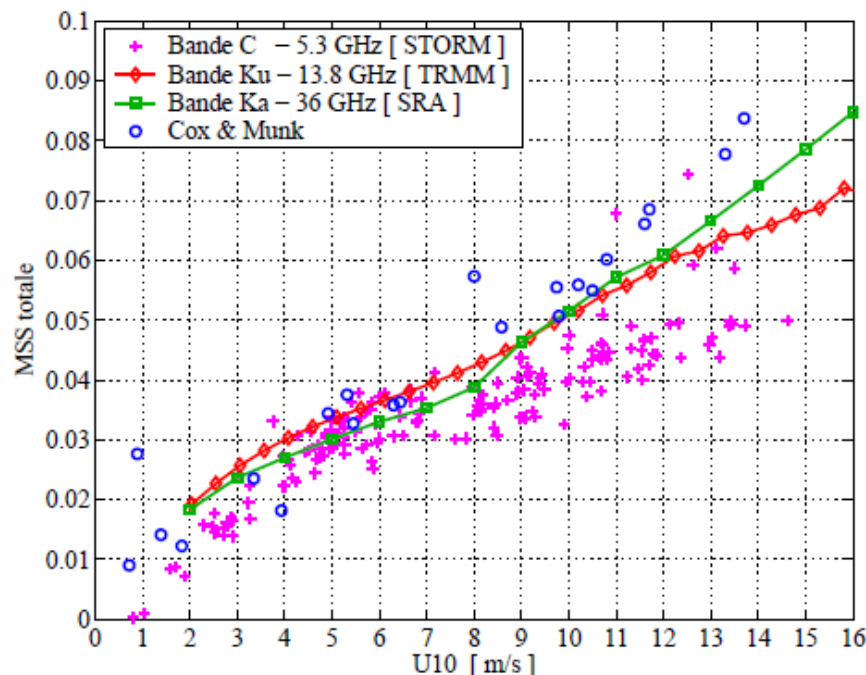
→ The Student law and G04 are two different approaches to the same concern, which could be expressed in a simple way as “accounting for the curvature effects”. In principle when accounting for this effect, the  $mss$  inverted from radar near-nadir observations should correspond to the total (non-filtered)  $mss_T$ .

# Analytical models – Second order models – Student Law

- Assume a student distribution for probability density of wave slopes for wave (Guimbard, 2010)

The goal here is to relax the assumption that wave slopes have a Gaussian PDF and use a Student distribution instead, which includes an additional curvature parameter ( $n > 0$ ) and then uses the total  $mss$  instead the shape  $mss$ :

$$p(t = \tan^2(\theta) | n, mss_T) = \frac{n + 2}{mss_T(n + 1)} \left( 1 + \frac{t}{mss_T(n + 1)} \right)^{-(n+3)}$$



# Analytical models – Second order models – Student Law

## ▪ Student distribution for probability density of wave slopes for wave (Guimbard, 2010)

› We search for the minimum of

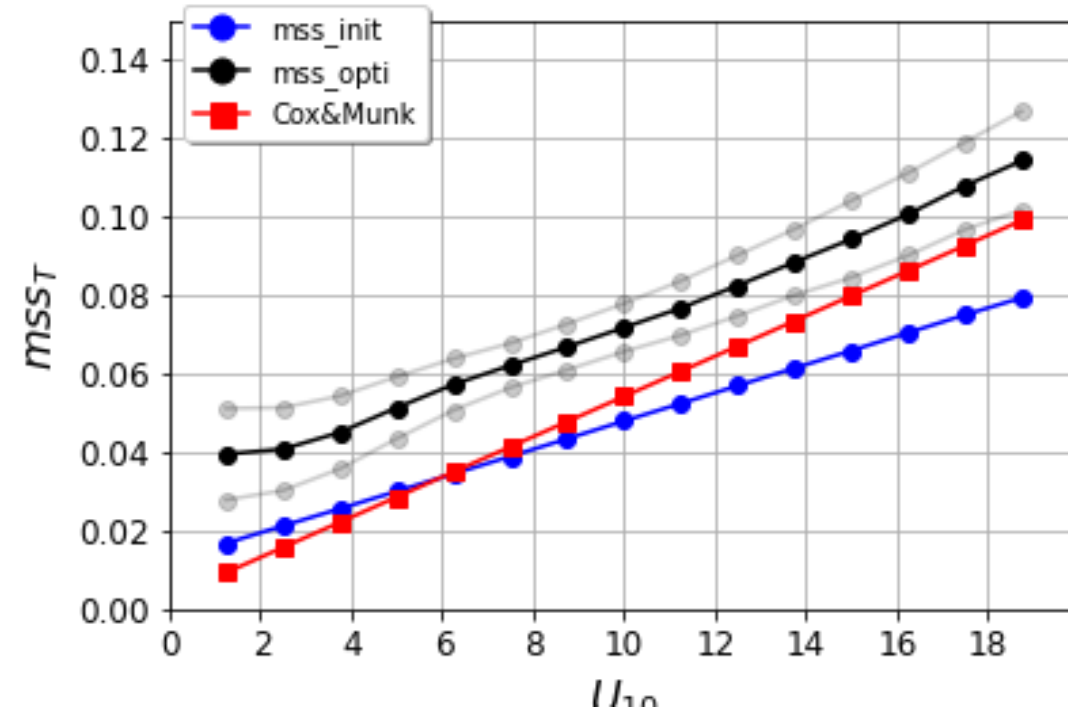
$$J(|R(0)|^2, n, mss_T | \theta) = \frac{1}{2} |\sigma_{dB}^0(\theta) - A(|R(0)|^2, mss_T, n) + 10(n+3) \log_{10} \left( 1 + \frac{\tan(\theta)^2}{mss_T (n+1)} \right)|^2$$

› We use a least square approach with the following constraints:

- Initialization
  - linear  $mss_T$  as a function of wind from Guimbard (2010)
  - $n$  follows a normal law depending on wind speed from Guimbard (2010)
  - $|R(0)|^2 = 0.6$
- Constraints
  - $0 \leq mss_T \leq 0.2$
  - $0 \leq n \leq 3$
  - $0 \leq R_0 \leq 1$

### Results:

- $mss_T$  has an expected behavior (linear increase with wind), but is overestimated compared to measurements from Cox & Munk
- $n$  does not converge properly
- Inverted  $|R(0)|^2$  values are overestimated



# Analytical models – Second order models – GO4

## ■ GO4 model omnidirectional (*Boisot et al., 2015*)

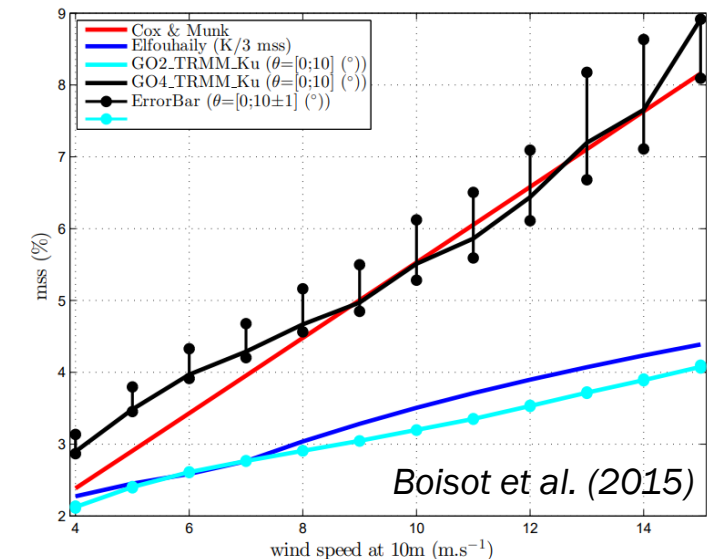
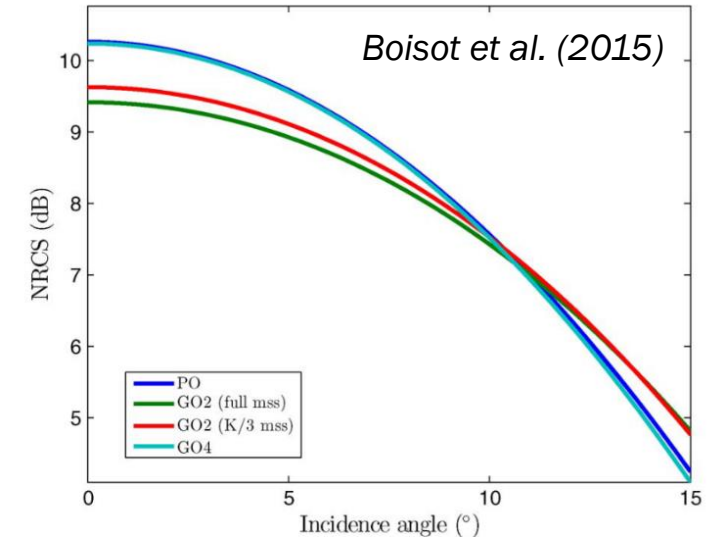
$$\sigma_{GO4,omni}^0(\theta; mss_T) = \sigma_{GO2,omni}^0(\theta; mss_T) \left[ 1 + \frac{\alpha}{4} \left( \frac{\tan^4 \theta}{mss_T^2} - 4 \frac{\tan^2 \theta}{mss_T} + 2 \right) \right]$$

with  $\alpha = \frac{mss}{Q_Z^2 mss_T^2}$  (+kurtosis param. assumed to be 0)

→ Better description of near-nadir microwave scattering from the sea-surface, with an accuracy comparable to the PO model.

→ Adds an effective “mean square curvature”  $mss$  (but ill-defined)

→ It uses the omnidirectional  $\sigma_{omni}^0$ . The directional  $mss$  is outside the scope of this study



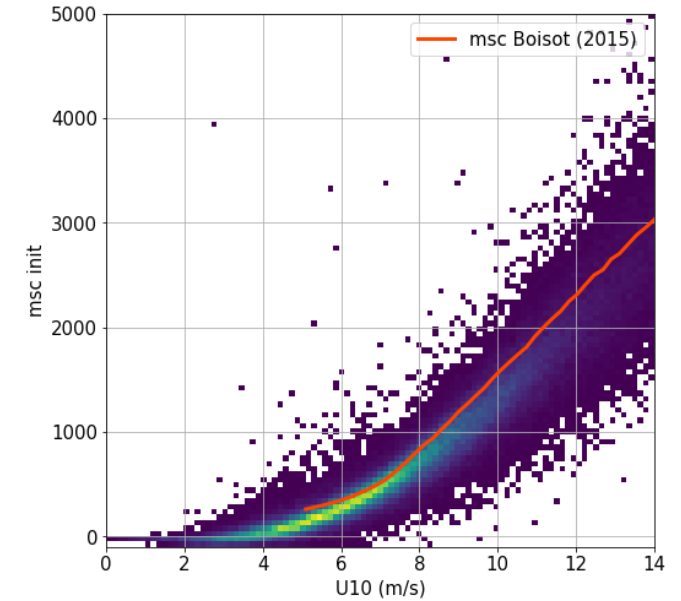
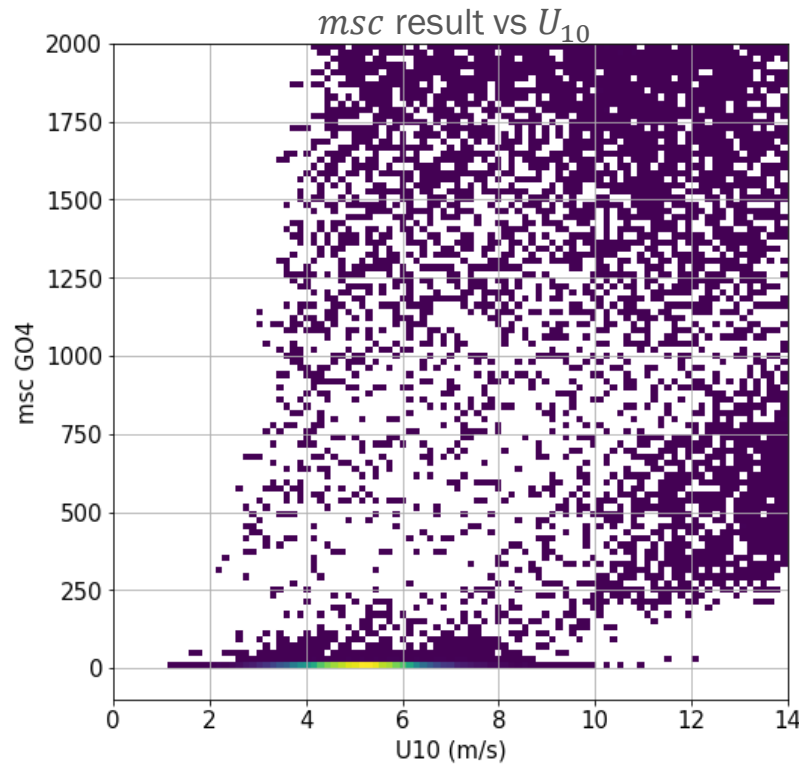
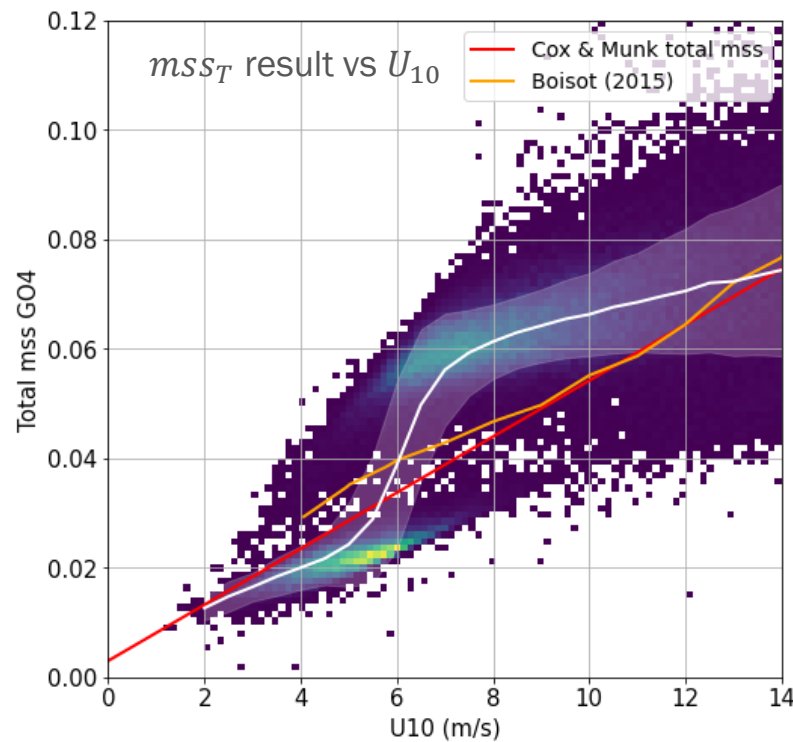
# Analytical models – Second order models – G04

## Inversion approach #1

→ Estimation is made by minimizing  $\sigma_{omni}^0 - \sigma_{G04}^0(\theta, mss, msc)$  with a least-square approach

→ As a first-guess we use  $mss_{CM}$  and  $msc_{init} = 8 \frac{2\pi}{\lambda} mss_{CM}^2 \left( \frac{\sigma_{nadir}^0 mss_{CM}}{R^2} - 1 \right)$

→ Inversion of two parameters :  $mss_T$  and  $msc$



- Poorly converging inversion
- $msc$  constrained to lower bound
- Two regimes of  $mss_T$ , not clearly understood

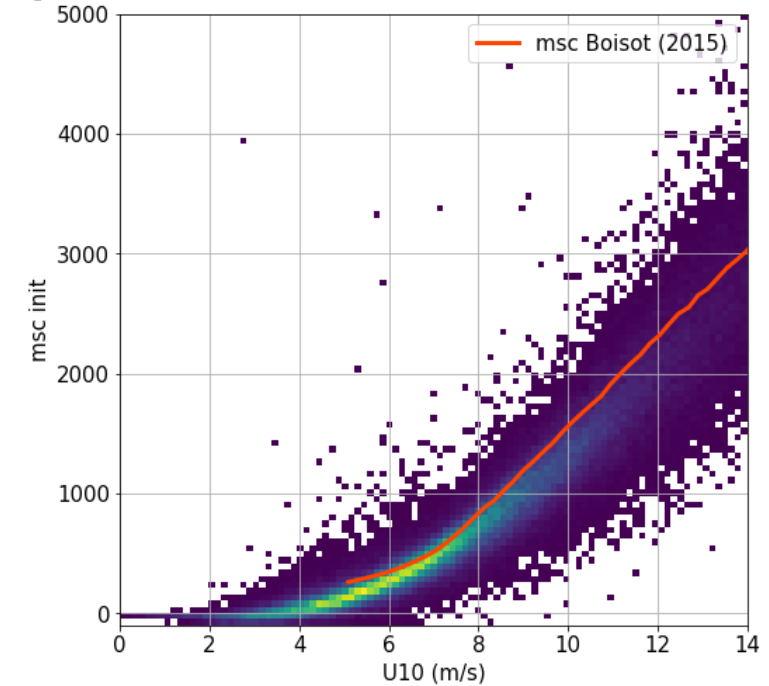
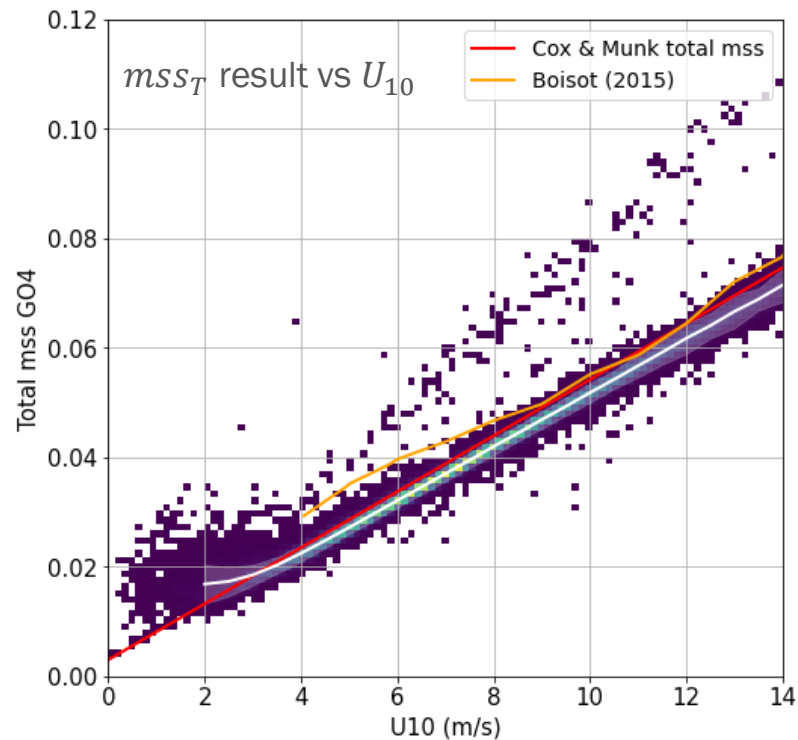
# Analytical models – Second order models – G04

## Inversion approach #2

→ Estimation is made by minimizing  $\sigma_{omni}^0 - \sigma_{G04}^0(\theta, mss, msc)$  with a least-square approach

→ As a first-guess we use  $mss_{CM}$  and  $msc_{init} = 8 \frac{2\pi}{\lambda} mss_{CM}^2 \left( \frac{\sigma_{nadir}^0 mss_{CM}}{R^2} - 1 \right)$

→ Inversion of one parameters :  $mss_T$  .  $msc$  is fixed to  $msc_{init}$



- Not ideal, as  $msc$  depends on  $mss_{CM}$  ...
- ... but significantly improved  $mss_T$  estimation
- Above 3 m/s wind speeds, remarkable agreements between TRMM estimates (Boisot et al., 2015) and Cox & Munk empirical model

# Conclusions

We demonstrated the capability to estimate, to some degrees, different values of  $mss$  with SWIM L2  $\sigma^0$  data

## $mss_{shape}$ (filtered $mss$ )

- › We find a solid estimation of total and directional  $mss_{shape}$  using the G02 model and a linear regression.
- › The inferred values are very consistent with the literature

## $mss_T$ (unfiltered $mss$ )

- › Student distribution for wave slopes PDF is a promising lead, that needs to be investigated further. At this stage, inversion offers reasonable  $mss$  values. There seem to be a bias compared to Cox & Munk measurements, but the dependance on wind speed is comparable.
- › Using the G04 model gives very good  $mss_T$  results, but with a strong assumption on  $m_{sc}$  which is fixed in the inversion. The inferred  $mss_T$  strongly agrees with TRMM values from *Boisot et al., (2015)* and optical measurements from *Cox & Munk (1953)*

→ For these finer models and complete inversions of all parameters ( $mss$ ,  $m_{sc}$ ,  $n$ ), we are likely reaching the current limit of  $\sigma^0$  mini-profile accuracy.

Solutions include either a selection of “high-quality” mini-profiles (with TBD criterions) and/or improvements in radiometric calibration

➔ Possibility to includes  $mss_{shape}$  and/or  $mss_T$  to SWIM L2 products

# BACKUP

# Student law vs G04

The G04 model uses a 4<sup>th</sup> order development of the correlation (instead of 2<sup>nd</sup> order for G02)

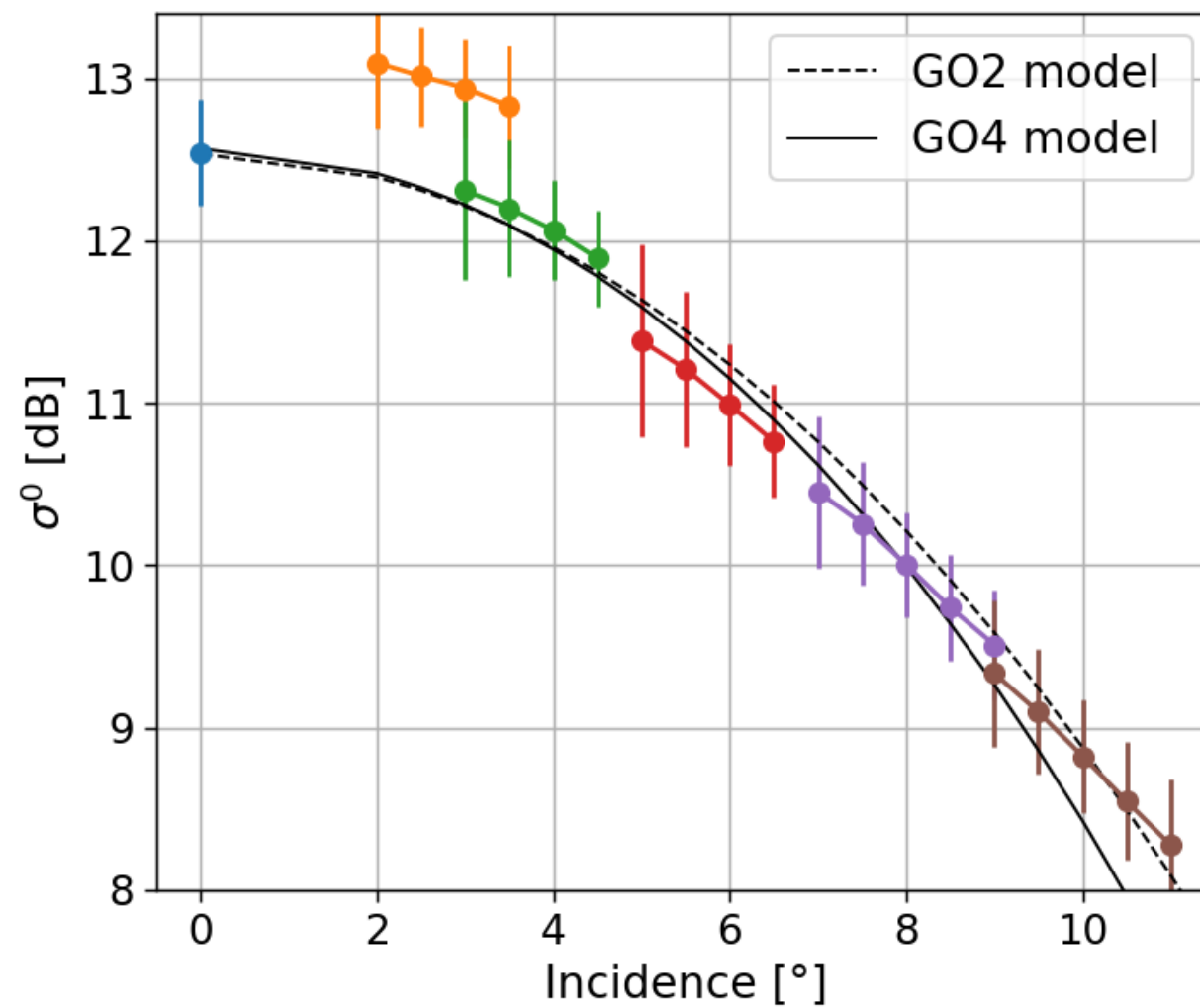
G02: 
$$\rho(r) = \rho(0) - \frac{mss_{shape}}{2} r^2 + o(r^2)$$

G04: 
$$\rho(r) = \rho(0) - \frac{mss_T}{2} r^2 + \frac{msc}{32} r^4 + o(r^4),$$
 msc is the spectral moment of order 4, related to the curvature.

(Guimbard, 2010) identified the Kurtosis of the student law,  $\kappa_4 = \frac{2mss_T^2}{n}$ , to the term  $\frac{msc}{(2K \cos \theta)^2}$ , arising from the use of the 4<sup>th</sup> order correlation while solving the Kirchhoff Integral ( $\propto \sigma^0$ ).

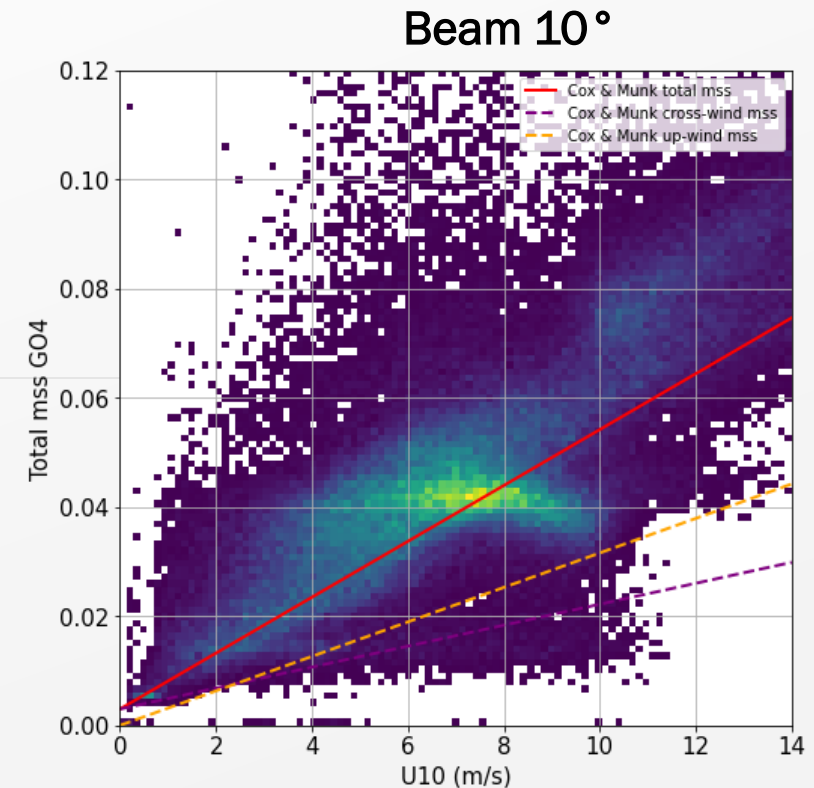
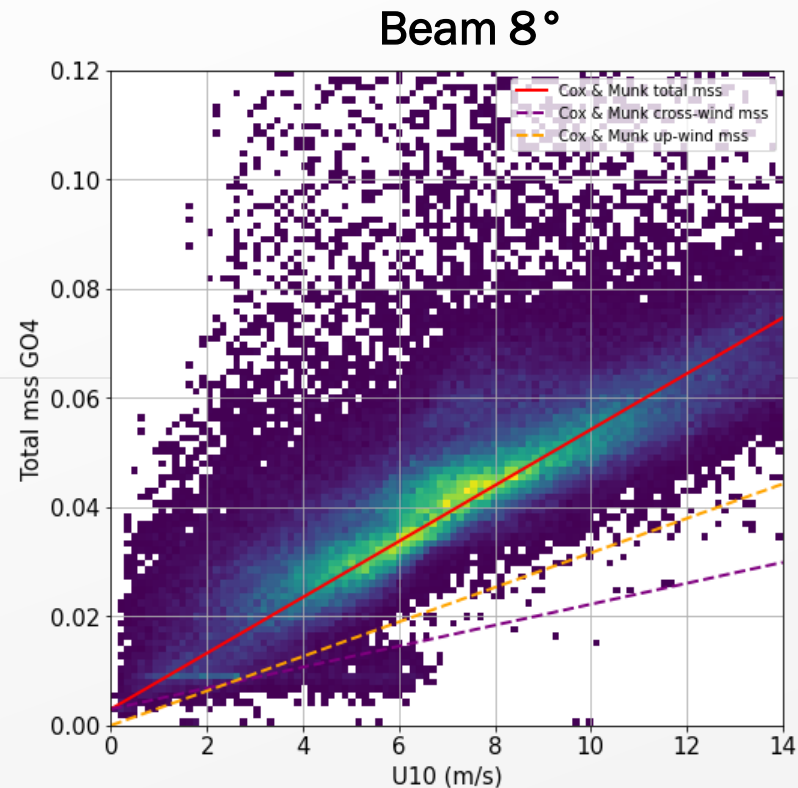
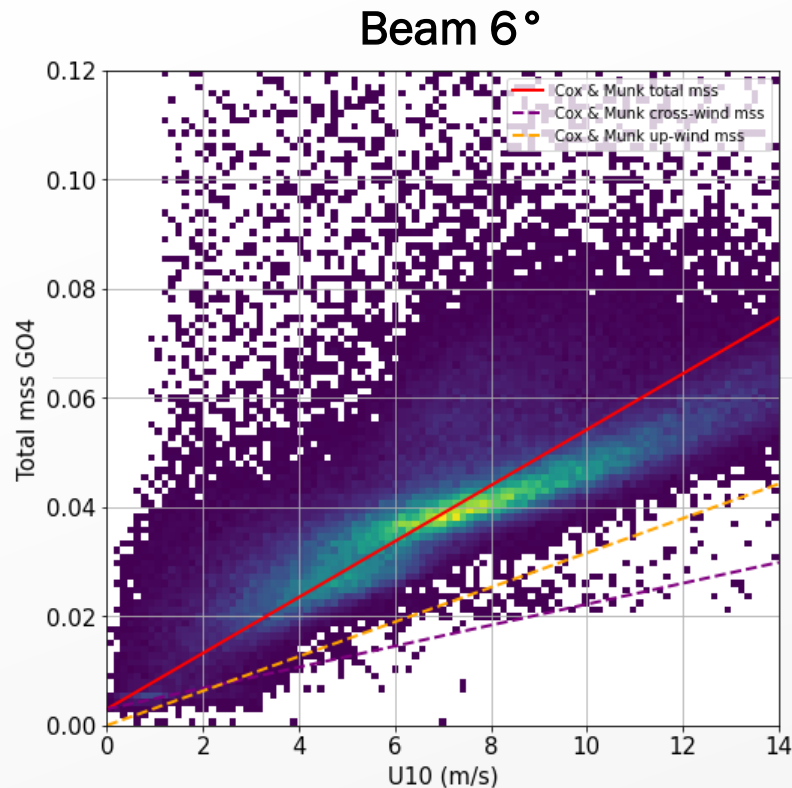
The G04 parameter  $\alpha = \frac{msc}{(2K \cos \theta)^2 mss_T^2}$  and the parameter  $n$  of the Student law are then straightforwardly related.

Note: Using the 2<sup>nd</sup> order expansion of  $\rho(r)$  and solving Kirchhoff Integral is strictly equivalent to choosing a Gaussian distribution for the probability density of wave slopes (Jackson, 1981: Physics Optical (PO) model). Therefore, the terminology G0(2) is often used, by abuse of language, to refer to the PO model under Gaussian assumption.



# Calcul de $mss$ à partir de données $\sigma^0$ SWIM

Calcul de  $mss_T$  isotrope à partir de données  $\sigma^0_{swim,iso}(\theta)$  et modèle GO4



— Cox & Munk

