An analytical method to propagate errors in the Altimeter system

P. Exertier, P. Bonnefond, N.D. Luong

OMP/GRGS, OP/SYRTE, France, NUCE University, Vietnam

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Introduction

- Altimetry needs accurate geodetic products like: the gravity field and reference frame

=> Analysing the errors is something like understanding the transfer of errors from their origin(s) to the result (e.g. the altitude of the sea surface above a reference)

- Numerical methods are extensively used to determine a solution which is constrained by different models (or approaches)

But it is necessary to evaluate the quality, from external sources, etc. => characterize the errors in time&space

- Analytical methods have the property of providing direct relations (equations) between input parameters (ε) and the solution; e.g. :

orbit (λ,ϕ, t) = fcnt(λ,ϕ, ε(geophysics, satellite, etc. , t))

They have been used to characterize the Radial Orbit Error(s), using the approach of Kaula, Rosborough, etc. (see *Kaula, Balmino, Colombo, Engelis, Rosborough, Scharroo, Schrama, Tapley, Visser, and Wagner*.

=> radial orbit errors coming from the gravity field essentially, in view of understanding the limits of the altimetry system (see e.g. the so-called Geographically Correlated Errors)

- Here, we develop an analytical approach which is dedicated to the transfer of errors **from the orbit** to geodetic products: the sea surface topography is a first target, before studying the stability of tracking station coordinates.

=> Inversely, we are not able to study the impact, on the orbit, of different error sources or approaches.

Method

- In addition to the 6 classical keplerian elements (aei…), as orbital parameters, we use a set of spherical coordinates (r'ϕ'λ', strictly dedicated to circular motion) in a privileged frame:

  => the precessing mean orbital plane which is oriented by (i) and Ω(t) given along the equator

d aei /dt = L(aei) δU / δ aie => 6 equations => Kaula solution

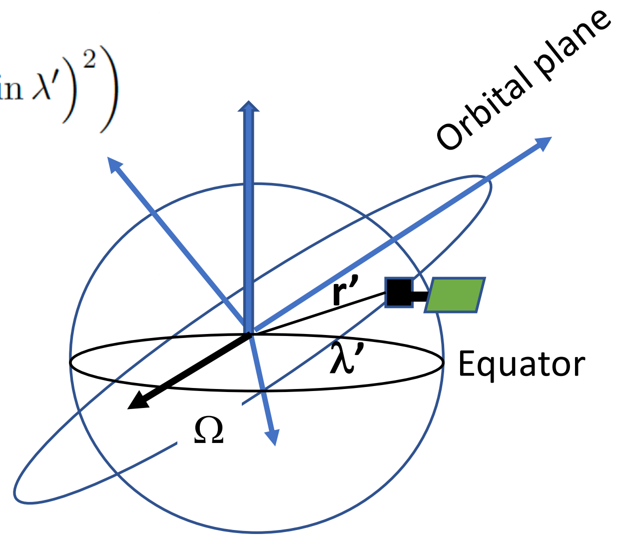
d² r'ϕ'λ' /dt² = G(r'ϕ'λ' ; r. ϕ. λ.) δU / δ r'ϕ'λ' => 3 equations of the second order in t => r'ϕ'λ' solution

$$T = \frac{1}{2}\left(\dot{r'}^2 \;+\; (r'\,\dot{\varphi}' - r'\dot{\Omega}\sin i\cos\lambda')^2 \;+\; \left(r'\cos\varphi'\,(\dot{\lambda}' - \dot{\Omega}\cos i) - r'\sin\varphi'\,\dot{\Omega}\sin i\sin\lambda'\right)^2\right)$$

$$\text{with}:\; L = T + U(r', \varphi', \lambda';\; i, \Omega)$$

$$\begin{cases} r' = r'_0 + \varepsilon r'_1 \\ \varphi' = 0 + \varepsilon\varphi'_1 \\ \lambda' = n(t - t_0) + \varepsilon\lambda'_1 \end{cases}$$

$$\begin{cases} r'_1 = \sum_{k=1}^{L} \dfrac{r_1^{ck} cos\,\psi_k + r_1^{sk} sin\,\psi_k}{\dot{\psi}_k} \\ \varphi'_1 = \sum_{k=1}^{L} \dfrac{\varphi_1^{ck} cos\,\psi_k + \varphi_1^{sk} sin\,\psi_k}{\dot{\psi}_k} \\ \lambda'_1 = \sum_{k=1}^{L} \dfrac{\lambda_1^{ck} cos\,\psi_k + \lambda_1^{sk} sin\,\psi_k}{\dot{\psi}_k} \end{cases}$$



Method-II

- the gravity field of the solid earth, and its time variable components

$$U_{n,m} = \frac{GM}{r}(\frac{ae}{r})^n P_{n,m}(\sin\phi)(C_{n,m}\cos m\lambda + S_{n,m}\sin\lambda)$$

where: $(C/S)_{n,m} = (C/S)_0 + (t - t_{ref})(C/S)_t + (C/S)_{\text{annual}} + (C/S)_{\text{semi-annual}}$

- the ocean tides (11 waves)

$$U^{+-}_{f,n,m} = \sum_f \sum_{+-} \frac{4\pi Gae\rho}{g}\left(\frac{ae}{r}\right)^n \left(\frac{2n+1}{1+k'_n}\right) P_{n,m}(\sin\phi)(C^{+-}_{f,n,m}\cos m\lambda + S^{+-}_{f,n,m}\sin\lambda)$$

- the solar radiation pressure considered as a field (U_{SRP})

$$\begin{aligned} U_{SRP} &= -\sigma\; r\; P_{1,0}(\cos\Psi) \\ \cos\Psi &= \sin\varphi\;\sin\varphi_\odot\; + \;\cos\varphi\;\cos\varphi_\odot\;\cos(\alpha_\odot - \lambda) \end{aligned}$$

with: $\sigma = \frac{S}{m}.Cr.C_{sol}$

and r is the Sun-Earth direction $(\varphi_\odot, \alpha_\odot)$, and from which $\partial U_{SRP}/\partial r$ leads to the well-known acceleration.

The analytical solution, here, is used to propagate (in time):

δ(Cnm,Snm, Cr, ...)

instead of

(Cnm,Snm, Cr, ...)

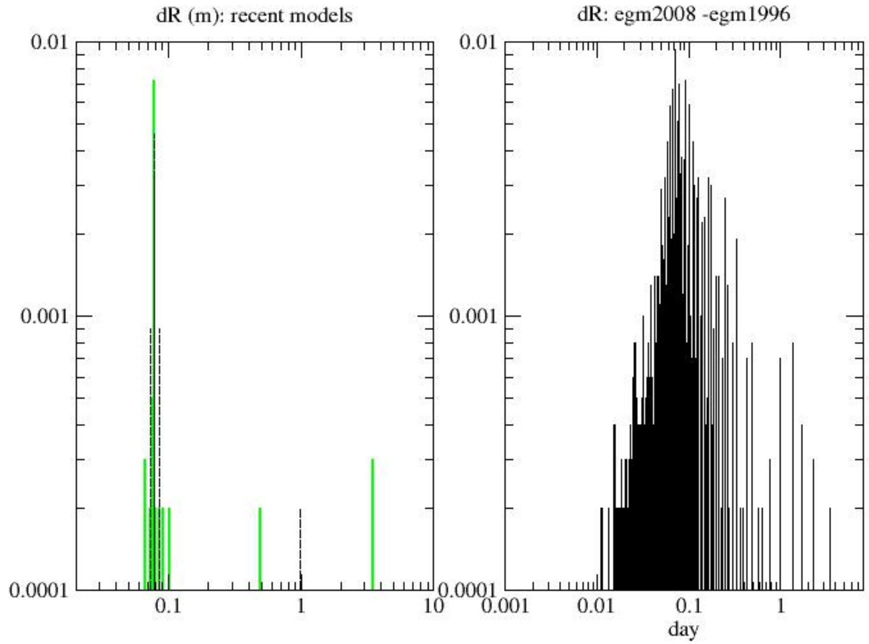
The "solution" directly gives orbit errors δ r′ϕ′λ′ (t) (equivalent to δ RNT_{sat})

Orbit dynamics

- One of the important issue is to characterize the ''errors'' **δ(Cnm,Snm, Cr, ...)** used here, notably:
  - δ(C_nm, S_nm) for the gravity field
  - δ(C_fnm, S_fnm /ocean wave f)
  - δ(Cr, S/m) for the Solar Radiation Pressure

- Gravity field model (constant and variable parts)
  - Variance/Covariance matrix
  - Differences between 2 models
  - Variable part of the geopotential (essentially annual and semi-annual terms)

  - Influence of the model on precise orbits of altimetry satellites; see for recent papers: *Couhert, Rudenko, Zelensky*
  - X-over analysis is used since years (see: *Rosborough, Engelis, Schrama*)



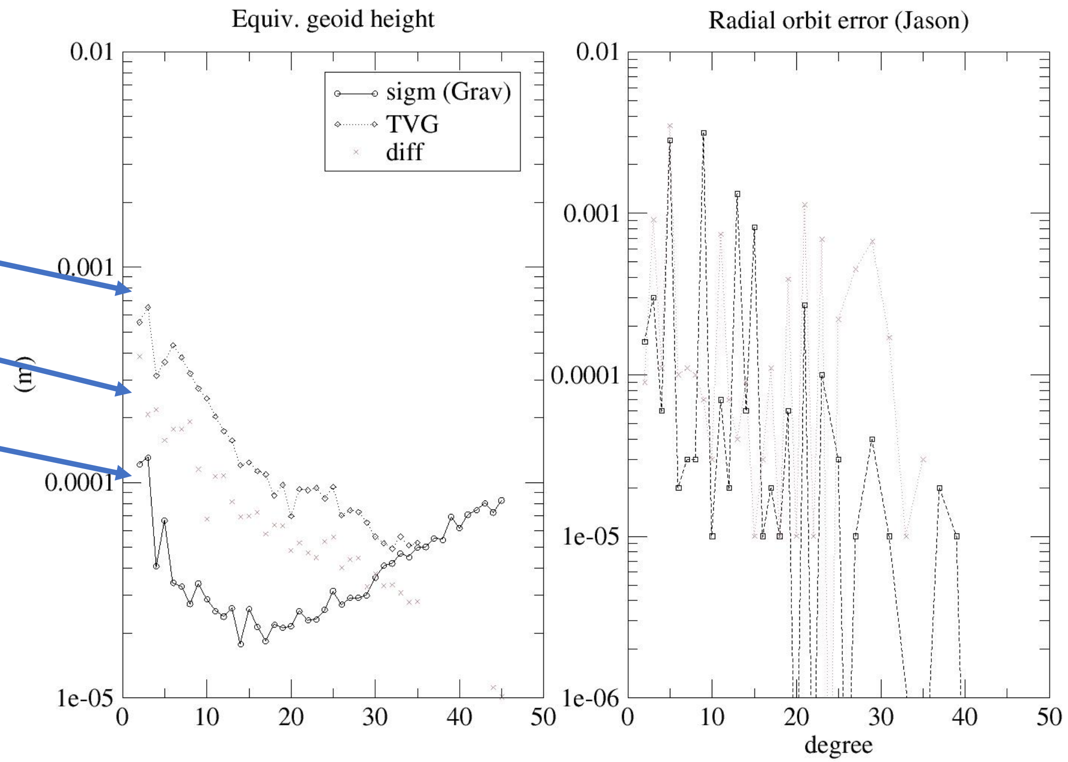
Orbit dynamics-II

Gravity field model (constant and variable parts)

Variable part of the geopotential, essentially annual and semi-annual terms: grgs-RL02

Differences between 2 models: eigen-GL04s1 and grgs-RL03-v2 (GDR-E products)

Variance/Covariance matrix (calibrated)



In addition:

degree 1 : δC_10, δC/S_11 ⇔ a few mm: 1.9, 1.3/1.5 at annual period

Solar Radiation Pressure

δ Cr : a few %

δ S/m / (S/m) : 0.007 with [Ω –Sun-longitude] (t)

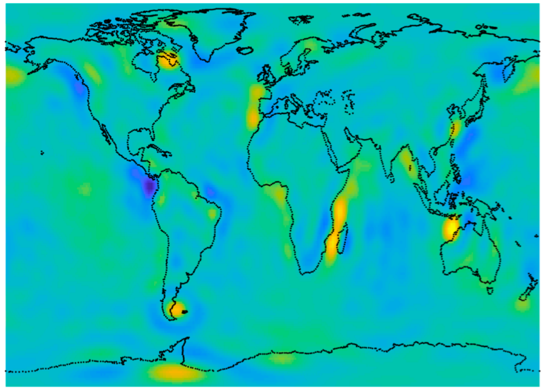
Orbit dynamics-III

Ocean tide model (e.g. FES and GOT):

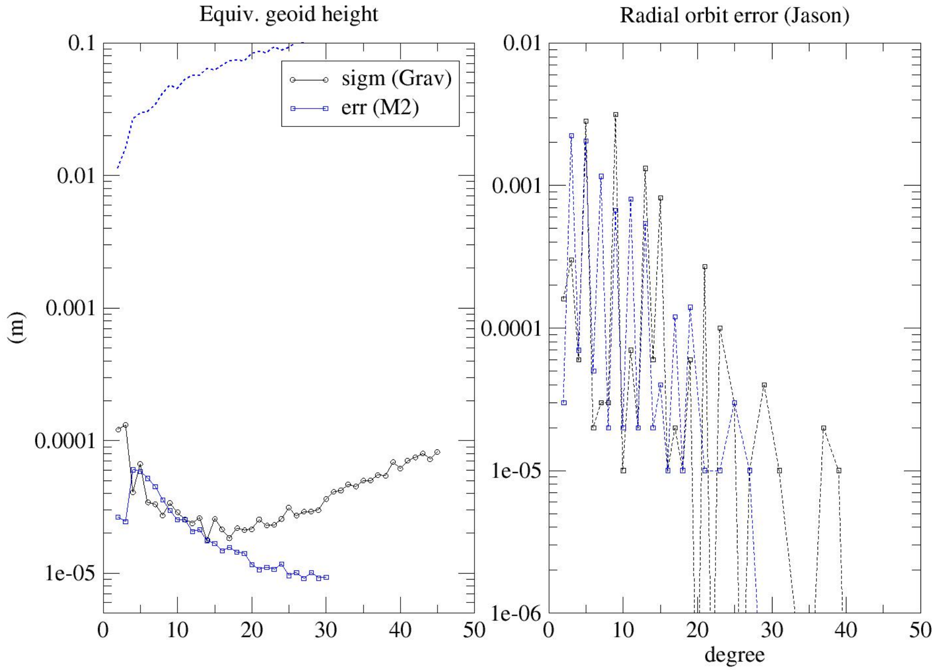
=> only part of discussions about orbit errors (see, however, the work of *Bettadpur & Eanes*)

		M2
Amplitude		~100 cm
Accuracy	< 1995	2.87 cm
GOT.99v2		1.47
CSR 4.0		1.64
FES 95		1.65
FES2012		1.30 cm
# FES2012 - 2014		< 1%

=> tide deep-ocean tide gaude
⇒ GRACE
Ray, Lyard: 2019, 2014



"errors" in water height => ± 8 mm



11 waves (d/o: 30)

Sa,Ssa,Mf
Q,O,P,S,K : diurnal
N,M,S,K : semi-diurnal

Models and tests

Model 1

eigen-gl04s1 : variance of SH coefficients, d/o 45

Radial orbit error (mm):

Total: mean:	0.0 ± 0.42	mean of RMS:	8.1
X-over: mean:	0.0 ± 0.2	… …	1.0
Asc/Desc: mean:	0.0/0.0 ± 0.4	… …	8.0

Model 2

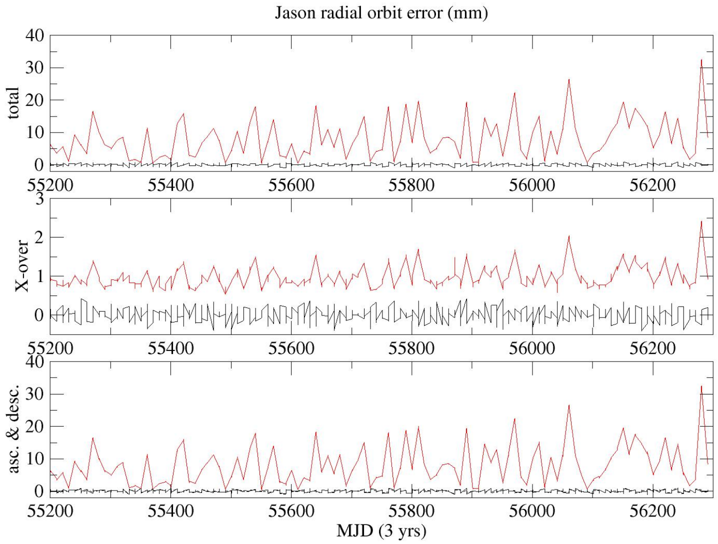
eigen-gl04s1 : variance of SH coefficients, d/o 45

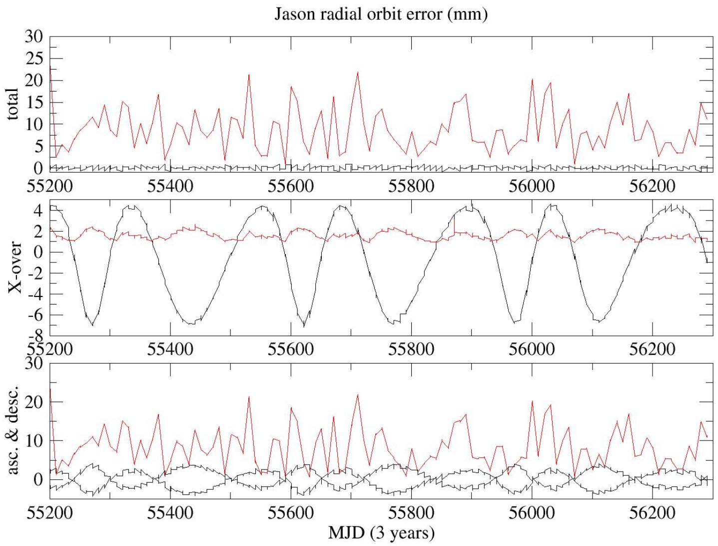
C/S11: 1.3/1.5 C10: 1.9 (annual, in mm)

SRP: δ S/m /(S/m) + δ Cr/Cr = 0.00694

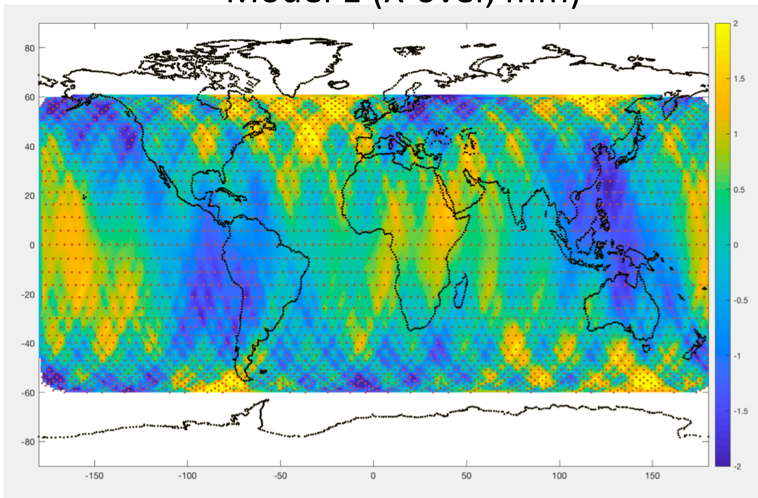
Radial orbit error (mm):

Total: mean:	0.0 ± 0.43	mean of RMS:	8.7
X-over: mean:	-0.6 ± 3.9	… …	1.6
Asc/Desc: mean:	-0.3/+0.3 ± 2.0	… …	8.3

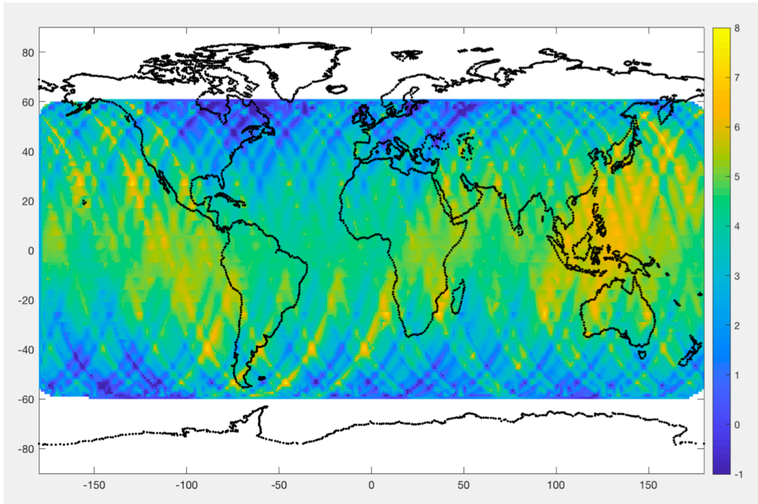




Model 1 (X-over, mm)



Model 2 (X-over, mm)



Model 1

eigen-gl04s1 : variance of coefficients, d/o 45

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Model 2

eigen-gl04s1 : variance of coefficients, d/o 45

C/S11: 1.3/1.5 C10: 1.9 (annual, in mm)

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Total: mean:	0.0 ± 0.43	mean of RMS:	8.7
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Asc/Desc: mean:	-0.3/+0.3 ± 2.0	… …	8.3

Models and tests-II

Model 3

eigen-gl04s1 : variance of SH coefficients, d/o 45
ocean tides: 11 waves (o/ 0, 1, 2 & d/ 30)

Radial orbit error (mm):

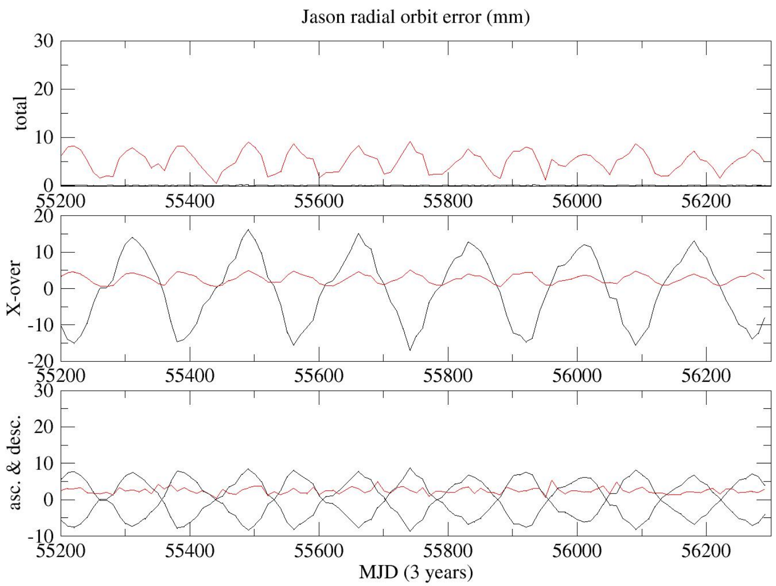
Total: mean:	0.0 ± 0.1	mean of RMS:	5.0
X-over: mean:	-0.7 ± 9.0	… …	2.6
Asc/Desc: mean:	-0.3/+0.3 ± 4.8	… …	2.4

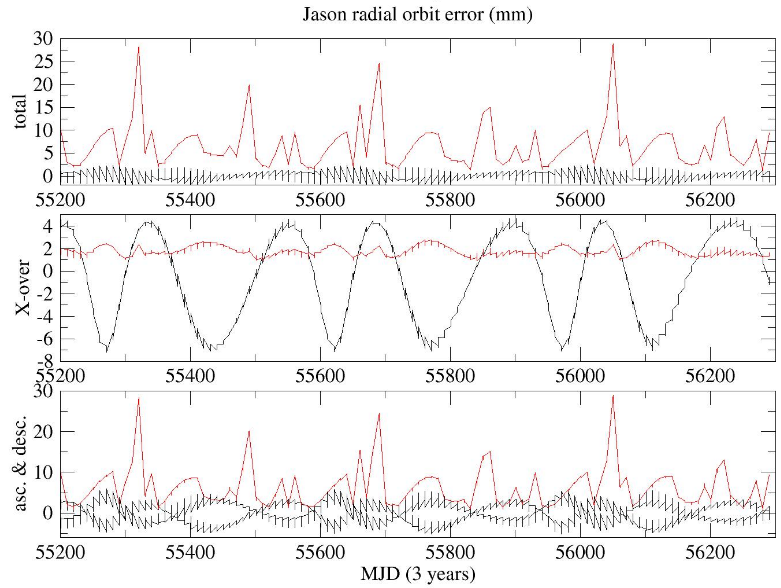
Model 4

grgs-RL02_MF : time var. coefficients, d/o 45
C/S11: 1.3/1.5 C10: 1.9 (annual, in mm)
SRP: δ S/m /(S/m) + δ Cr/Cr = 0.00694

Radial orbit error (mm):

Total: mean:	0.0 ± 1.2	mean of RMS:	6.7
X-over: mean:	-0.6 ± 4.0	… …	1.7
Asc/Desc: mean:	-0.3/+0.3 ± 2.4	… …	6.3





Model 3

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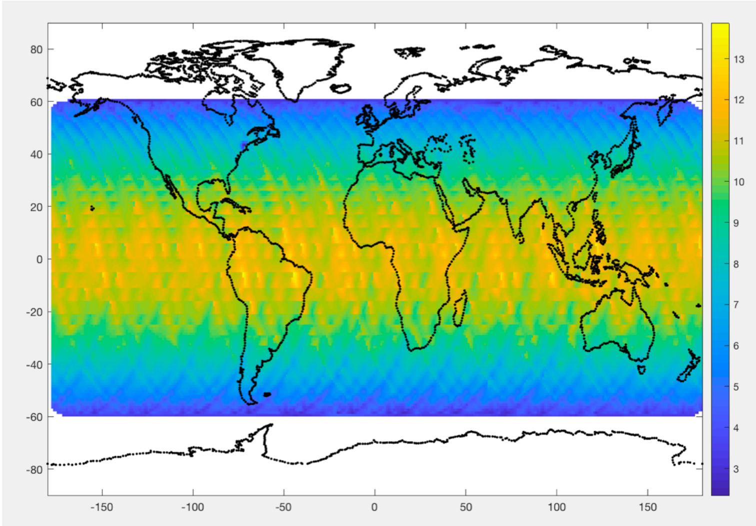
Radial orbit error (mm):

Total: mean:	0.0 ± 0.1	mean of rms: 5.0
X-over: mean:	-0.7 ± 9.0	… …: 2.6
Asc/Desc: mean:	-0.3/+0.3 ± 4.8	… …: 2.4

Model 3 (X-over, mm)



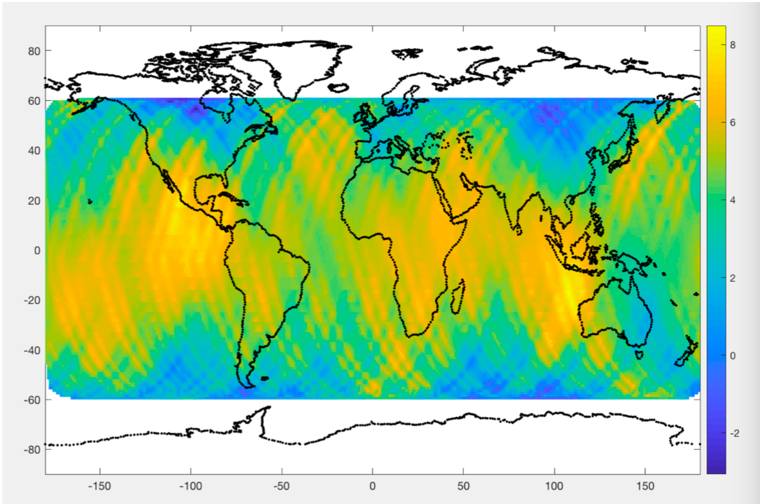
Model 4

grgs-RL02_MF : time var. coefficients, d/o 45
C/S11: 1.3/1.5 C10: 1.9 (annual, in mm)
SRP: δ S/m /(S/m) + δ Cr/Cr = 0.00694

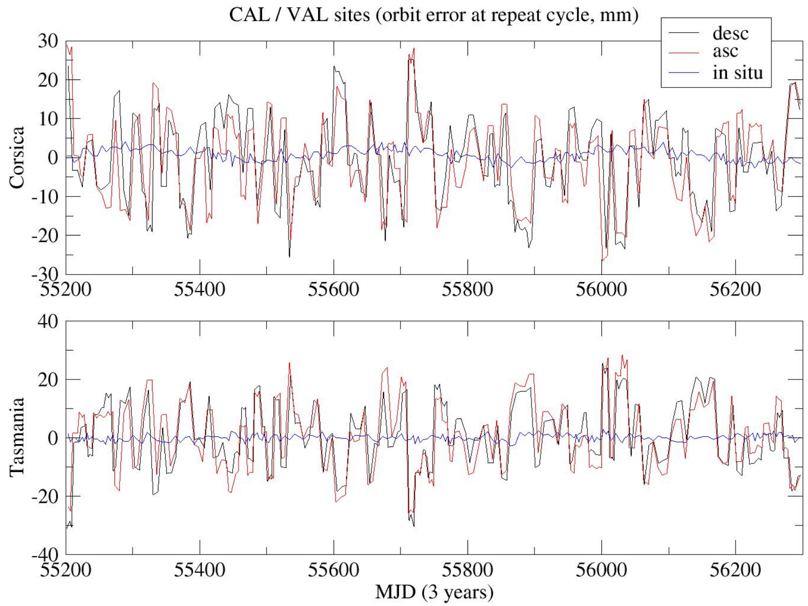
Radial orbit error (mm):

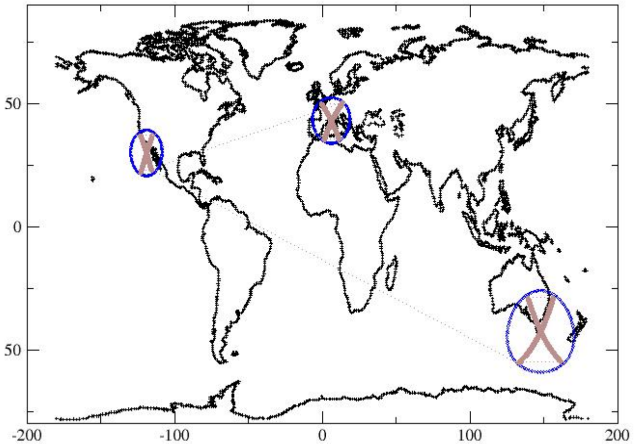
Total: mean:	0.0 ± 1.2	mean of rms: 6.7
X-over: mean:	-0.6 ± 4.0	… …: 1.7
Asc/Desc: mean:	-0.3/+0.3 ± 2.4	… …: 6.3

Model 4 (X-over, mm)



Radial orbit errors at CAL/VAL sites





Model 2

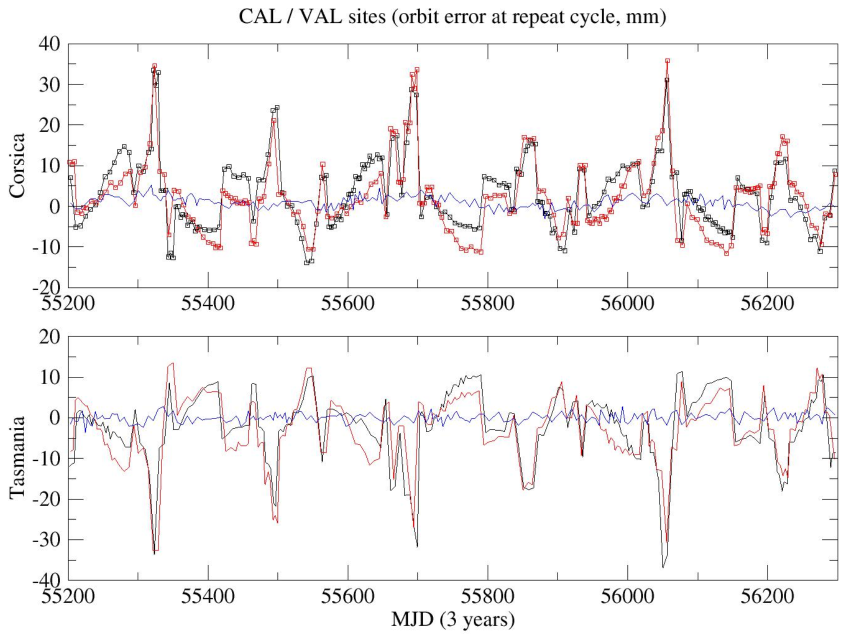
eigen-gl04s1 : variance of SH coefficients, d/o 45

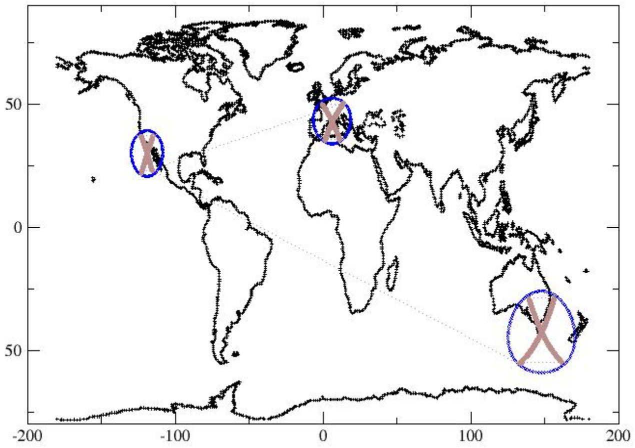
C/S11: 1.3/1.5 C10: 1.9 (annual, in mm)

SRP: δ S/m /(S/m) + δ Cr/Cr = 0.00694

Corsica (repeat arc, mm)			Tasmania (id.)	
	mean	RMS	mean	RMS
Desc. arc:	-0.70	11.7	0.46	11.9
Asc. arc:	-0.10	11.8	0.57	12.3

CAL/VAL sites-II





Model 4

grgs-RL02_MF : time var. coefficients, d/o 45

C/S11: 1.3/1.5 C10: 1.9 (annual, in mm)

SRP: δ S/m /(S/m) + δ Cr/Cr = 0.00694

	Corsica (repeat arc, mm)		Tasmania (id.)	
	mean	RMS	mean	RMS
Desc. arc:	3.10	8.8	-2.30	8.8
Asc. arc:	2.50	8.9	-2.84	8.1

Summary (theory)

The present analytical approach is used to study the orbit errors due to several and potential origines:

- Gravity field: constant and time variable parts
- Ocean tide model: 11 waves
- Solar Radiation Pressure

After validating the firts order analytical solution (spherical coordinates in the mean orbital plane) at 10^{-3}, errors in the ‘‘geodynamical’’ parameters are introduced in the solution equations in order to propagate the signals.

Errors in the ‘‘geodynamical’’ parameters are defined following different techniques:

- Variance of SH coefficients (with white noise) <= geopotential
- Part (10%) of the time variable geopotential
- Differences between existing models
- Global analysis of coefficient accuracy <= tide model
- Empirical results <= solar radiation pressure

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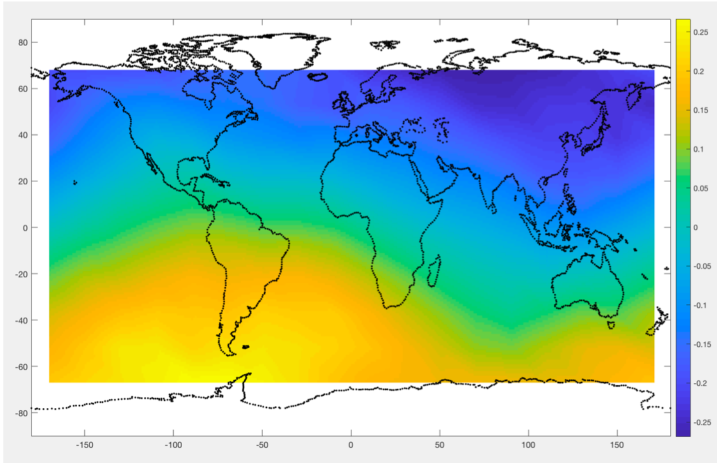
Conclusion (results)



The present analytical approach is applied in 4 different situations:

- constant errors (variance of the geopotential)
- a slowly changing SPR (draconitic period)
- a part of the annual and semi-annual changes (hydrology) of the TVG field
- < 1% error in tide waves (global ocean tide model)

Global radial orbit error is determined at the level of 5 to 8 mm RMS (depending on the ''model'')
At CAL/VAL sites, the orbit errors (repeat arcs) are of 11 mm RMS with constant part of 1 to 3 mm

The regional part of Sea Surface Height errors coming from the orbit errors is of 1 to 8 mm
Over a 3 years period, the radial drift rate is reaching 0.3 mm/yr (depending on the ''model'')

A global test will be performed in order to cumulate all the different signals presented here: TVG, ocean tides and SRP

The propagation of orbit errors will be applied to study the stability of tracking station coordinates.

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